ABSTRACT

Simulations Alfvén Wave Current Drive in Cylindrical Geometry [1]

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Current profile control in the RFP has been shown to reduce significantly MHD fluctuations and restore good flux surfaces. We are investigating the Alfvén wave behavior at the shear Alfvén resonance and the current driven by its dynamo effect as a means to control the current profile. We have confirmed several features obtained previously analytically[4]. The resonance width scales with the Lundquist number, S, as $S^{-1/3}$. In addition, the current reverses direction around the resonance surface and thus has the effect of flattening the current profile while driving little net current compared with the current in the opposing channels. We use the ratio of the oscillating current to the mean driven current, $S\left\langle (V\times B)_{||}/\eta\right\rangle_\omega / \left\langle |J_{||}|\right\rangle_\omega$, as our figure of merit. This quantity is 1 for ohmic current drive, and for a single Alfvén mode, is dependent on the wave amplitude. The harmonic nature of the perturbation and the relative phasing of B_θ and B_z of the wave at the resonance also affect this efficiency. This behavior will be compared with that of the tearing mode. Initial results on the effects of non-linear amplitudes and a spectrum of overlapping resonances will also be presented with the goal of optimizing the figure of merit.

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[4] S. Rauff and J. A. Tataronis, Phys. Rev. E, vol. 52, page 4311 (1995)

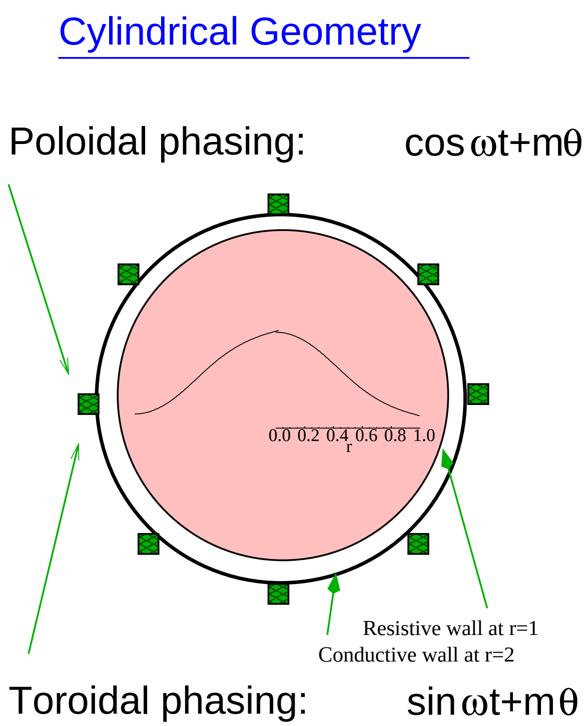
Introduction

We have approached the question of Alfvén wave current drive from a self-consistent angle. Where past approaches have looked at various linear analytic models, or numerically solved linear systems, we have taken a fully nonlinear resistive MHD code and used it to self-consistently study the propagation of Alfvén waves in a cylindrical geometry. At this stage we have investigated the behavior of a single $k_{||}$ at three different amplitudes and for three different Lundquist numbers(τ_r/τ_a), and have shown that there is a small amount of net current driven by a single wave.

OUTLINE

1. Experimental picture
2. Theoretical Background
   (a) 1-D Ideal MHD resonance
   (b) Analogy to tearing modes
   (c) Code Description
3. Simulation Results
   (a) Scan of the Lundquist Number
   (b) Scan of the amplitude
   (c) Effect on the current profile
4. Summary and Conclusions

The Wave is Launched by a Periodic Boundary Electric Field.



- A poloidal array of coils are excited with poloidal and toroidal currents with a 90° relative phase
- The frequency is chosen to localize deposition according to resonance condition

★ This is implemented in the `Debs` code as time dependent boundary conditions on the poloidal and toroidal electric fields.

