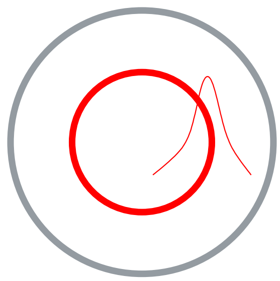
ABSTRACT

Calculations of Alfvén Wave Current Drive in Cylindrical Geometry



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Alfvén Waves (AW) may be driven from the edge of an RFP to produce a local current that may be used to control the current profile. This localization of the current drive (CD) effect is determined by the shear AW resonance. It has been suggested that AWCD may have an efficiency comparable to Ohmic, $\eta_{ohm} \equiv 1/\eta J$, through the interaction of its dynamo generated electric field, $E_f \equiv -\widetilde{V} \times \widetilde{B}$. We present results of an analysis of the efficiency of AWCD as defined by $\eta_{AW} \equiv < J > / < J \cdot E >$, where $<>$ denotes a time and flux surface average, and the electric field is given by the resistive form of Ohm's Law, $E = -V \times B + \eta J$. We will comment on the relative strengths of the dissipation due to the Ohmic and dynamo generate electric fields. Both global and local behavior will be discussed.

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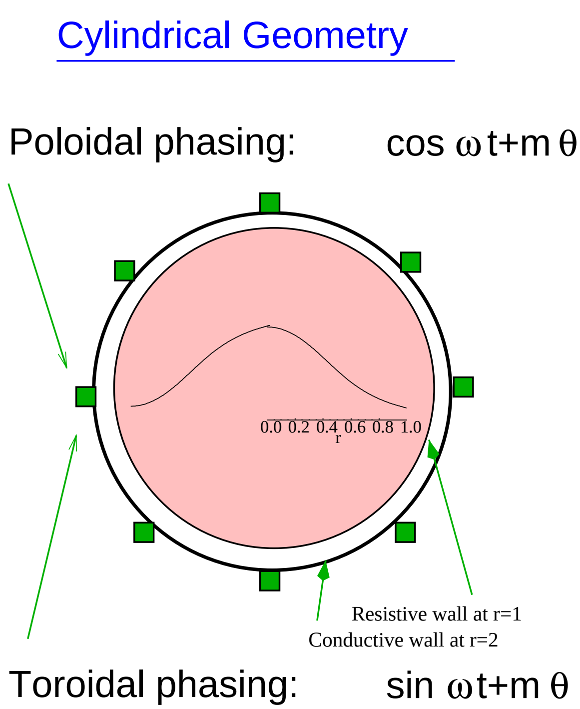
Introduction

We have approached the question of Alfvén wave current drive from a self-consistent angle. Where past approaches have looked at various linear analytic models, or numerically solved linear systems, we have taken a fully nonlinear resistive MHD code and used it to self-consistently study the propagation of Alfvén waves in a cylindrical geometry. At this stage we have investigated the behavior of a single $k_{||}$ at three different amplitudes and for three different Lundquist numbers(τ_r/τ_a), and have shown that there is a small amount of net current driven by a single wave.

OUTLINE

1. Theoretical Background
   (a) 1-D Ideal MHD resonance
   (b) Code Description
   (c) Boundary Conditions
2. Current Drive Efficiency
3. Simulation Results
   (a) Coupling Optimization
   (b) Scan of the Lundquist Number
   (c) Effects of viscosity
   (d) Scan of the amplitude
   (e) Non-linear suppression of efficiency
4. Summary and Conclusions

The Wave is Launched by a Periodic Boundary Electric Field.



- A poloidal array of coils are excited with poloidal and toroidal currents with a 90^o relative phase
- The frequency is chosen to localize deposition according to resonance condition

★ This is implemented in the `Debs` code as time dependent boundary conditions on the poloidal and toroidal electric fields.

The Theoretic Model is Resistive MHD

The ideal equations show the presence of a singularity.

