International Journal of Bifurcation and Chaos, Vol. 27, No. 14 (2017) 1750221 (14 pages)

DOI: 10.1142/S0218127417502212

A Tribute to J. C. Sprott

Fahimeh Nazarimehr and Sajad Jafari*
Biomedical Engineering Department,
Amirkabir University of Technology,
Tehran 15875-4413, Iran
**sajadjafari83@gmail.com*

Guanrong Chen
Department of Electronic Engineering,
City University of Hong Kong, Hong Kong, P. R. China

Tomasz Kapitaniak
Division of Dynamics, Lodz University of Technology,
Stefanowskiego 1/15, 90-924 Lodz, Poland

Nikolay V. Kuznetsov and Gennady A. Leonov
Faculty of Mathematics and Mechanics,
Saint-Petersburg State University, Russia

Chunbiao Li
Collaborative Innovation Center on Forecast
and Evaluation of Meteorological Disasters,
Nanjing University of Information Science and Technology,
Nanjing 210044, P. R. China

Zhouchao Wei
School of Mathematics and Physics,
China University of Geosciences,
Wuhan 430074, P. R. China

Received October 5, 2017

In honor of his 75th birthday, we review the prominent works of Professor Julien Clinton Sprott in chaos and nonlinear dynamics. We categorize his works into three important groups. The first and most important group is identifying new dynamical systems with special properties. He has proposed different chaotic maps, flows, complex variable systems, nonautonomous systems, partial differential equations, fractional-order systems, delay differential systems, spatiotemporal systems, artificial neural networks, and chaotic electrical circuits. He has also studied dynamical properties of complex systems such as bifurcations and basins of attraction. He has done work on generating fractal art. He has examined models of real-world systems that exhibit chaos. The second group of his works comprise control and synchronization of chaos. Finally, the third group is extracting dynamical properties of systems using time-series analysis. This paper highlights the impact of Sprott's work on the promotion of nonlinear dynamics.

Keywords: Chaos; dynamical system; dynamical property; control; synchronization.

*Author for correspondence

1. Introduction

JULIEN CLINTON SPROTT earned his bachelor's degree from MIT in 1964 and his PhD in physics from the University of Wisconsin in 1969. He is an Emeritus Professor of Physics at the University of Wisconsin-Madison. He is the author or coauthor of about 500 papers and a dozen books on topics including plasma physics, chaos, dynamical systems, and fractals. After a 25-year career in experimental plasma physics, he became interested in chaos in 1988 when George Rowlands from the University of Warwick gave a colloquium on the topic for the Physics Department at his university.

Using the primitive personal computers of the day, he developed programs to automate the search for new chaotic systems in an attempt to quantify how common are chaotic solutions in various types of dynamical systems [Sprott, 1993b, 1993c]. That work led to a search for the algebraically simplest examples of chaotic systems of various types and to the identification of many new chaotic systems with special properties, including many that are algebraically simpler than any that were previously known [Sprott, 1994b, 1997a, 1997b; Linz & Sprott, 1999; Sprott, 2010]. His search and analysis programs have been continually refined and widely used for many important subsequent studies.

Professor Sprott has been especially helpful to students and junior scientists in developing their ideas and getting them published. He enjoys his collaborations with such people and humbly admits that most of the ideas for his many recent publications came from his coauthors. The rest of this paper will discuss this research and its importance.

2. Review of Publications

Nonlinear dynamics is an important area of research that has received great attention in recent years, especially since the advent of inexpensive and powerful personal computers. Chaotic behavior is one of the most important phenomena in nonlinear systems. Structural properties of systems that are capable of generating chaotic attractors are a challenging subject in this area. For example, hidden and self-excited attractors have been defined and studied by Leonov and Kuznetsov [Kuznetsov *et al.*, 2011; Kuznetsov *et al.*, 2010; Leonov & Kuznetsov, 2011a, 2011b, 2014; Leonov *et al.*, 2014; Leonov *et al.*, 2011, 2012; Leonov & Kuznetsov, 2013a, 2013b; Bragin *et al.*, 2011]. Self-excited attractors are ones whose basin of attraction is associated with an unstable equilibrium, while hidden attractors have basins of attraction that are not associated with any equilibria. Another example is time-reversible chaotic flows in which the governing equations of the system do not change under the transformation $t \rightarrow -t$ except for changing the sign of one or more of the variables [Sprott, 2015c]. Continuous-time systems with dimensions greater than three can exhibit hyperchaos in which two or more of the Lyapunov exponents are positive.

Sprott has classified strange attractors into three principal classes, hyperbolic, Lorenz-type, and quasi-attractors and discussed the possibility of finding a rigorous mathematical model describing the chaotic attractors [Elhadj & Sprott, 2012d]. In 1993 he investigated a subset of maps and flows to determine what fraction of them had chaotic solutions [Sprott, 1993b] and estimated the most probable dimension of their strange attractors [Sprott, 1994a]. He also showed how chaos evolves from being relatively rare in low-dimensional systems to generic in high-dimensional artificial neural networks [Dechert *et al.*, 1999] and its persistence (smooth variation of Lyapunov exponents with parameter changes) [Albers *et al.*, 2006; Elhadj & Sprott, 2014].

He described his methods for efficiently searching large classes of discrete and continuous-time systems to find those often tiny regions of parameter space that admit chaos [Sprott, 1993a]. Partial differential equations (PDEs) and systems of many ordinary differential equations as well as discrete-time neural networks can also generate high-dimensional chaos [Sprott, 2008b]. He identified those parameters for various common chaotic systems that give the most complex chaotic attractors (the largest attractor dimension) [Sprott, 2007a]. Social networks in the real world show weak chaos, and networks of ODEs can model them [Chlouverakis & Sprott, 2007]. Because of the large number of new chaotic systems being published, he proposed a standard for the publication of new chaotic systems that is often used by reviewers in evaluating submissions [Sprott, 2011c].

Sprott discussed the concept of robust chaos (the absence of periodic windows or coexisting attractors) [Elhadj & Sprott, 2008c] and proposed a method for generating robust chaos in 2D maps [Zeraoulia & Sprott, 2011]. He showed that increasing the dimension of the typical dynamical system leads to increasing the number of positive Lyapunov exponents and decreasing the number of periodic windows [Albers *et al.*, 2006; Albers & Sprott, 2006c]. He determined that bifurcations from fixed points of large dynamical systems are typically due to complex eigenvalues leading to a Neimark–Sacker bifurcation [Albers & Sprott, 2006a]. He has reviewed 2D rational mappings, hyperbolic structure, structural stability, some questions about periodic, homoclinic, and heteroclinic orbits, classification of chaos, and some questions about chaotification of dynamical systems [Elhadj & Sprott, 2011c]. He has studied structural stability as a condition for robustness of invariant sets in 3D quadratic mappings [Zeraoulia & Sprott, 2012]. He has shown the existence of invariant sets (universal basins of attraction) for typical nonlinear, high-dimensional dynamical systems [Elhadj & Sprott, 2013b]. He has also shown that special nonlinearities with partial information in the variable can lead to chaos (by omitting amplitude or polarity information in the feedback loop) [Li *et al.*, 2016c]. In the following subsections, various aspects of his studies are described in more detail.