The Theoretic Model is Resistive MHD

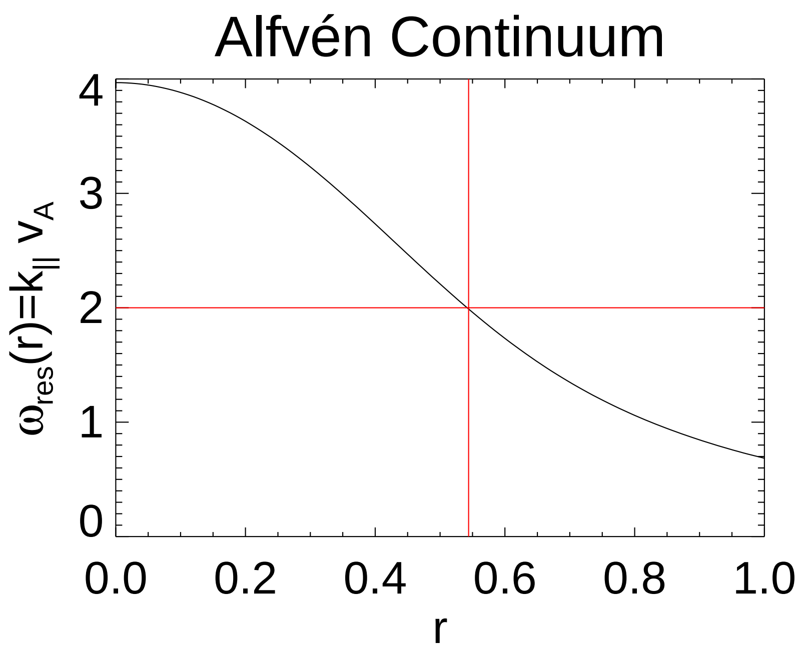
The ideal equations show the presence of a singularity.

$$\boxed{\begin{aligned} \frac{d}{dr}\left[A\frac{d}{dr}(r\xi)\right] - C(r\xi) &= 0 \\ A(r) &\equiv \left[\frac{\rho(V_a^2+V_s^2)}{r}\right]\frac{(\omega^2-\omega_A^2)(\omega^2-\omega_h^2)}{(\omega^2-\omega_f^2)(\omega^2-\omega_s^2)} \end{aligned}}$$

Where $A(r)$ is the Alfvén resonance term and ξ is the radial displacement.

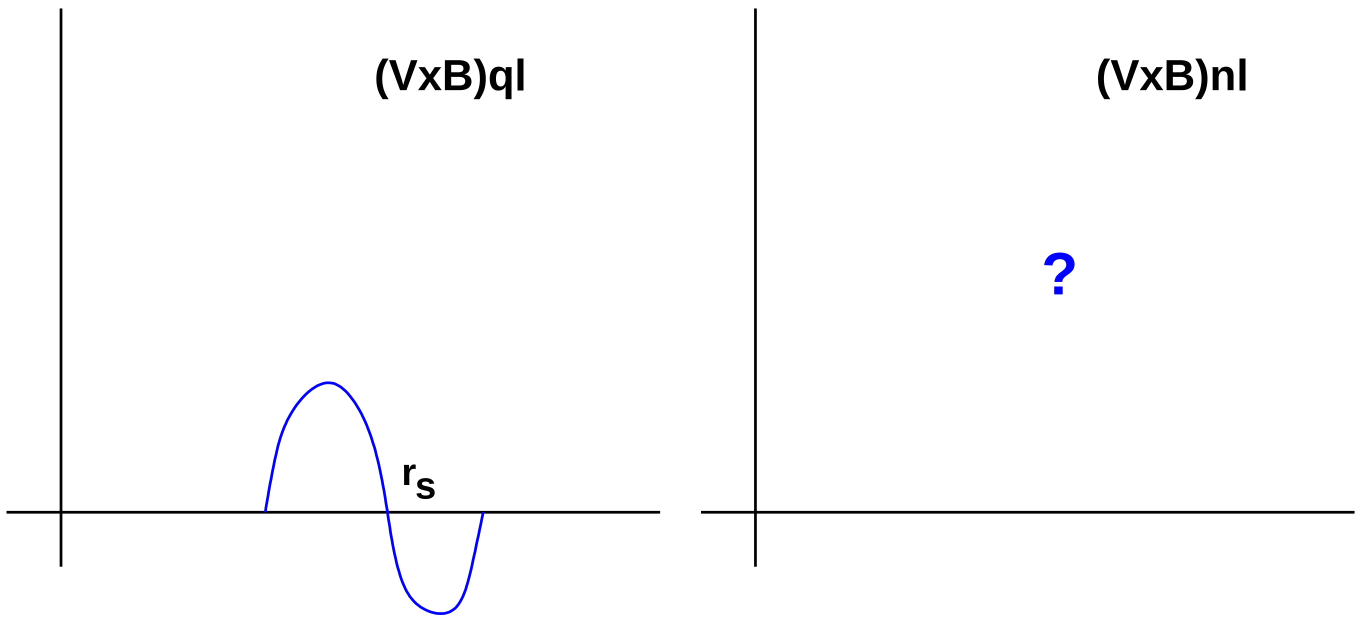
The singularity at $A = 0$ is resolved by the finite resistivity of the resistive MHD model.