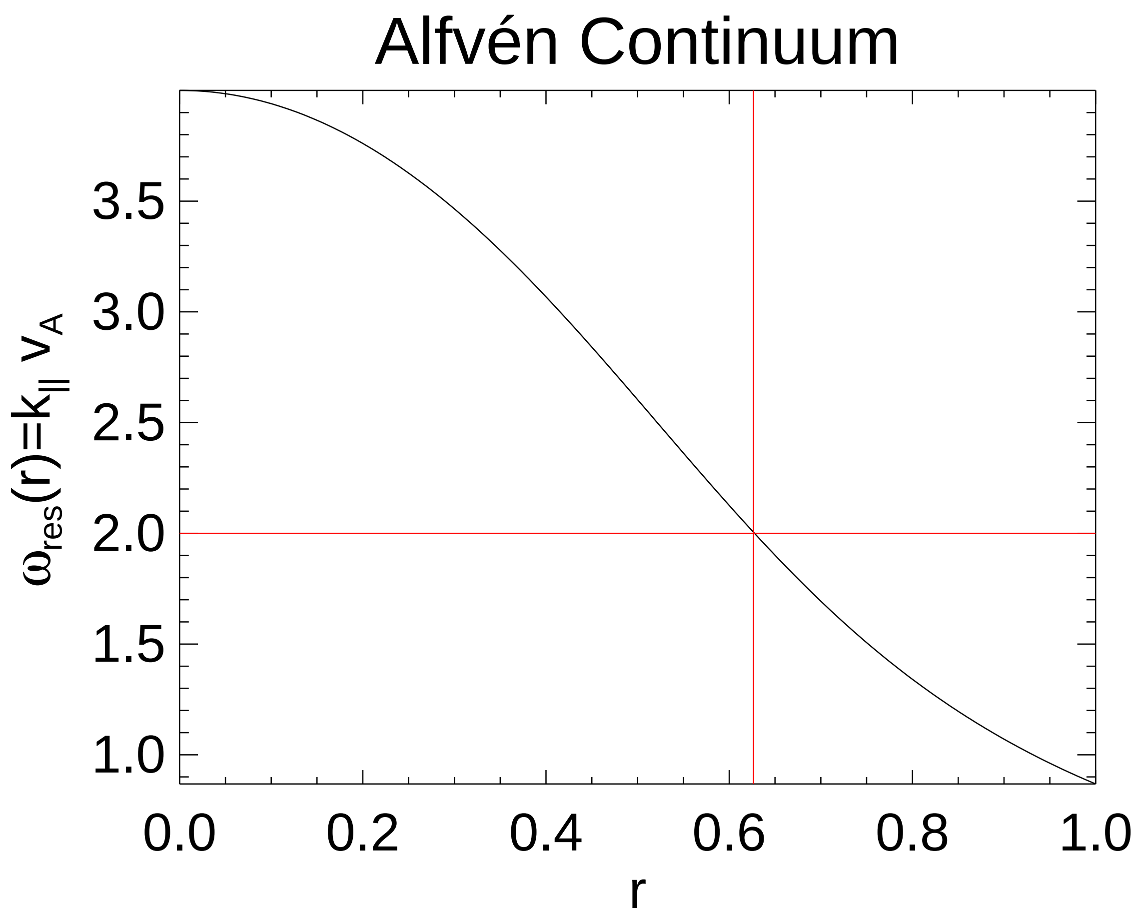
$$\frac{d}{dr}\left[A\frac{d}{dr}(r\xi)\right] - C(r\xi) = 0$$

$$A(r) \equiv \left[\frac{\rho(V_a^2+V_s^2)}{r}\right]\frac{(\omega^2-\omega_A^2)(\omega^2-\omega_h^2)}{(\omega^2-\omega_f^2)(\omega^2-\omega_s^2)}$$

Where $A(r)$ is the Alfvén resonance term and ξ is the radial displacement.

The singularity at $A = 0$ is resolved by the finite resistivity of the resistive MHD model.

The Alfvén Resonance varies spatially



- The Alfvén continuum is shown without the effects of density for an $m = 2$, $k = 0$ mode.
- The scale is in units of $\omega_A \equiv /(B/\sqrt{\rho})$.
- Waves can be damped, resonant, or freely propagating depending on frequency.

The Resistive MHD Equations

✰ The non-linear resistive MHD equations are solved in three dimensions by the initial value code, `Debs`.

$$\rho\,(\partial \mathbf{v}/\partial t + \mathbf{v}\cdot\boldsymbol{\nabla}\mathbf{v}) = \mathbf{J}\times\mathbf{B} - \frac{\beta_0}{2}\boldsymbol{\nabla}p + P_{\mathrm{m}}\boldsymbol{\nabla}^2\mathbf{v}$$

$$\partial \mathbf{A}/\partial t = -\mathbf{E}$$

$$\mathbf{E} + S\,\mathbf{V}\times\mathbf{B} = \eta\,\mathbf{J}$$

❋ In the `Debs` code, these quantities are non-dimensionalized as shown in the table below.