2.1. *Dynamical systems and their properties*

This subsection surveys some of Sprott's work in generating new chaotic systems and investigating their behavior.

2.1.1. *Chaotic maps*

Some of the simplest examples of chaos arise in iterated maps in which time advances in discrete steps. Sprott has found some new maps with specific fractal attractors [Elhadj & Sprott, 2008b, 2010c]. A 2D multifold map with a period-doubling route to chaos has been proposed [Elhadj & Sprott, 2008c] as well as an unusual map with a rational function [Elhadj & Sprott, 2011b]. Piecewise-linear maps are important because they sometimes facilitate analytic analysis [Elhadj & Sprott, 2011a]. A robust homoclinic chaotic system involving a piecewise-smooth map which is a combination of the Hénon and Lozi systems has been discussed [Elhadj & Sprott, 2012f] as well as a piecewise-linear map with the same nonlinearity as Chua's circuit and capable of generating single and multiscroll chaotic and hyperchaotic attractors [Elhadj & Sprott, 2009c]. Sprott proposed a criteria for 2D quadratic maps that can ensure the solution is either chaotic or nonchaotic [Elhadj & Sprott, 2009a]. The dynamics of a 2D Hénon-like mapping with an unknown bounded function has been discussed [Elhadj & Sprott, 2012f],and a rigorous proof that the exact value of the overall period of a particular 2D chaotic attractor from an iterated map is two was provided by partitioning and quantizing the map into disconnected sets [Elhadj & Sprott, 2013a]. Also, a chaotic 2D map surrounded by nested invariant tori has been proposed [Elhadj & Sprott, 2012a], and it was shown that S-unimodal maps are Collet–Eckmann in a limited interval of their bifurcation parameter [Elhadj & Sprott, 2012g]. A monomial Hénon mapping was introduced using a 2D monomial term in the classical Hénon map [Elhadj & Sprott, 2013c].

2.1.2. *Chaotic flows*

Sprott has extensively studied chaotic systems of ODEs [Sprott, 2010]. In 1994, he published a classic paper describing 19 chaotic flows that were simpler than any previously known with only five terms and two quadratic nonlinearities or six terms and one quadratic nonlinearity [Sprott, 1994b]. In 1996, he identified the algebraically simplest dissipative chaotic flow with five terms and a single quadratic nonlinearity [Sprott, 1997a]. Some other simple chaotic jerk flows with quadratic, cubic, and exponential nonlinearities were introduced [Sprott, 1997b; Munmuangsaen *et al.*, 2011]. He also found the simplest 3D chaotic system with only one nonlinearity in the form of the modulus of the dynamical variable [Linz & Sprott, 1999; Sprott, 2006a].

A unified chaotic system containing Lorenz, Chua, and some other systems was introduced [Elhadj & Sprott, 2010a]. Hyperjerk systems have been investigated with their chaotic and hyperchaotic attractors [Chlouverakis & Sprott, 2006]. A new category of 3D diffeomorphisms with constant Jacobian and different behaviors was proposed [Elhadj & Sprott, 2009b]. Sprott has investigated the time evolution of some maps and flows in terms of higher-order moments [Rowlands & Sprott, 2008]. A chaotic jerk system with a piecewise-exponential nonlinearity has been introduced [Sun & Sprott, 2009b]. Sprott investigated the equivalence between 4D autonomous dynamical systems and hyperjerk systems [Elhadj & Sprott, 2013e]. Some conditions for the nonexistence of homoclinic and heteroclinic orbits and Shilnikov chaos in n-dimensional smooth flows have been presented [Elhadj & Sprott, 2012e] and a method for generating n-scroll chaotic attractors in 3D flows was proposed [Elhadj & Sprott, 2013d].

Sprott also proposed different types of systems with hidden attractors such as systems without any equilibria [Jafari *et al.*, 2013; Jafari *et al.*, 2015b], systems with one stable equilibrium [Molaie *et al.*, 2013; Sprott *et al.*, 2015; Baghdadi *et al.*, 2015], systems with lines of equilibria [Jafari *et al.*, 2015b; Jafari & Sprott, 2013, 2015], systems with a plane of equilibria [Jafari *et al.*, 2016b], systems with surfaces of equilibria [Jafari *et al.*, 2016c], systems with a curve of equilibria [Barati *et al.*, 2016], and systems with circular and square equilibria [Gotthans *et al.*, 2016]. Dynamical behaviors of the Moffatt system and its hidden chaotic attractors have been demonstrated [Wei *et al.*, 2017].

A 4D system without any equilibria with multistability has been investigated [Li & Sprott, 2014b]. He has proposed a four-wing chaotic system with symmetric bistability [Li *et al.*, 2015b]. He showed that the Wang–Chen system which has a chaotic attractor with one stable equilibrium has a coexisting limit cycle, giving a system with three types of coexisting attractors [Sprott *et al.*, 2013]. A new chaotic system with multistability was proposed [Li *et al.*, 2016a], and amplitude and phase control methods are applied to it. A 3D flow with involutional symmetry has been proposed [Sprott, 2014b] as well as a new 3D chaotic flow with many regions of multistability [Li & Sprott, 2013b]. Sprott has proposed two autonomous 3D chaotic flows with a continuously adjustable attractor dimension [Munmuangsaen *et al.*, 2015]. He proposed a 3D chaotic flow, all of whose terms except one are quadratic [Li & Sprott, 2014a]. He showed that the extreme multistability which is discussed in [Hens *et al.*, 2012] can be achieved in any dynamical system by adding extraneous variables and using their initial conditions in place of the existing parameters or as additional parameters [Sprott & Li, 2014].

He described a new system with a hidden attractor and proposed a likelihood score in a Gaussian Mixture Model which can estimate the system's parameter [Lao *et al.*, 2014]. He proposed a hyperchaotic system with a line of equilibria [Li *et al.*, 2014a]. It has two symmetric limit cycles that merge together to form a strange attractor. A simple 3D time-reversible system of ODEs has been described which has nested conservative quasi-periodic tori coexisting with a dissipative chaotic attractor [Sprott, 2014a]. A piecewise-linear hyperchaotic system with a signum nonlinearity and absolute values has been presented [Li *et al.*, 2014b]. A simple time-reversible 3D chaotic flow with a polynomial nonlinearity has been described [Sprott, 2015c]. Some 3D chaotic flows with hidden attractors and multistability have been proposed [Sprott, 2015b]. Using a chaotic hyperbolic system and a synchronization technique, he has proposed a method for generating chaotic hyperbolic systems [Elhadj & Sprott, 2012c], and he introduced a 3D chaotic jerk system with a single nonhyperbolic equilibrium [Wei *et al.*, 2015].