The Alfvén Resonance varies spatially



- The Alfvén continuum is shown without the effects of density for an $m = 2, k = 0$ mode.
- The scale is in units of $\omega_A \equiv /(B/\sqrt{\rho})$.
- Waves can be damped, resonant, or freely propagating depending on frequency.

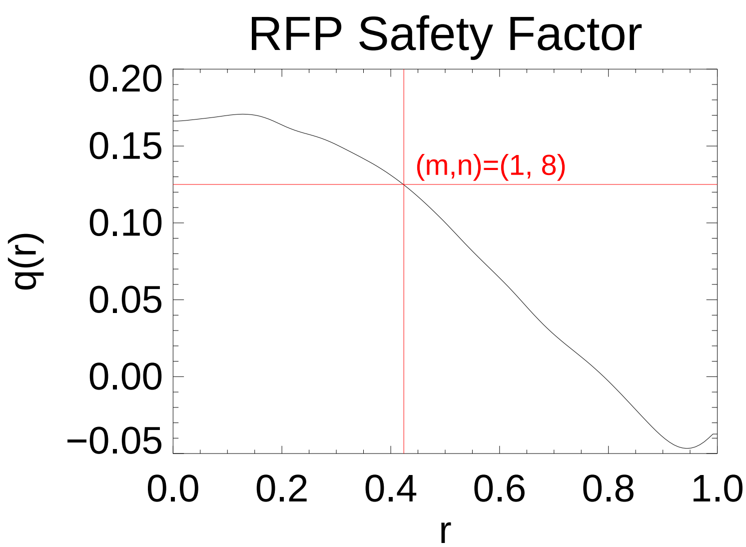
The Global Non-Linear behavior of Alfvén waves is unknown.