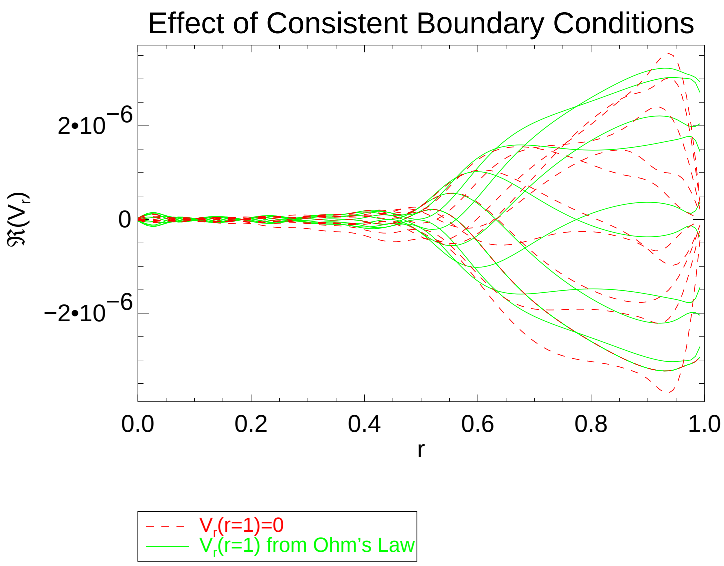
✻ In the simulations presented here, the magnetic Prandtl number, $P_{\mathrm{m}} \equiv \nu/\eta$, has the range of $40 \rightarrow 160$. The Lundquist number, $S \equiv \tau_{\mathrm{r}}/\tau_{\mathrm{Alfvén}}$, is scanned from 1×10^4 to 64×10^4. The perturbation is always $m = 2, n = 0$, and the amplitude is scanned from 2.5% of the loop voltage to 2500%.

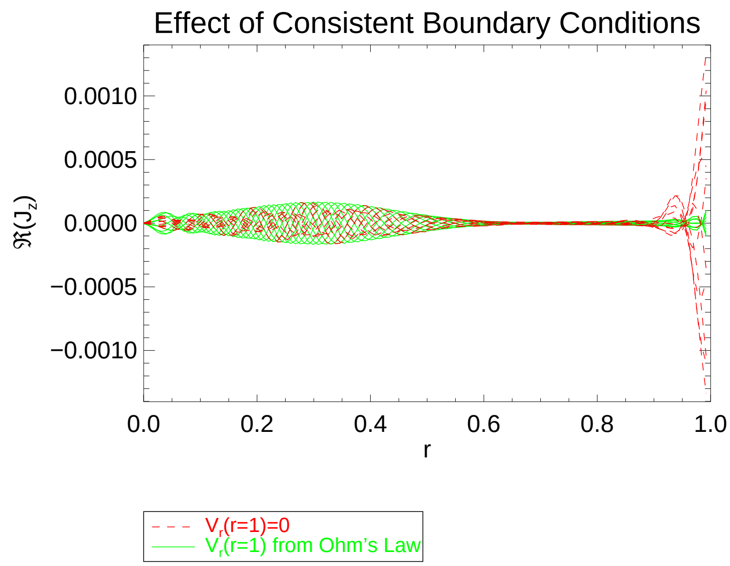
Description of the `Debs` Code

- An initial value three dimensional code that solves the non-linear resistive MHD equations in cylindrical geometry (Schnack et al., 1987).
- The equations are normalized as below:
- The code can be used to study small amplitude linear waves, large amplitude non-linear effects, and the interaction of waves with tearing mode turbulence all in the same framework.

Variable	Symbol	[[units]]	MST values
Major radius	R	$[[R_o]]$	150 cm
Aspect ratio	A	$[[R_o/a]]$	2.9
Lundquist number	S	$[[\tau_r/\tau_a]]$	10^6
Magnetic field	$\mathbf{B}$	$[[B_o]]$	625 gauss
Length(minor radius)	r	$[[a]]$	52 cm
Ion density	ρ	$[[\rho_o]]$	$0.4\times10^{14}\mathrm{m_i}/\mathrm{Z_{eff}}\,\mathrm{cm}^{-3}$
Alfvén time	τ_a	$[[a\sqrt{4\pi\rho}/B_o]]$	$\approx 1\,\mu$s
Time(resistive time)	τ_r	$[[4\pi a^2/\eta c^2]]$	≈ 1 s
Vector potential	$\mathbf{A}$	$[[a\,B_o]]$	3.3×10^5 gauss-cm
Current density	$\mathbf{J}$	$[[(c/4\pi)\,B_o/a]]$	$0.12\,\mathrm{MA/m^2}$
Electric field	$\mathbf{E}$	$[[aB_o/c\tau_r]]$	0.53 V/m
Velocity	$\mathbf{V}$	$[[a/\tau_a]]$	0.5×10^6 m/s