Sprott showed that perpetual points are limited in finding dissipativity of systems [Jafari *et al.*, 2015a] as proposed by Prasad [2015]. Four categories of chaotic flows with and without fixed points and perpetual points have been discussed [Nazarimehr *et al.*, 2017b]. It was shown that perpetual points are not sufficient for locating hidden attractors as originally hoped [Nazarimehr *et al.*, 2017c]. Different behaviors of the Kingni–Jafari system have been investigated [Wei *et al.*, 2016], especially its unstable behaviors.

He explored a new regime of chaotic flows with offset boosting [Li & Sprott, 2016] and a 4D hyperjerk system with an exponential nonlinearity and its realization with field programmable analog arrays [Dalkiran & Sprott, 2016]. Flows with spatially-periodic functions such as the sine can generate elegant chaotic behaviors and deterministic fractional Brownian motion [Sprott & Chlouverakis, 2007]. He proposed a chaotic system with self-reproduction using a sine function that leads to an infinite number of multistable attractors that can be used as a model for memory [Li *et al.*, 2017b]. Asymmetric dynamical systems can generate symmetric coexisting attractors, which is called conditional symmetry, and chaotic flows with such conditional symmetry have been presented [Li *et al.*, 2017d]. He proposed a new chaotic flow with an infinite number of strange attractors [Li *et al.*, 2017c] and described a periodically-forced system with an infinite number of coexisting limit cycles, attracting tori, and strange attractors [Sprott *et al.*, 2017]. He proposed a chaotic flow, all of whose terms are quadratic except one which is linear and whose coefficient controls the amplitude and frequency of oscillations without inducing bifurcations [Li *et al.*, 2017a].

Sprott has examined simplifications of the Lorenz system and proposed a chaotic system with two fewer terms and a single parameter [Sprott, 2009b; Sun & Sprott, 2009a] and a fractional-order simplified Lorenz system [Sun *et al.*, 2010]. Hyperchaotic behavior of a new simplified Lorenz system with a sinusoidally-forced parameter and feedback control were investigated [Sun *et al.*, 2012]. He also investigated the fractal basin of the Lorenz model [Djellit *et al.*, 2011] and of a modified Lorenz system [Djellit *et al.*, 2015]. He examined the Lorenz system in a new region of parameter space [Li & Sprott, 2014d] and found a previously unknown regime of multistability. He proposed a partial and complete piecewise-linear version of the Lorenz system which has an independent total amplitude control parameter [Li *et al.*, 2015c]. The Lorenz system (with slightly changed parameters) exhibits tristability with two stable equilibria and one strange attractor, and its basins of attraction were examined using a 3D printer [Xiong *et al.*, 2017]. Also, a modified Chua circuit with one, two, and three scrolls was introduced [Elhadj & Sprott, 2010b]. He has shown that the Lorenz and Chen systems have the same attractor in $t \to \infty$ and $t \to -\infty$, respectively, and some of the behaviors of these two models which are equivalent have been investigated [Sprott, 2015a]. He studied the synchronization of two Rössler systems with a temporally-periodic coupling term [Buscarino *et al.*, 2017].

2.1.3. *Chaotic systems with complex variables*

Complex-variable chaotic systems have been relatively less studied, but Sprott has identified the simplest nonautonomous [Marshall & Sprott, 2009] and autonomous [Marshall & Sprott, 2010] examples of such systems.

2.1.4. *Nonautonomous chaotic systems*

Externally driven systems generally have a more complex behavior and a higher-dimension attractor than an autonomous system with the same number of variables. Chaotic flows with a sinusoidal drive and a cubic nonlinearity [Gottlieb & Sprott, 2001] and a signum nonlinearity [Sun & Sprott, 2010] have been examined. Also, the logistic map with a time-varying parameter has been discussed [Mirus & Sprott, 1999b; Elhadj & Sprott, 2008a].

2.1.5. *Fractional-order chaotic systems*

Fractional-order chaotic systems are of much current interest. Sprott investigated an electronic chaotic oscillator and a mechanical chaotic jerk model with fractional order $2+\varepsilon$ where $0 < \varepsilon < 1$ [Ahmad & Sprott, 2003] as well as a fractional-order simplified Lorenz system [Sun *et al.*, 2010] and synchronized fractional-order systems [Chen *et al.*, 2012a; Chen *et al.*, 2012b].

2.1.6. *Spatiotemporal chaos*

Mathematical models in ecology often need to incorporate spatial dependence to accurately model real-world systems [Wildenberg *et al.*, 2006; Sprott, 2012]. There are many studies of chaos in systems described by partial differential equations (PDEs). Chaotic behaviors of Lotka–Volterra models [Sprott *et al.*, 2005b; Vano *et al.*, 2006] and their high-dimensional extensions [Wildenberg *et al.*, 2006] have been studied. A spatiotemporal chaotic system with cyclic symmetry has been investigated [Chlouverakis & Sprott, 2007]. Sprott has discussed the Kuramoto–Sivashinsky equation as a simple chaotic partial differential equation and searched for even simpler examples [Brummitt & Sprott, 2009].

2.1.7. *Other chaotic systems*

Delay in discrete and continuous systems can cause more complex behaviors [Sprott, 2006b, 2007b]. In addition, Sprott studied the use of artificial neural networks for generating chaos [Sprott, 1998].

2.1.8. *Bifurcations*

Bifurcations are important in nonlinear dynamical systems, and Sprott has done much work in this area. He showed that the most probable route to chaos in high-dimensional artificial neural networks is the quasi-periodic route and that increasing the dimension of a system makes chaotic solutions more likely [Albers *et al.*, 1998]. He showed that the most probable route to chaos in high-order systems is through Neimark–Sacker bifurcations [Albers & Sprott, 2006b].

2.1.9. *Basins of attraction*

Complex dynamical systems can generate different attractors, each with a basin of attraction. Such systems are multistable, and they are important in the engineering applications since a small perturbation can cause a huge shift in the system's state from one attractor to another. Sprott has investigated multistability in many systems such as the delayed Hénon map [Sprott, 2006b]. He has shown that symmetric chaotic flows such as the Rössler system have regimes of multistability not previously known [Sprott & Li, 2017], and he has categorized basins of attractions into four classes depending on their size and extent [Sprott & Xiong, 2015]. Coexistence of some strange attractors and limit cycles in symmetric systems has been investigated [Li *et al.*, 2015a].

2.1.10. *Fractal art*

A byproduct of his search for new chaotic maps and flows has been thousands of examples of fractal strange attractors and their basins of attraction, many of which have aesthetic appeal. Sprott has automated the search for visually interesting fractals and has published many of the most appealing ones along with the algorithms for their production [Sprott, 1992, 1998, 2001; Sprott & Pickover, 1997; Sprott, 2004a; Taylor & Sprott, 2008; Draves *et al.*, 2008; Sprott & Pickover, 1995; Sprott, 1996; Chapman *et al.*, 2005]. He was able to demonstrate a relation between the aesthetic appeal and their fractal dimension [Sprott, 1993a; Aks & Sprott, 1996; Draves *et al.*, 2008]. He had one of the first fractal galleries on the Web (http://sprott.physics.wisc.edu/fractals.htm), and for over 20 years a new "Fractal of the Day" has been regularly produced by his programs. He is currently writing a book, *Elegant Fractals*: *Automated Generation of Computer Art*, to be published in 2018.