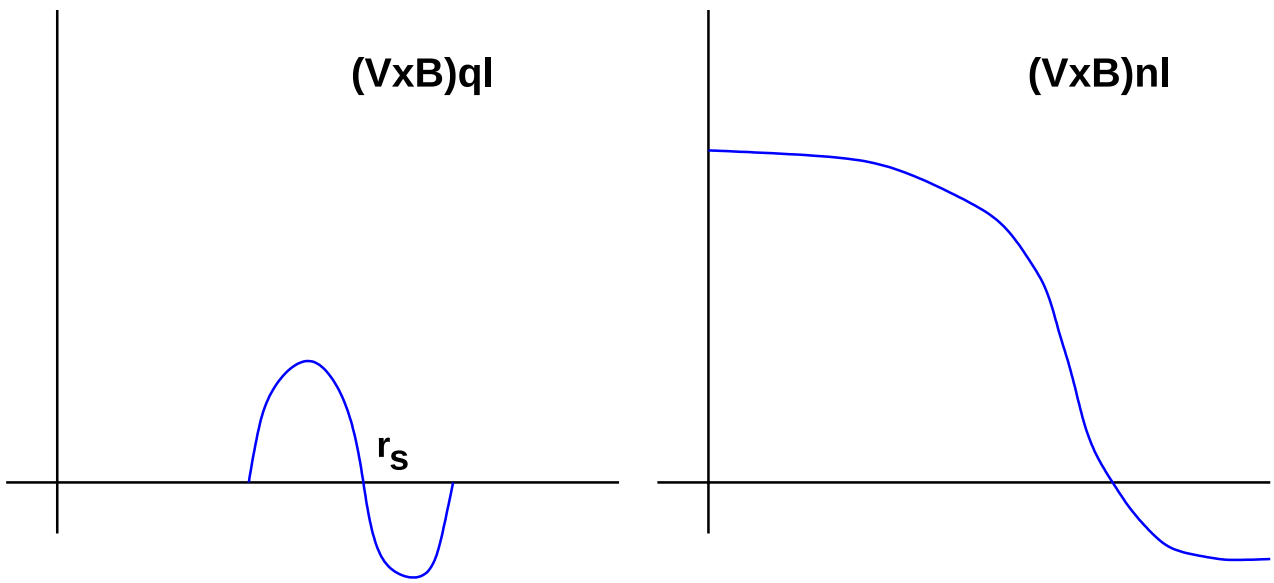


$k_{||} = \omega / v_A$ determines resonance.

- The resonance is localized by the $q(r)$ profile
- Can a nonlinear combination of of Alfvén waves create a global dynamo effect?

Linear Tearing Modes are local
Non-linear are global





$k_{||} = 0$ determines resonance.

By analogy to tearing modes, we theorize it may be possible to create a global effect with a spectrum of waves at large ($O \sim 1\%$) amplitude.

The Debs Code is used to simulate the Wave

The evolution of the magnetic field is governed by Faraday's law of induction, Eq. (1), and Ohm's Law, Eq. (2), and the velocity by the momentum equation, Eq. (3).

$$\partial \mathbf{A}/\partial t = -\mathbf{E} \tag{1}$$

$$\mathbf{E} + S\,\mathbf{V} \times \mathbf{B} = \eta\,\mathbf{J} \tag{2}$$

$$\rho\,\partial \mathbf{V}/\partial t = -S\,\mathbf{V} \cdot \nabla \mathbf{V} + S\,\mathbf{J} \times \mathbf{B} + \nu \nabla^2 \mathbf{V} \tag{3}$$

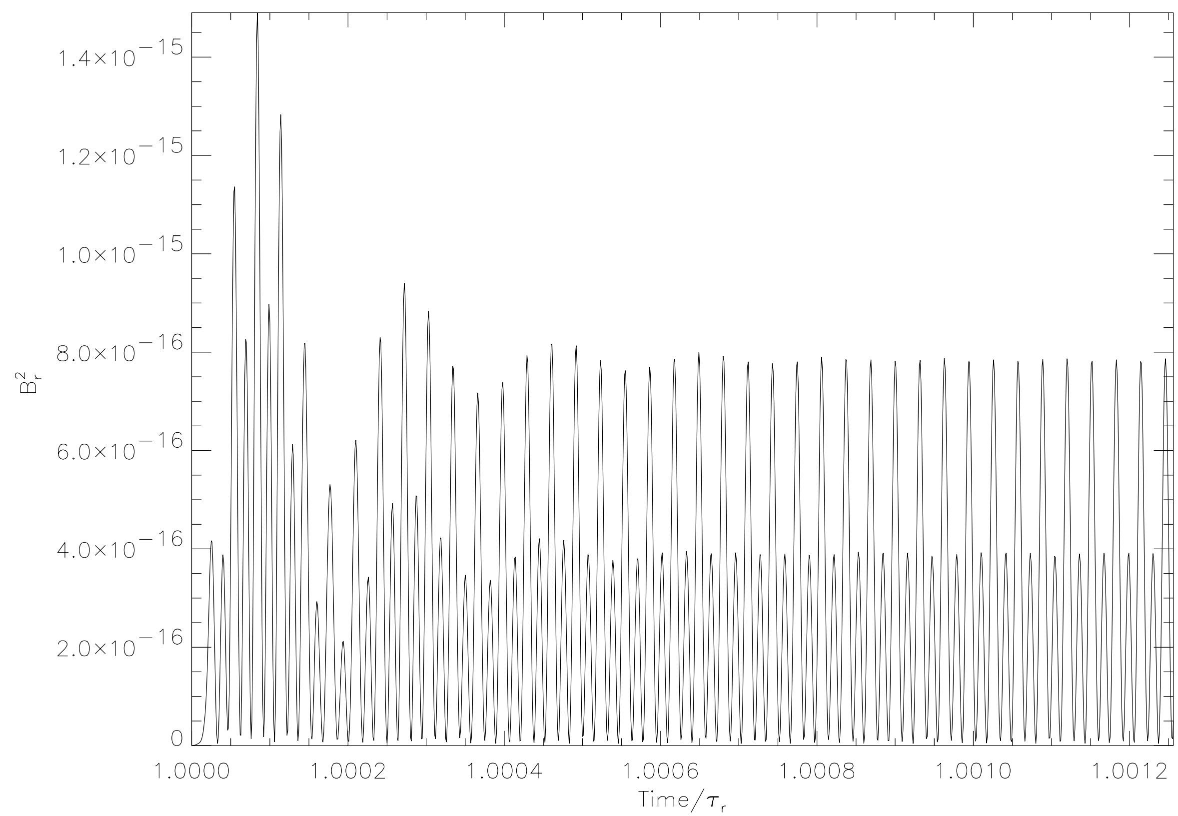
These quantities are non-dimensionalized as shown in the table below.