Treatment of the Boundary



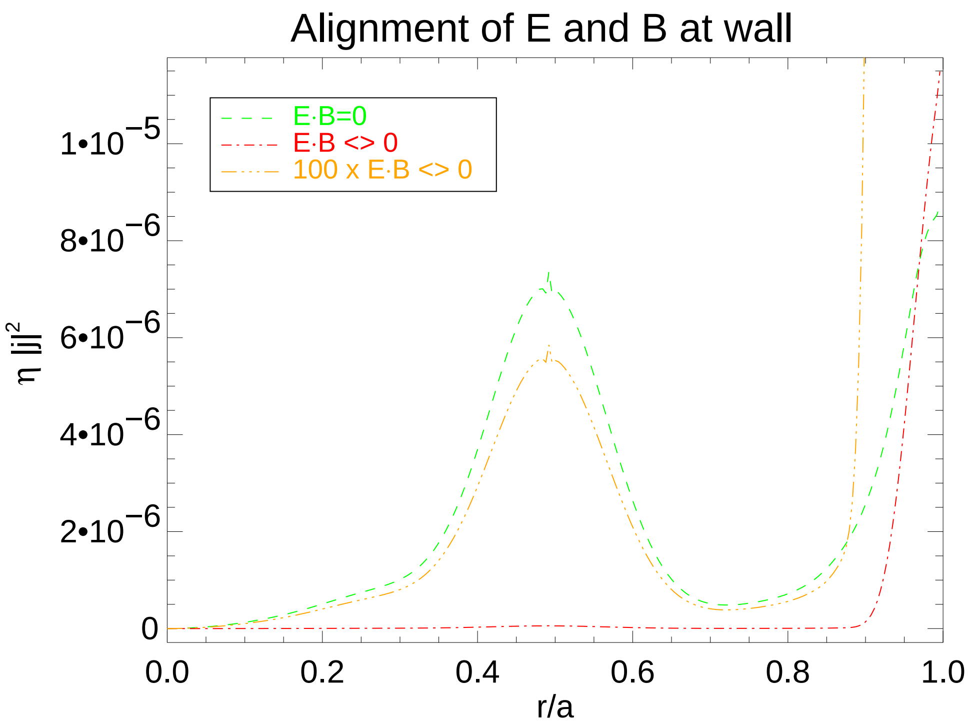


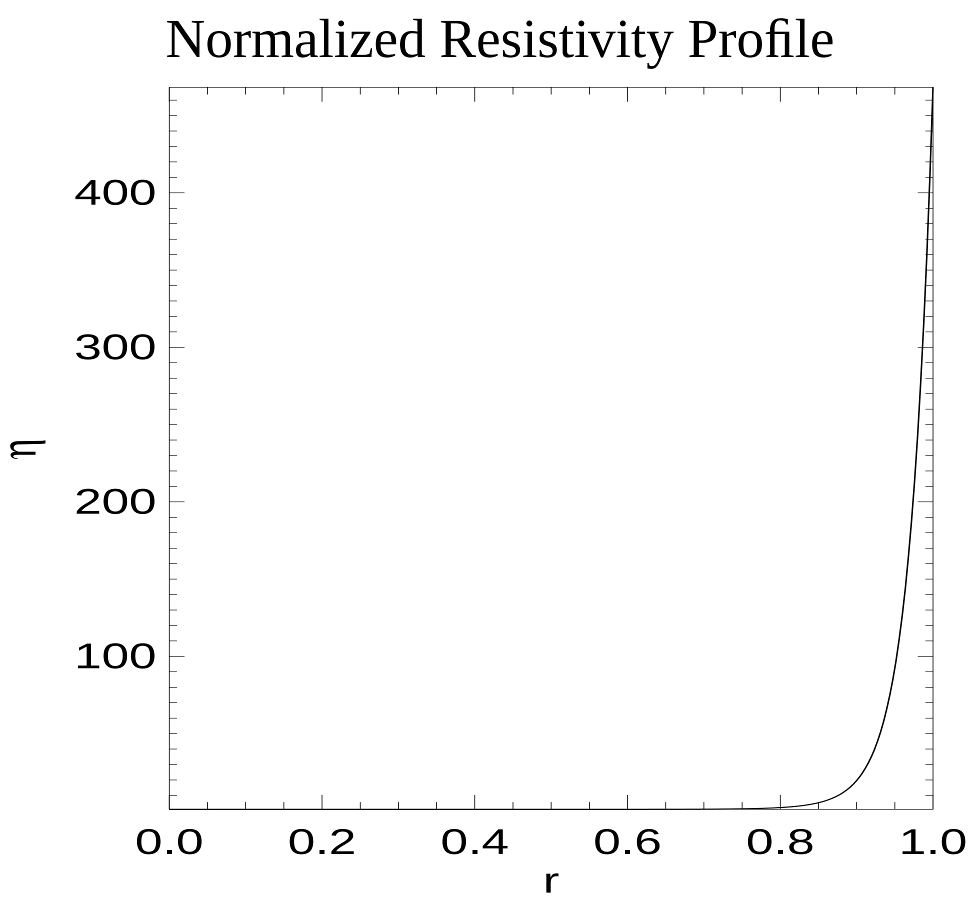
- An edge enhanced resistive boundary is both realistic and necessary for coupling of the edge perturbations to the bulk plasma.
- $pi/4$ phased electric fields are chosen for helicity optimization.
- The boundary condition on radial velocity, v_r, is chosen consistent with ideal Ohm's Law. The $v_r = 0$ condition causes large edge gradient in field quantities as the diffusive terms compensate.