2.1.11. *Chaotic electrical circuits*

Sprott has had a long interest in electronic circuits dating back to his teenage years as an amateur radio operator, and his first book was an electronics text [Sprott, 1981]. Thus it was natural for him to propose many new examples of chaotic electrical circuits using only resistors, capacitors, diodes, and inverting operational amplifiers mostly based on the simple equations found in his search for new chaotic systems [Sprott, 2000a, 2000b; Kiers *et al.*, 2004a; Kiers *et al.*, 2004b; Piper & Sprott, 2010; Sprott, 2011b; Buscarino *et al.*, 2016; Sprott, 2010]. He has been especially interested in finding the simplest circuits that oscillate chaotically.

2.1.12. *Chaotic models*

Sprott has also been interested in models of real-world systems that exhibit chaos. For example, he has studied models of plasmas [Watts *et al.*, 1994; Horton *et al.*, 2001; Zhdankin & Sprott, 2013], molecular dynamics [Sprott *et al.*, 2014; Patra *et al.*, 2015; Hoover *et al.*, 2015; Hoover *et al.*, 2016a, 2016c; Patra *et al.*, 2016; Hoover *et al.*, 2016b], reaction–diffusion fronts [Suwannasen *et al.*, 2016], financial markets [Sprott, 2004b; Gomes & Sprott, 2017], human behavior [Warren *et al.*, 2002], romance [Sprott, 2004c, 2008a; Jafari *et al.*, 2016a], happiness [Sprott, 2005; Tabatabaei *et al.*, 2014], substance abuse [Warren *et al.*, 2003], attention deficit disorders [Baghdadi *et al.*, 2015], postural sway [Ghomashchi *et al.*, 2010; Ghomashchi *et al.*, 2011], ecology [Wildenberg *et al.*, 2006; Sprott, 2008a; Zhdankin & Sprott, 2010; Bolliger *et al.*, 2003; Sprott, 2011a, 2012; Sprott *et al.*, 2002; Sprott *et al.*, 2005a], music [Meloon & Sprott, 1997], neural systems [Molaie *et al.*, 2014a], memory [Aram *et al.*, 2017], and learning [Sprott, 2013].

He also proposed a model of the world in which Newton's first and second laws hold, but where the third law takes the form that the forces between any two objects are equal in magnitude *and* direction as might apply in biological systems such as a fox chasing a rabbit, and he found a rich variety of behaviors including chaos for the two-body problem in two spatial dimensions [Sprott, 2009a].

After modeling a complex system, estimating its parameters for different behaviors is desired. Sprott has proposed a new cost function to estimate parameters of chaotic systems with a statistical model of its phase space [Shekofteh *et al.*, 2015] and has provided a new viewpoint for the estimation of model parameters [Jafari *et al.*, 2014].

2.2. *Control and synchronization of chaotic systems*

Chaos can be either a virtue or a vice, but in either case, it is useful to have some control over it, and many practical applications such as secure communications require the synchronization of two chaotic systems. Sprott's interest in this problem arose from a desire to improve the confinement of plasmas by controlling their ubiquitous magnetic fluctuations. He proposed a control method using periodic parametric perturbations [Mirus & Sprott, 1999a, 1999b]. He also described a matrix controller for flows to allow a transition between asymptotically stable limit sets and hyperchaos [Elhadj & Sprott, 2012b]. Amplitude control of chaotic systems can be helpful in detecting multistability and hidden attractors [Li & Sprott, 2014c; Li *et al.*, 2016b]. A method for amplitude control of chaotic systems via a control function has been proposed [Li & Sprott, 2013a]. He also presented a general method for amplitude control of chaotic systems by changing the degree of some terms in the governing equations [Li *et al.*, 2015d].

Sprott has studied the synchronization and anti-synchronization of chaotic systems [Chen *et al.*, 2013]. He also discussed the synchronization of chaotic electronic circuits [Bai *et al.*, 2002] and proposed a universal nonlinear control law for synchronization of 3D quadratic flows [Elhadj & Sprott, 2012h]. A sliding mode control for synchronizing integer-order chaotic systems and a class of fractional-order chaotic system were also presented [Chen *et al.*, 2012a; Chen *et al.*, 2012b]. He has proposed an adaptive modified hybrid function projective synchronization for chaotic and hyperchaotic systems with complex variables and parameters [Liu *et al.*, 2016].

2.3. *Time-series analysis*

Like so many others, Sprott has been interested in finding evidence of chaos in real-world systems through the analysis of a time series of experimental data. He proposed a method based on singular-value decomposition to model a system using its time series [Rowlands & Sprott, 1992]. He has improved the method for calculating the correlation dimension from measured data [Sprott & Rowlands, 2001] and investigated the relation between the correlation dimension and the Kaplan–Yorke dimension [Chlouverakis & Sprott, 2005]. He proposed a mechanism for detecting weak chaos in complex ecologies [Sprott *et al.*, 2005a]. He described a method to estimate the optimal time-delay embedding dimension [Maus & Sprott, 2011] and Lyapunov exponent spectra [Maus & Sprott, 2013] using an artificial neural network. Estimating Lyapunov exponents from a time series is an important problem since it can be used to predict tipping (bifurcation) points [Nazarimehr *et al.*, 2017a]. He described a method for noise reduction in respiratory sounds [Molaie *et al.*, 2014b] and proposed using the rate of divergence to differentiate four types of voice signals [Calawerts *et al.*, 2017]. He also proposed a nonlinear model with exogenous inputs (NARX) based on a novel recurrent fuzzy function to predict a chaotic time series [Goudarzi *et al.*, 2016].

3. Discussion

Only a small fraction of low-dimensional chaotic systems exhibit chaos. Professor Sprott has teased out many new examples of chaotic systems over the past three decades. He has published important books such as *Chaos and Time-Series Analysis* [Sprott, 2003], and *Elegant Chaos*: *Algebraically Simple Chaotic Flows* [Sprott, 2010]. He has developed programs to search for chaotic solutions in different structures of dynamical systems and to analyze their properties. He has proposed many special chaotic systems such as those with hidden attractors and used a variety of tools to study their dynamics. He has modeled many real-world systems and explored methods for chaos control and synchronization. We thank Professor Sprott for his brilliant works over the years.

4. Congratulatory Messages to Professor Sprott

In the following, the authors provide some memories of their collaborations with Professor Sprott.