Variable	Symbol	[[units]]	MST values
Length(minor radius)	r	$[[a]]$	52 cm
Major radius	R	$[[R_o]]$	150 cm
Aspect ratio	A	$[[R_o/a]]$	2.9
Magnetic field	$\mathbf{B}$	$[[B_o]]$	625 gauss
Ion density	ρ	$[[\rho_o]]$	$0.4{\times}10^{14}\mathrm{m_i}/\mathrm{Z_{eff}}\ \mathrm{cm}^{-3}$
Alfvén time	τ_a	$[[a\sqrt{4\pi\rho}/B_o]]$	$\approx 1\ \mu$s
Time(resistive time)	τ_r	$[[4\pi a^2/\eta c^2]]$	≈ 1 s
Lundquist number	S	$[[\tau_r/\tau_a]]$	10^6
Vector potential	$\mathbf{A}$	$[[a\ B_o]]$	3.3×10^5 gauss-cm
Current density	$\mathbf{J}$	$[[(c/4\pi)\ B_o/a]]$	$0.12\,\mathrm{MA/m^2}$
Electric field	$\mathbf{E}$	$[[aB_o/c\tau_r]]$	0.53 V/m
Velocity	$\mathbf{V}$	$[[a/\tau_a]]$	0.5×10^6 m/s

The non-dimensionalized equations are solve in a cylindrical geometry with a perfectly conducting wall. The mesh is finite element in the radial ($\hat{r}$) direction and spectral with Fourier harmonics in the poloidal ($\hat{\theta}$) and axial ($\hat{z}$) directions. For the cases shown, $n_r = 251$, $n_\theta = 8$, $n_z = 4$, $dt = 0.01\tau_{\text{Alfvén}}$, $S_{\text{wall}} = 1000$.