$$v_r(a) = \frac{(\mathbf{E} \times \mathbf{B}) \cdot \hat{\mathbf{r}}}{B^2}(a)$$

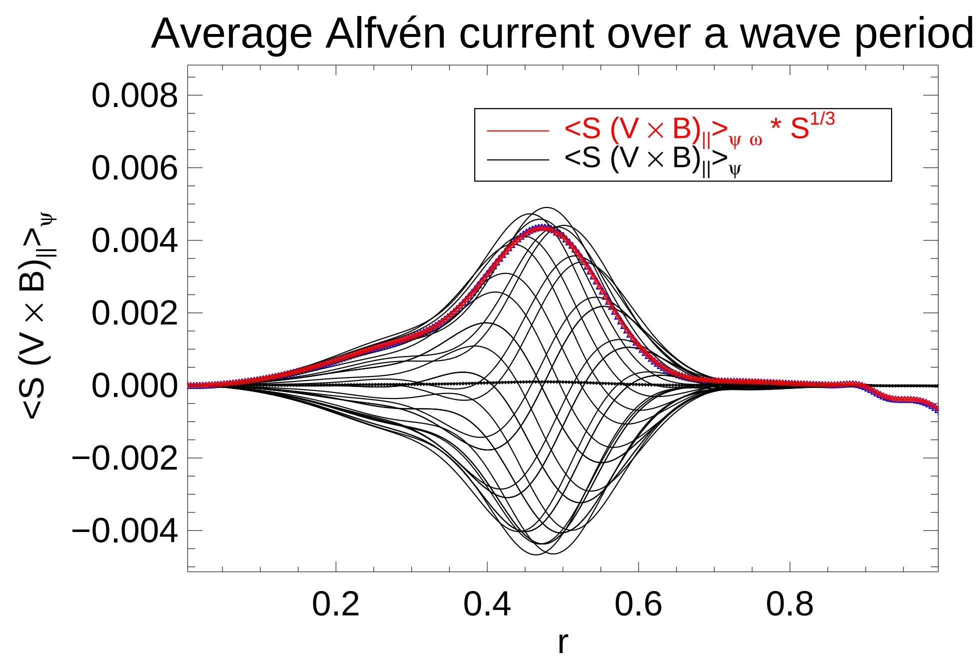
- The electric perturbation is perpendicular to edge magnetic field to avoid driving edge currents.

$$\mathbf{E} \cdot \mathbf{B}|_{r/a=1} = 0$$


Alignment of E and B at wall
E·B=0
E·B <> 0
100 x E·B <> 0
$\eta\,|j|^2$
r/a


Normalized Resistivity Profile
η
r

Temporal and Spatial Response of the Dynamo



Instances of $\langle S\,(V\times B)_{||}\rangle_\psi$ compared with its wave average.

- The perturbation driven at the wall excites a resonance response at the radius where $F(r) = 0$.
- When the Lundquist number is sufficiently large, ($S = 8\times10^4$ in the above plot), the width of the resonant response is in the asymptotic limit and the resonance is "far" from the walls.
- Even after averaging over a flux surface, the dynamo still is dominated by an odd character the oscillates harmonically in time at twice the wave frequency.
- Averaging over time pulls out the even constant response which is seen to scale as $S^{-1/3}$ compared to the odd harmonic part. It is this part that will drive net current.

Expressions for Current Drive Efficiency

$$\eta_{\text{eff}} \equiv \frac{J}{P_d}$$

$$\eta_{\text{Ohm}} = \frac{J}{\eta J^2}$$

$$\eta_{\text{RF}} = \frac{\left\langle J_{||} \right\rangle}{-\nu \mathbf{v} \cdot \nabla^2 \mathbf{v} + \eta \left(\left\langle J_{||}^2 \right\rangle + \left\langle \widetilde{J}^2 \right\rangle \right)}$$