Fahimeh Nazarimehr: Chaos and nonlinear dynamics is a research field with many interesting problems. It is a great honor for me to learn from some of the best scientists in this area. I have collaborated with Professor Sprott and learned much from him. It is my pleasure to see him healthy and remain an enthusiastic pioneer in the field.

Sajad Jafari: During my PhD exam, I had no idea about working under the supervision of Professor Hashemi Golpayegani in the field of chaos and nonlinear dynamics. I was afraid of starting something new. However, it was a great luck of my life when he gave me some papers from Professor Sprott and told me to read these papers which are from a wise scientist. That was the beginning of my story with that kind Professor from whom I learned much and owe a lot. I wish him health and happiness forever.

Guanrong Chen: Before I met Clint in Hong Kong, I knew he was excellent in chaotic time-series analysis from his popular book *Chaos and Time-Series Analysis*. Through the years after meeting him, I gradually and surprisingly found that he is superb in constructing and searching for new kinds of chaotic attractors from lower-dimensional, simply-structured autonomous systems, with some unusual and amazing properties that are unexplained by modern mathematical chaos theory. I consider him an asset to our chaos community.

Tomasz Kapitaniak: Clint proposed a number of simple looking, low-dimensional dynamical systems. With these examples, dynamics look simple, strange attractors are not too strange, and hidden attractors are not too deeply hidden, i.e. easy to uncover.

Gennady A. Leonov and Nikolay V. Kuznetsov: The elegant and meaningful examples of dynamic systems proposed by Professor Sprott are opening the veil of the mystery of chaos and motivate us to develop new analytical methods for the investigation of chaotic dynamics.

Chunbiao Li: I feel that chaos and multistablity is MAGIC when I am working with Professor Sprott. The year of 2012 was a great period in my life since it brought me to work with him. During my time in Madison, we discussed chaos almost every day, and sometimes we played tennis. I wrote a poem to describe that wonderful time. "We played tennis under the cloud; we discussed academic questions under the rain; we said goodbye day after day, but your figure is in my memory every day. I am a visiting scholar from abroad. While I am in your sun, all in all, I hope the days continue and forever be there; I hope the sky and cloud bring my best wishes to you everywhere."

Zhouchao Wei: First of all, I, together with my friends, wish to send our best wishes to Professor Sprott on the occasion of his 75th birthday. I have known Professor Sprott since my first year as a graduate student at Guangzhou in the fall of 2007. Professor Sprott has made great contributions to chaos research. He was always encouraging, even when we did not agree. (He was much more rigorous than me.) He was a true mentor, and I am grateful for the experience and all the ideas he provided me.

References

Ahmad, W. M. & Sprott, J. C. [2003] "Chaos in fractional-order autonomous nonlinear systems," *Chaos Solit. Fract.* **16**, 339–351.

Aks, D. J. & Sprott, J. C. [1996] "Quantifying aesthetic preference for chaotic patterns," *Empir. Stud. Arts* **14**, 1–16.

Albers, D. J., Sprott, J. C. & Dechert, W. D. [1998] "Routes to chaos in neural networks with random weights," *Int. J. Bifurcation and Chaos* **8**, 1463–1478.

Albers, D. J., Sprott, J. C. & Crutchfield, J. P. [2006] "Persistent chaos in high dimensions," *Phys. Rev. E* **74**, 057201.

Albers, D. J. & Sprott, J. C. [2006a] "Probability of local bifurcation type from a fixed point: A random matrix perspective," *J. Stat. Phys.* **125**, 885–921.

Albers, D. J. & Sprott, J. C. [2006b] "Routes to chaos in high-dimensional dynamical systems: A qualitative numerical study," *Physica D* **223**, 194–207.

Albers, D. J. & Sprott, J. C. [2006c] "Structural stability and hyperbolicity violation in high-dimensional dynamical systems," *Nonlinearity* **19**, 1801–1847.

Aram, Z., Jafari, S., Ma, J., Sprott, J. C., Zendehrouh, S. & Pham, V.-T. [2017] "Using chaotic artificial neural networks to model memory in the brain," *Commun. Nonlin. Sci. Numer. Simul.* **44**, 449–459.

Baghdadi, G., Jafari, S., Sprott, J. C., Towhidkhah, F. & Golpayegani, S. M. R. H. [2015] "A chaotic model of sustaining attention problem in attention deficit disorder," *Commun. Nonlin. Sci. Numer. Simul.* **20**, 174–185.

Bai, E.-W., Lonngren, K. E. & Sprott, J. C. [2002] "On the synchronization of a class of electronic circuits that exhibit chaos," *Chaos Solit. Fract.* **13**, 1515–1521.

Barati, K., Jafari, S., Sprott, J. C. & Pham, V.-T. [2016] "Simple chaotic flows with a curve of equilibria," *Int. J. Bifurcation and Chaos* **26**, 1630034-1–6.

Bolliger, J., Sprott, J. C. & Mladenoff, D. J. [2003] "Self-organization and complexity in historical landscape patterns," *Oikos* **100**, 541–553.

Bragin, V. O., Vagaitsev, V. I., Kuznetsov, N. V. & Leonov, G. A. [2011] "Algorithms for finding hidden oscillations in nonlinear systems. The Aizerman and Kalman conjectures and Chua's circuits," *J. Comp. Syst. Sci. Int.* **50**, 511–543.

Brummitt, C. D. & Sprott, J. C. [2009] "A search for the simplest chaotic partial differential equation," *Phys. Lett. A* **373**, 2717–2721.

Buscarino, A., Corradino, C., Fortuna, L., Frasca, M. & Sprott, J. C. [2016] "Nonideal behavior of analog multipliers for chaos generation," *IEEE Trans. Circuits Syst.-II* **63**, 396–400.

Buscarino, A., Frasca, M., Branciforte, M., Fortuna, L. & Sprott, J. C. [2017] "Synchronization of two Rössler systems with switching coupling," *Nonlin. Dyn.* **88**, 673–683.

Calawerts, W. M., Lin, L., Sprott, J. C. & Jiang, J. J. [2017] "Using rate of divergence as an objective measure to differentiate between voice signal types based on the amount of disorder in the signal," *J. Voice* **31**, 16–23.

Chapman, R. S., Sprott, J. C. & Pickover, C. A. [2005] *Images of a Complex World*: *The Art and Poetry of Chaos* (World Scientific, Singapore).

Chen, D., Zhang, R., Sprott, J. C., Chen, H. & Ma, X. [2012a] "Synchronization between integer-order chaotic systems and a class of fractional-order chaotic systems via sliding mode control," *Chaos* **22**, 023130.

Chen, D., Zhang, R., Sprott, J. C. & Ma, X. [2012b] "Synchronization between integer-order chaotic systems and a class of fractional-order chaotic system based on fuzzy sliding mode control," *Nonlin. Dyn.* **70**, 1549–1561.

Chen, D., Zhao, W., Sprott, J. C. & Ma, X. [2013] "Application of Takagi–Sugeno fuzzy model to a class of chaotic synchronization and anti-synchronization," *Nonlin. Dyn.* **73**, 1495–1505.