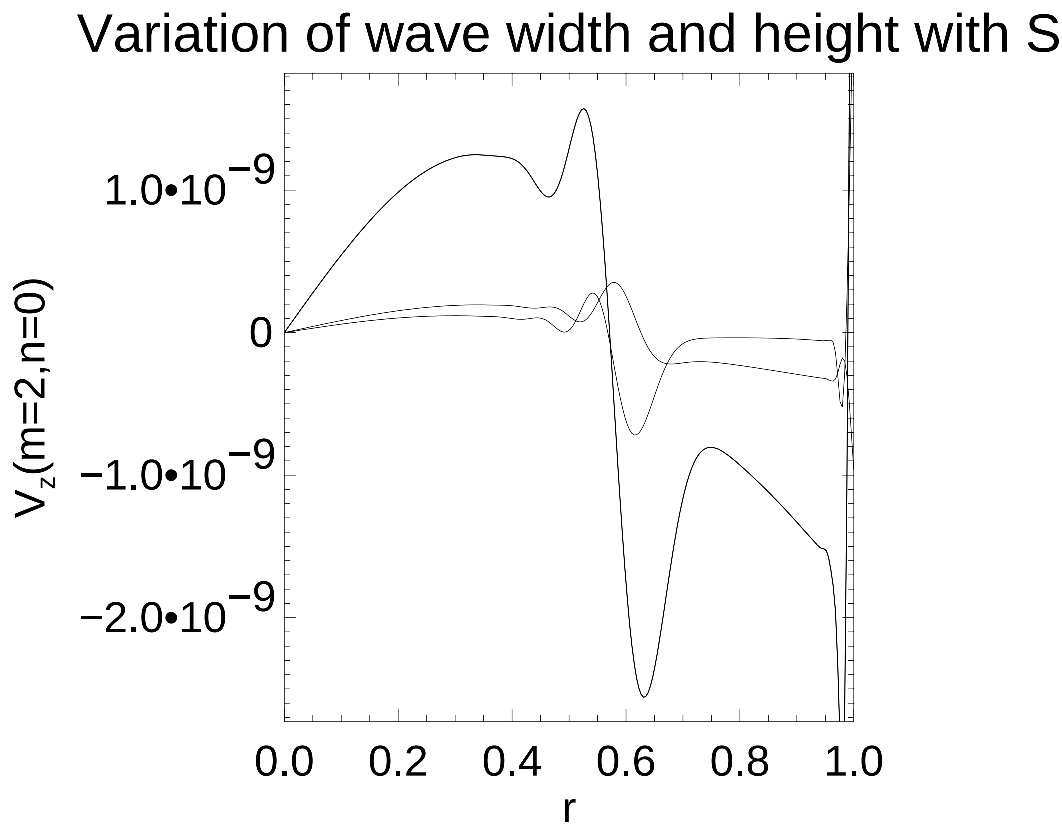
A relaxed paramagnetic Equilibrium is used.

The Code is run to a steady State



The radial magnetic energy versus time shows that the wave's eigenfunction has been established (note the beating harmonic).

The Linear Wave Height and Width decrease with S.