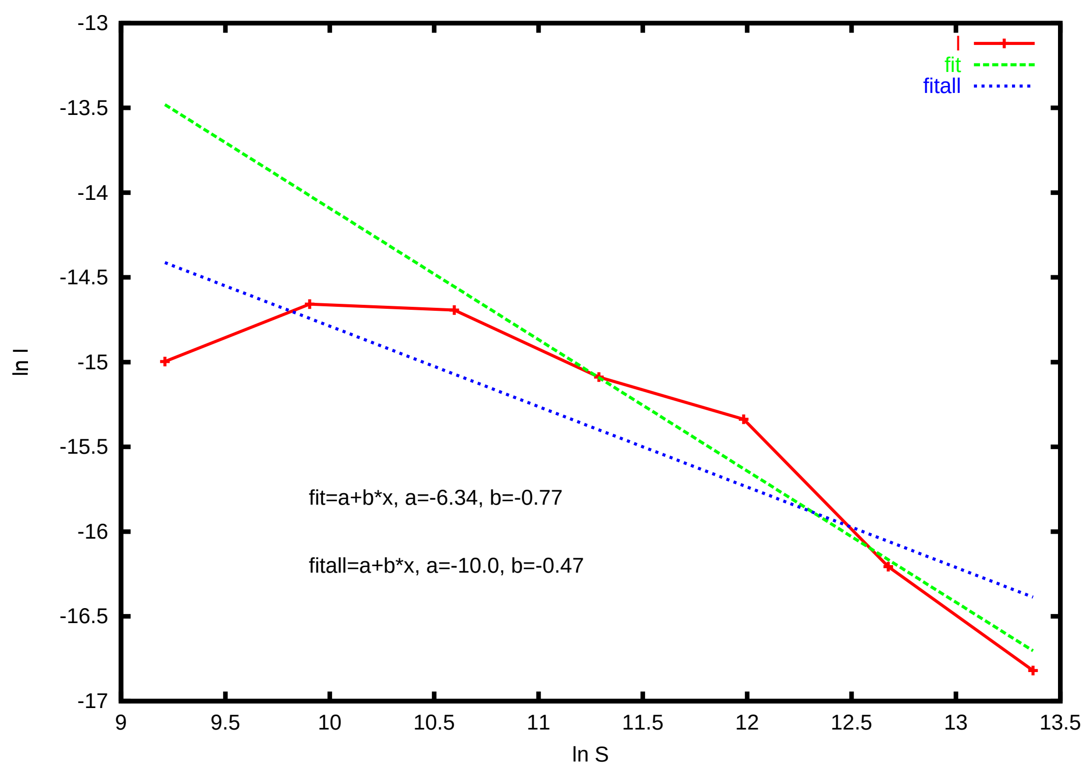
- Current Drive efficiency is given as the ratio of driven current, J, to deposited power, P_d. The units are $[[L_0/(\eta_0 B_0)]]$.
- Ohmicly driven current has power dissipated only resistively.
- For Alfvén waves, power is deposited by fluctuations both viscously and resistively in addition to the resistive dissipation by the axisymmetric driven current, $J_{||} \equiv S \widetilde{v} \times \widetilde{b}$.
- Note that resistive power deposition is given properly by ηj^2 in a moving media. The terms from $\mathbf{V} \times \mathbf{B} \cdot \mathbf{J}$ do not dissipate energy but merely transfer it between the velocity and magnetic fields.

Lundquist number scaling of Current

- Theory in a slab (Mett and Taylor, 1992) predicts:

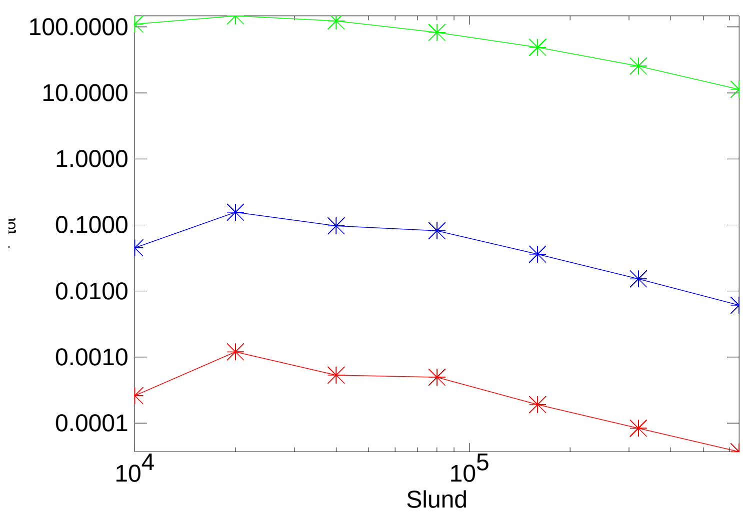
$$I \approx S^{-2/3}$$

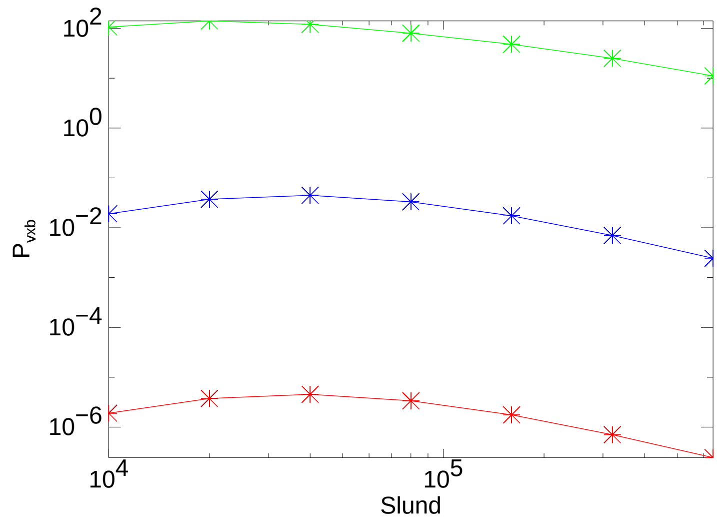
- In the simulation, the current scales as $S^{-0.77}$. when fitting the asymptotic region ($\ln S > 10.5$).



The Dynamo Dissipated Power $\propto E_\perp^4(a)$.

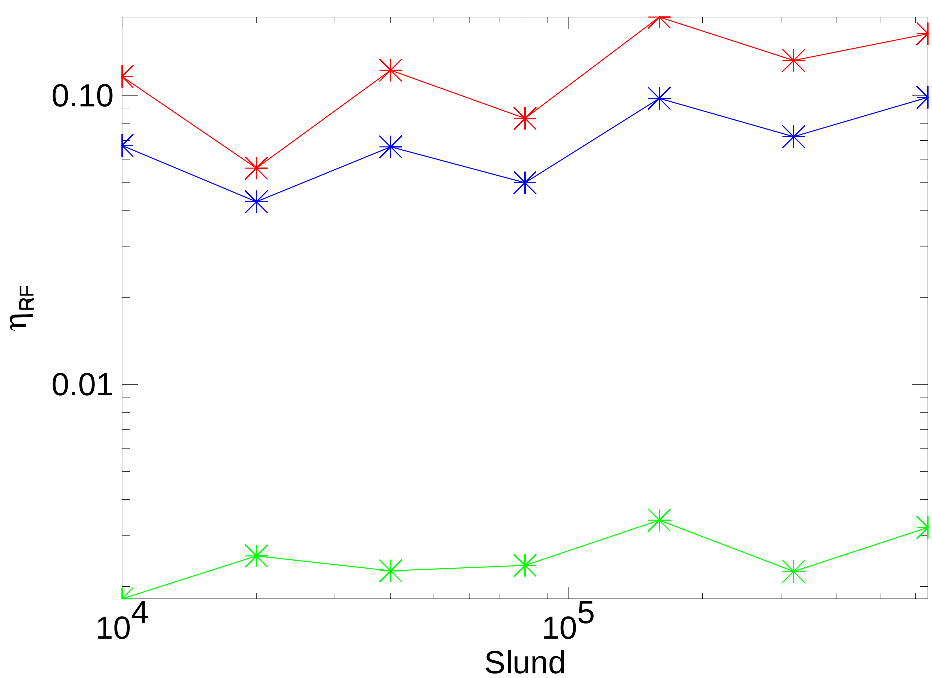
- Legend: $E_\perp(a) = 1$, $E_\perp(a) = 10$, $E_\perp(a) = 100$
- At small amplitudes, dynamo dissipated power is negligible, at large amplitudes, it dominates