Chlouverakis, K. E. & Sprott, J. C. [2005] "A comparison of correlation and Lyapunov dimensions," *Physica D* **200**, 156–164.

Chlouverakis, K. E. & Sprott, J. C. [2006] "Chaotic hyperjerk systems," *Chaos Solit. Fract.* **28**, 739–746.

Chlouverakis, K. E. & Sprott, J. C. [2007] "Hyperlabyrinth chaos: From chaotic walks to spatiotemporal chaos," *Chaos* **17**, 023110.

Dalkiran, F. Y. & Sprott, J. C. [2016] "Simple chaotic hyperjerk system," *Int. J. Bifurcation and Chaos* **26**, 1650189-1–6.

Dechert, W. D., Sprott, J. C. & Albers, D. J. [1999] "On the probability of chaos in large dynamical systems: A Monte Carlo study," *J. Econ. Dyn. Contr.* **23**, 1197–1206.

Djellit, I., Sprott, J. C. & Ferchichi, M. R. [2011] "Fractal basins in the Lorenz model," *Chinese Phys. Lett.* **28**, 060501.

Djellit, I., Kilani, B. & Sprott, J. C. [2015] "Broken symmetry in modified Lorenz model," *Int. J. Dyn. Syst. Diff. Eqs.* **5**, 136–148.

Draves, S., Abraham, R., Viotti, P., Abraham, F. D. & Sprott, J. C. [2008] "The aesthetics and fractal dimension of electric sheep," *Int. J. Bifurcation and Chaos* **18**, 1243–1248.

Elhadj, Z. & Sprott, J. C. [2008a] "The effect of modulating a parameter in the logistic map," *Chaos* **18**, 023119.

Elhadj, Z. & Sprott, J. C. [2008b] "A minimal 2D quadratic map with quasi-periodic route to chaos," *Int. J. Bifurcation and Chaos* **18**, 1567–1577.

Elhadj, Z. & Sprott, J. C. [2008c] "On the robustness of chaos in dynamical systems: Theories and applications," *Front. Phys. China* **3**, 195–204.

Elhadj, Z. & Sprott, J. C. [2008d] "A two-dimensional discrete mapping with C^∞ multifold chaotic attractors," *Electron. J. Theor. Phys.* **5**, 111–124.

Elhadj, Z. & Sprott, C. J. [2009a] "Some criteria for chaos and no chaos in the quadratic map of the plane," *Facta Universitatis-Series*: *Electronics and Energetics* **22**, 105–117.

Elhadj, Z. & Sprott, J. C. [2009b] "Classification of three-dimensional quadratic diffeomorphisms with constant Jacobian," *Front. Phys. China* **4**, 111–121.

Elhadj, Z. & Sprott, J. C. [2009c] "The discrete hyperchaotic double scroll," *Int. J. Bifurcation and Chaos* **19**, 1023–1027.

Elhadj, Z. & Sprott, C. J. [2010a] "The unified chaotic system describing the Lorenz and Chua systems," *Facta Universitatis-Series*: *Electronics and Energetics* **23**, 345–355.

Elhadj, Z. & Sprott, J. C. [2010b] "Generating 3-scroll attractors from one Chua circuit," *Int. J. Bifurcation and Chaos* **20**, 135–144.

Elhadj, Z. & Sprott, J. C. [2010c] "A new simple 2D piecewise linear map," *J. Syst. Sci. Compl.* **23**, 379–389.

Elhadj, Z. & Sprott, J. C. [2011a] "Chaotifying 2D piecewise-linear maps via a piecewise-linear controller function," *Nonlin. Oscill.* **13**, 352–360.

Elhadj, Z. & Sprott, J. C. [2011b] "On the dynamics of a new simple 2D rational discrete mapping," *Int. J. Bifurcation and Chaos* **21**, 155–160.

Elhadj, Z. & Sprott, J. C. [2011c] "Some open problems in chaos theory and dynamics," *Int. J. Open Prob. Compt. Math.* **4**, 1–9.

Elhadj, Z. & Sprott, J. C. [2012a] "An example of a fully bounded chaotic sea that surrounds an infinite set of nested invariant tori," *Palestine J. Math.* **1**, 71–73.

Elhadj, Z. & Sprott, J. C. [2012b] "A general approach for hyperchaotifying n-dimensional continuous-time systems," *Sci. Tech. J. Sci. Technol.* **1**, 106–109.

Elhadj, Z. & Sprott, J. C. [2012c] "Hyperbolification of dynamical systems: The case of continuous-time systems," *J. Exp. Theor. Phys.* **115**, 356–360.

Elhadj, Z. & Sprott, J. C. [2012d] "Is a unifying chaotic dynamical system possible?" *Int. J. Open Prob. Comp. Sci. Math.* **5**, 75–78.

Elhadj, Z. & Sprott, J. C. [2012e] "Non-existence of Shilnikov chaos in continuous-time systems," *Appl. Math. Mech.* **33**, 371–374.

Elhadj, Z. & Sprott, J. C. [2012f] "A unified piecewise smooth chaotic mapping that contains the Hénon and the Lozi systems," *Ann. Rev. Chaos Th. Bif. Dyn. Syst.* **1**, 50–60.

Elhadj, Z. & Sprott, J. C. [2012g] "A proof that s-unimodal maps are collet-Eckmann maps in a specific range of their bifurcation parameters," *Acta Universitatis Apulensis* **34**, 51–55.

Elhadj, Z. & Sprott, J. C. [2012h] "A universal nonlinear control law for the synchronization of arbitrary 3D continuous-time quadratic systems," *Adv. Syst. Sci. Appl.* **12**, 347–352.

Elhadj, Z. & Sprott, J. C. [2013a] "A rigorous determination of the overall period in the structure of a chaotic attractor," *Int. J. Bifurcation and Chaos* **23**, 1350046-1–4.

Elhadj, Z. & Sprott, J. C. [2013b] "About universal basins of attraction in high-dimensional systems," *Int. J. Bifurcation and Chaos* **23**, 1350197-1–7.

Elhadj, Z. & Sprott, J. C. [2013c] "On a conjecture about monomial Hénon mappings," *Int. J. Open Prob. Comp. Sci. Math.* **6**, 55–60.

Elhadj, Z. & Sprott, J. C. [2013d] "Simplest 3D continuous-time quadratic systems as candidates for generating multiscroll chaotic attractors," *Int. J. Bifurcation and Chaos* **23**, 1350120-1–6.

Elhadj, Z. & Sprott, J. C. [2013e] "Transformation of 4D dynamical systems to hyperjerk form," *Palestine J. Math.* **2**, 38–45.

Elhadj, Z. & Sprott, J. C. [2014] "Confirmation of persistent chaos in high dimensions," *Palestine J. Math.* **3**.

Ghomashchi, H., Esteki, A., Sprott, J. C. & Nasrabadi, A. M. [2010] "Identification of dynamic patterns of body sway during quiet standing: Is it a nonlinear process?" *Int. J. Bifurcation and Chaos* **20**, 1269–1278.