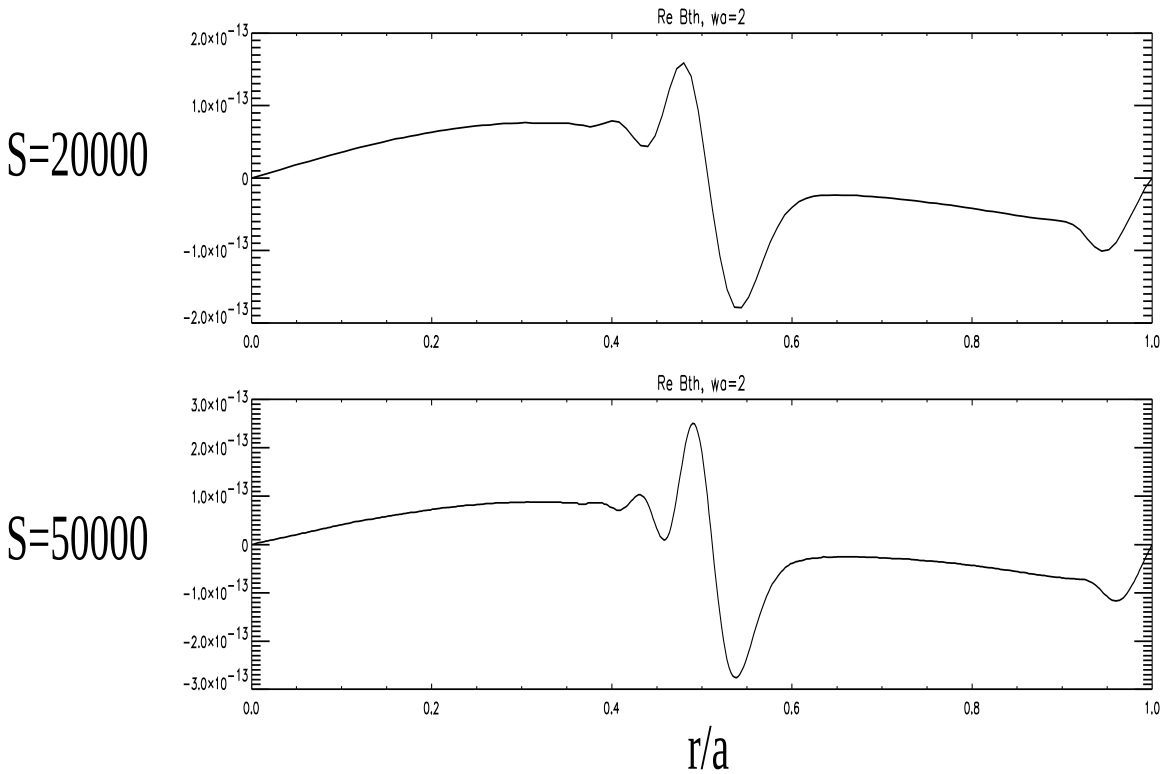


S	Width	[[Height]]
10^5	0.101	$[[4.13 \times 10^{-}9]]$
5×10^5	0.075	$[[9.94 \times 10^{-}10]]$
10^6	0.097	$[[5.73 \times 10^{-}10]]$

Scalings for height and width using just the first two cases are approximately $S^{-0.15}$ and $S^{-2.4}$ which do not agree with the theoretical scaling of $S^{-1/3}$.

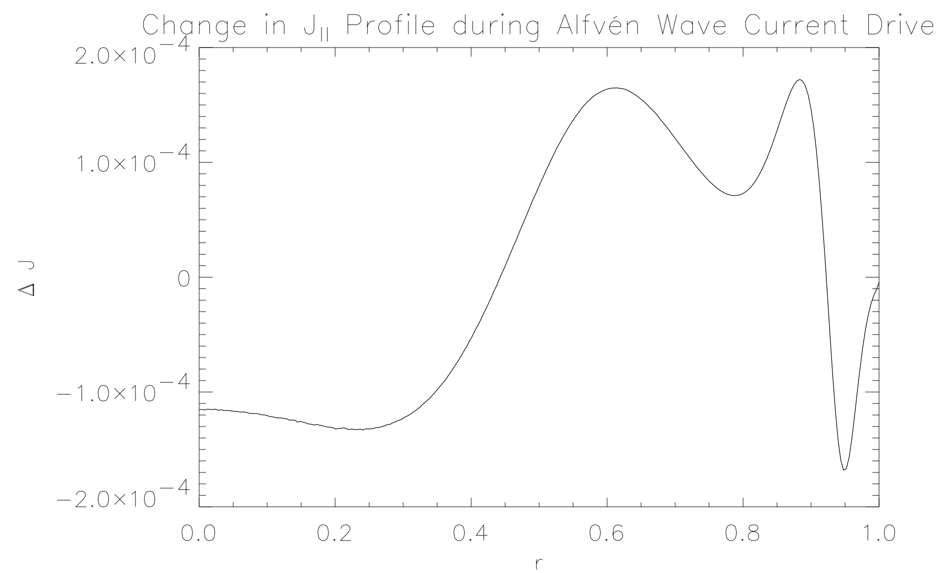
Note, however, that past cases at *lower* S have with resonances further from the wall have shown the correct scaling. It is suspected that the numerical viscosity used in these cases for stability was too large and has affected the scaling.

Case demonstrating correct Scaling of Resonance Width.



- Plots of B_θ for $m = 2$ for two values of S
- The resonance narrows with higher Lundquist number.
- Scaling $\approx S^{-0.31}$, near the theoretical, $S^{-1/3}$ (Tataronis)