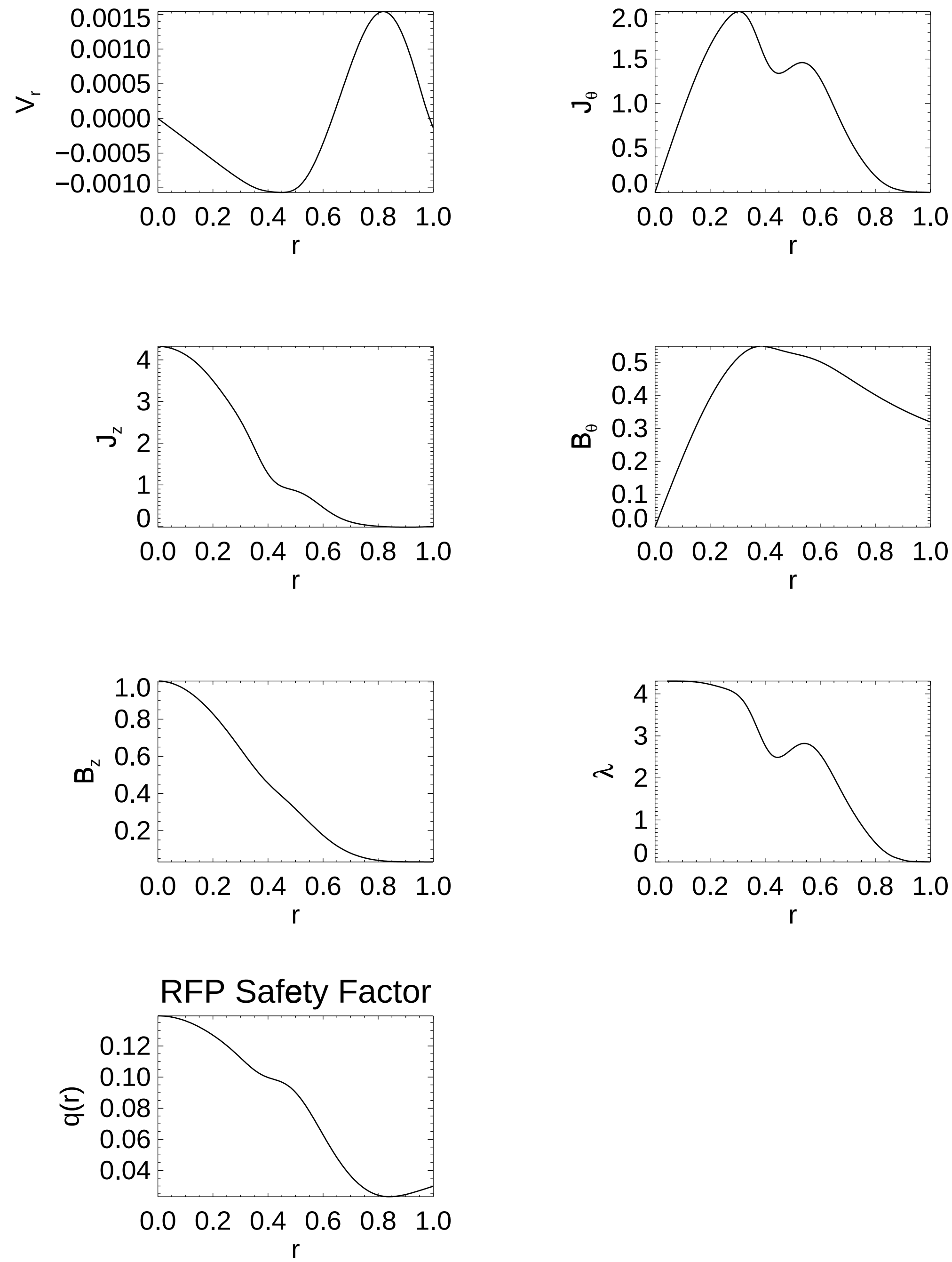


Non-linear effects supress the efficiency

- Legend: $E_{\perp}(a) = 1$, $E_{\perp}(a) = 10$, $E_{\perp}(a) = 100$
- Though the current decreases with *Slund*, the efficiency is somewhat constant because the power deposition also decreases. This is at odds with Mett and Taylor, and could indicate equilibriation with input power has not been reached.
- At large driving amplitudes, the efficiency drops off because of power dissipated by the dynamo term.



Non-linear feedback on equilibrium



- At large driven amplitudes the wave affects the equilibrium.
- The effects is primarily to modify the shape of the profile, compared to the actual amount of current driven.

Summary

- The current scales as $S^{-0.77}$.
- Corrected numerical and physical boundary conditions have improved coupling to resonance to nearly 100%.
- Most of the dynamo is AC.

Conclusions

- At low (linear) amplitudes, P_d for Alfvén waves is dominated by viscous damping and resistive resistive dissipation from the odd (time oscillations) component of the flux averaged dynamo. As the driving amplitude of the antenna increases, dissipation from the AC odd driven current dominates.
- Most of the axisymmetric current driven is odd and periodic at the second harmonic of the driven frequency. It is larger than the even non-periodic current in the ratio of $S^{1/3}$ in agreement with theory for a slab (Mett and Taylor, 1992).
- Consequently, at larger amplitudes, the observed effect is to flatten the current profile around the resonance surface but to not change the net current.
- Balance between input power and resistive relaxation of the system may not have been reached. This could be responsible for the dependence of total power deposition on dissipation. However, our P_d includes the significant term, $\eta\left(\left\langle J_{||}^2\right\rangle\right)$.

Future Possible Areas of Research

- Stochastic propagation of wave in a fully realized RFP.
- Drive a realistic antenna spectrum of waves.
- AWCD suppression of island growth.
- Investigation of toroidal effects.

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