Ghomashchi, H., Esteki, A., Nasrabadi, A. M., Sprott, J. C. & BahrPeyma, F. [2011] "Dynamic patterns of postural fluctuations during quiet standing: A recurrence quantification approach," *Int. J. Bifurcation and Chaos* **21**, 1163–1172.

Gomes, O. & Sprott, J. C. [2017] "Sentiment-driven limit cycles and chaos," *J. Evol. Econ.*, pp. 1–32.

Gotthans, T., Sprott, J. C. & Petrzela, J. [2016] "Simple chaotic flow with circle and square equilibrium," *Int. J. Bifurcation and Chaos* **26**, 1650137-1–8.

Gottlieb, H. P. W. & Sprott, J. C. [2001] "Simplest driven conservative chaotic oscillator," *Phys. Lett. A* **291**, 385–388.

Goudarzi, S., Jafari, S., Moradi, M. H. & Sprott, J. C. [2016] "Narx prediction of some rare chaotic flows: Recurrent fuzzy functions approach," *Phys. Lett. A* **380**, 696–706.

Hens, C. R., Banerjee, R., Feudel, U. & Dana, S. K. [2012] "How to obtain extreme multistability in coupled dynamical systems," *Phys. Rev. E* **85**, 035202.

Hoover, W. G., Sprott, J. C. & Patra, P. K. [2015] "Ergodic time-reversible chaos for Gibbs' canonical oscillator," *Phys. Lett. A* **379**, 2935–2940.

Hoover, W. G., Hoover, C. G. & Sprott, J. C. [2016a] "Nonequilibrium systems: Hard disks and harmonic oscillators near and far from equilibrium," *Mol. Simul.* **42**, 1300–1316.

Hoover, W. G., Sprott, J. C. & Hoover, C. G. [2016b] "Adaptive Runge–Kutta integration for stiff systems: Comparing Nosé and Nosé–Hoover dynamics for the harmonic oscillator," *Am. J. Phys.* **84**, 786–794.

Hoover, W. G., Sprott, J. C. & Hoover, C. G. [2016c] "Ergodicity of a singly-thermostated harmonic oscillator," *Commun. Nonlin. Sci. Numer. Simul.* **32**, 234–240.

Horton, W., Weigel, R. S. & Sprott, J. C. [2001] "Chaos and the limits of predictability for the solar-wind-driven Magnetosphere–Ionosphere system," *Phys. Plasmas* **8**, 2946–2952.

Jafari, S. & Sprott, J. C. [2013] "Simple chaotic flows with a line equilibrium," *Chaos Solit. Fract.* **57**, 79–84.

Jafari, S., Sprott, J. C. & Golpayegani, S. M. R. H. [2013] "Elementary quadratic chaotic flows with no equilibria," *Phys. Lett. A* **377**, 699–702.

Jafari, S., Sprott, J. C., Pham, V.-T., Golpayegani, S. M. R. H. & Jafari, A. H. [2014] "A new cost function for parameter estimation of chaotic systems using return maps as fingerprints," *Int. J. Bifurcation and Chaos* **24**, 1450134-1–18.

Jafari, S. & Sprott, J. C. [2015] "Erratum to: "Simple chaotic flows with a line equilibrium" [chaos, solitons and fractals 57 (2013) 79–84]," *Chaos Solit. Fract.* **77**, 341–342.

Jafari, S., Nazarimehr, F., Sprott, J. C. & Golpayegani, S. M. R. H. [2015a] "Limitation of perpetual points for confirming conservation in dynamical systems," *Int. J. Bifurcation and Chaos* **25**, 1550182-1–6.

Jafari, S., Sprott, J. C. & Nazarimehr, F. [2015b] "Recent new examples of hidden attractors," *Eur. Phys. J.* **224**, 1469–1476.

Jafari, S., Sprott, J. C. & Golpayegani, S. M. R. H. [2016a] "Layla and Majnun: A complex love story," *Nonlin. Dyn.* **83**, 615–622.

Jafari, S., Sprott, J. C. & Molaie, M. [2016b] "A simple chaotic flow with a plane of equilibria," *Int. J. Bifurcation and Chaos* **26**, 1650098-1–6.

Jafari, S., Sprott, J. C., Pham, V.-T., Volos, C. & Li, C. [2016c] "Simple chaotic 3D flows with surfaces of equilibria," *Nonlin. Dyn.* **86**, 1349–1358.

Kiers, K., Klein, T., Kolb, J., Price, S. & Sprott, J. C. [2004a] "Chaos in a nonlinear analog computer," *Int. J. Bifurcation and Chaos* **14**, 2867–2873.

Kiers, K., Schmidt, D. & Sprott, J. C. [2004b] "Precision measurements of a simple chaotic circuit," *Am. J. Phys.* **72**, 503–509.

Kuznetsov, N. V., Leonov, G. A. & Vagaitsev, V. I. [2010] "Analytical-numerical method for attractor localization of generalized Chua's system," *IFAC Proc. Vol.* **43**, 29–33.

Kuznetsov, N. V., Leonov, G. A. & Seledzhi, S. M. [2011] "Hidden oscillations in nonlinear control systems," *IFAC Proc. Vol.* **44**, 2506–2510.

Lao, S.-K., Shekofteh, Y., Jafari, S. & Sprott, J. C. [2014] "Cost function based on Gaussian mixture model for parameter estimation of a chaotic circuit with a hidden attractor," *Int. J. Bifurcation and Chaos* **24**, 1450010-1–11.

Leonov, G. A., Kuznetsov, N. V. & Vagaitsev, V. I. [2011] "Localization of hidden Chua's attractors," *Phys. Lett. A* **375**, 2230–2233.

Leonov, G. A. & Kuznetsov, N. V. [2011a] "Algorithms for searching for hidden oscillations in the Aizerman and Kalman problems," *Dokl. Math.* **84**, 475–481.

Leonov, G. A. & Kuznetsov, N. V. [2011b] "Analytical-numerical methods for investigation of hidden oscillations in nonlinear control systems," *IFAC Proc. Vol.* **44**, 2494–2505.

Leonov, G. A., Kuznetsov, N. V. & Vagaitsev, V. I. [2012] "Hidden attractor in smooth Chua systems," *Physica D* **241**, 1482–1486.

Leonov, G. A. & Kuznetsov, N. V. [2013a] "Analytical-numerical methods for hidden attractors' localization: The 16th Hilbert problem, Aizerman and Kalman conjectures, and Chua circuits," 41–64.

Leonov, G. A. & Kuznetsov, N. V. [2013b] "Hidden attractors in dynamical systems. From hidden oscillations in Hilbert–Kolmogorov, Aizerman, and Kalman problems to hidden chaotic attractor in Chua circuits," *Int. J. Bifurcation and Chaos* **23**, 1330002-1–69.