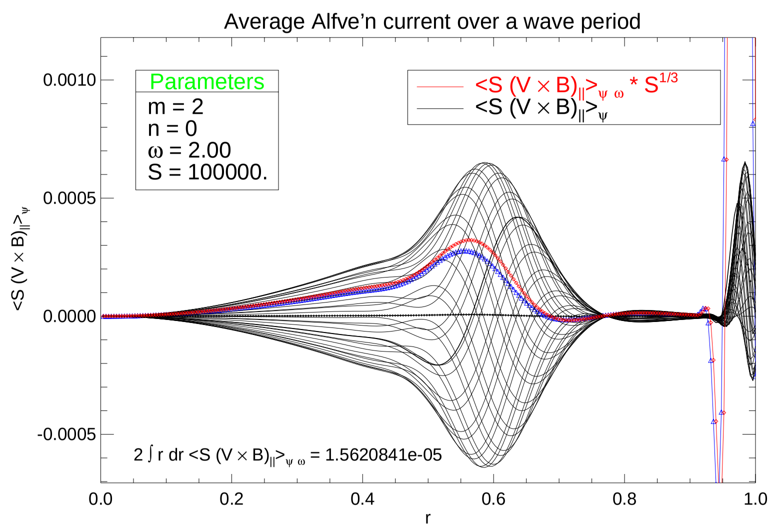
The Alfvén waves primarily redistributes the current around the layer



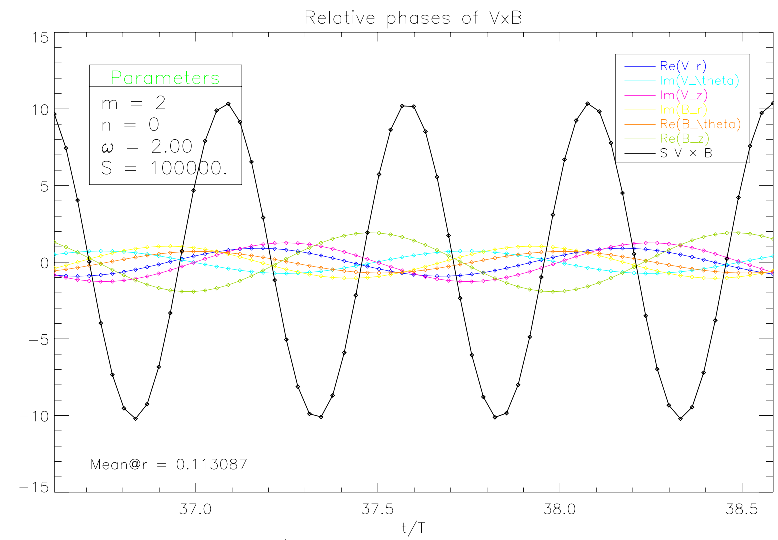
The current created by the dynamo effect is the result of both back EMF and diffusive processes. Its profile differs somewhat from the time averaged. Larger field amplitudes are needed to see the effect on the instaneous profile.

The case shown is for $E(r = r_{\text{wall}}) = 10^{-4}$ and $S = 10^5$. Note that the although there is some local magnetization around the resonance surface, the volume averaged current is about an order of magnitude less. This agrees with the analytic predictions of R. Torasso(see Poster BP1.128).

Temporal Phase conspires to produce small Dynamo



31 samples of <S (V × B)||>ψ compared with the average over a wave period.



Normalized to values at resonance for r=0.570

- The flux surface average dynamo is finite.
- But a time average reduces it magnitude by a large factor.
- A plot of temporal phases shows that at the resonance the velocity and magnetic field are nearly out of phase in time.

Near the wall, the temporal phases produce a large skin current, but as the wave penetrates, plasma effects push the velocity and magnetic field nearly out of phase in time.

Summary

- The scaling of the resonance dimensions with S was thrown off by too much viscosity.
- Much more current is deposited in the edge than at the resonance even with enhance resistivity at the wall.
- Most of the dynamo is AC and tend to redistribute rather than drive current.

Conclusions

The linear modelling phase is nearing completion. The simulations are easily extended to include several modes at non-linear amplitudes and the effects of the `MHD` turbulence. After the linear behavior is well understood, the differences the non-linear effects have will be discernible.

Future Directions

- Adjust the viscosity and resolution to minimize broadening from numerical viscosity.
- Quantify the scaling of $\Delta J_{||}$ with S and E_{wall}
- Compare current deposition with theoretic models and account for differences
- Introduce several waves at once at large amplitudes to check non-linear effects.

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