Leonov, G. A. & Kuznetsov, N. V. [2014] "Hidden oscillations in dynamical systems. 16 Hilbert's problem, Aizerman's and Kalman's conjectures, hidden attractors in Chua's circuits," *J. Math. Sci.* **201**, 645–662.

Leonov, G. A., Kuznetsov, N. V., Kiseleva, M. A., Solovyeva, E. P. & Zaretskiy, A. M. [2014] "Hidden oscillations in mathematical model of drilling system actuated by induction motor with a wound rotor," *Nonlin. Dyn.* **77**, 277–288.

Li, C. & Sprott, J. C. [2013a] "Amplitude control approach for chaotic signals," *Nonlin. Dyn.* **73**, 1335–1341.

Li, C. & Sprott, J. C. [2013b] "Multistability in a butterfly flow," *Int. J. Bifurcation and Chaos* **23**, 1350199-1–10.

Li, C. & Sprott, J. C. [2014a] "Chaotic flows with a single nonquadratic term," *Phys. Lett. A* **378**, 178–183.

Li, C. & Sprott, J. C. [2014b] "Coexisting hidden attractors in a 4D simplified Lorenz system," *Int. J. Bifurcation and Chaos* **24**, 1450034-1–12.

Li, C. & Sprott, J. C. [2014c] "Finding coexisting attractors using amplitude control," *Nonlin. Dyn.* **78**, 2059–2064.

Li, C. & Sprott, J. C. [2014d] "Multistability in the Lorenz system: A broken butterfly," *Int. J. Bifurcation and Chaos* **24**, 1450131-1–7.

Li, C., Sprott, J. C. & Thio, W. [2014a] "Bistability in a hyperchaotic system with a line equilibrium," *J. Exp. Theor. Phys.* **118**, 494–500.

Li, C., Sprott, J. C., Thio, W. & Zhu, H. [2014b] "A new piecewise linear hyperchaotic circuit," *IEEE Trans. Circuits Syst.-II* **61**, 977–981.

Li, C., Hu, W., Sprott, J. C. & Wang, X. [2015a] "Multistability in symmetric chaotic systems," *Eur. Phys. J.* **224**, 1493–1506.

Li, C., Pehlivan, I., Sprott, J. C. & Akgul, A. [2015b] "A novel four-wing strange attractor born in bistability," *IEICE Electron. Express* **12**, 20141116.

Li, C., Sprott, J. C. & Thio, W. [2015c] "Linearization of the Lorenz system," *Phys. Lett. A* **379**, 888–893.

Li, C., Sprott, J. C., Yuan, Z. & Li, H. [2015d] "Constructing chaotic systems with total amplitude control," *Int. J. Bifurcation and Chaos* **25**, 1530025-1–14.

Li, C., Pehlivan, I. & Sprott, J. C. [2016a] "Amplitude-phase control of a novel chaotic attractor," *Turkish J. Electr. Engin. Comput. Sci.* **24**, 1–11.

Li, C., Sprott, J. C. & Xing, H. [2016b] "Crisis in amplitude control hides in multistability," *Int. J. Bifurcation and Chaos* **26**, 1650233-1–11.

Li, C., Sprott, J. C. & Xing, H. [2016c] "Hypogenetic chaotic jerk flows," *Phys. Lett. A* **380**, 1172–1177.

Li, C., Sprott, J. C., Akgul, A., Iu, H. H. C. & Zhao, Y. [2017a] "A new chaotic oscillator with free control," *Chaos* **27**, 083101.

Li, C., Sprott, J. C., Hu, W. & Xu, Y. [2017b] "Infinite multistability in a self-reproducing chaotic system," *Int. J. Bifurcation and Chaos* **27**, 1750160-1–11.

Li, C., Sprott, J. C. & Mei, Y. [2017c] "An infinite 2D lattice of strange attractors," *Nonlin. Dyn.* **89**, 2629–2639.

Li, C., Sprott, J. C. & Xing, H. [2017d] "Constructing chaotic systems with conditional symmetry," *Nonlin. Dyn.* **87**, 1351–1358.

Li, C. & Sprott, J. C. [2016] "Variable-boostable chaotic flows," *Optik-Int. J. Light and Electron. Opt.* **127**, 10389–10398.

Linz, S. J. & Sprott, J. C. [1999] "Elementary chaotic flow," *Phys. Lett. A* **259**, 240–245.

Liu, J., Liu, S. & Sprott, J. C. [2016] "Adaptive complex modified hybrid function projective synchronization of different dimensional complex chaos with uncertain complex parameters," *Nonlin. Dyn.* **83**, 1109–1121.

Marshall, D. & Sprott, J. C. [2009] "Simple driven chaotic oscillators with complex variables," *Chaos* **19**, 013124.

Marshall, D. & Sprott, J. C. [2010] "Simple conservative, autonomous, second-order chaotic complex variable systems," *Int. J. Bifurcation and Chaos* **20**, 697–702.

Maus, A. & Sprott, J. C. [2011] "Neural network method for determining embedding dimension of a time series," *Commun. Nonlin. Sci. Numer. Simul.* **16**, 3294–3302.

Maus, A. & Sprott, J. C. [2013] "Evaluating Lyapunov exponent spectra with neural networks," *Chaos Solit. Fract.* **51**, 13–21.

Meloon, B. & Sprott, J. C. [1997] "Quantification of determinism in music using iterated function systems," *Empir. Stud. Arts* **15**, 3–13.

Mirus, K. A. & Sprott, J. C. [1999a] "Controlling chaos in a high dimensional system with periodic parametric perturbations," *Phys. Lett. A* **254**, 275–278.

Mirus, K. A. & Sprott, J. C. [1999b] "Controlling chaos in low- and high-dimensional systems with periodic parametric perturbations," *Phys. Rev. E* **59**, 5313.

Molaie, M., Jafari, S., Sprott, J. C. & Golpayegani, S. M. R. H. [2013] "Simple chaotic flows with one stable equilibrium," *Int. J. Bifurcation and Chaos* **23**, 1350188-1–7.

Molaie, M., Falahian, R., Gharibzadeh, S., Jafari, S. & Sprott, J. C. [2014a] "Artificial neural networks: Powerful tools for modeling chaotic behavior in the nervous system," *Front. Comput. Neurosci.* **8**, 241–243.

Molaie, M., Jafari, S., Moradi, M. H., Sprott, J. C. & Golpayegani, S. M. R. H. [2014b] "A chaotic viewpoint on noise reduction from respiratory sounds," *Biomed. Sign. Process. Contr.* **10**, 245–249.

Munmuangsaen, B., Srisuchinwong, B. & Sprott, J. C. [2011] "Generalization of the simplest autonomous chaotic system," *Phys. Lett. A* **375**, 1445–1450.

Munmuangsaen, B., Sprott, J. C., Thio, W. J.-C., Buscarino, A. & Fortuna, L. [2015] "A simple chaotic flow with a continuously adjustable attractor dimension," *Int. J. Bifurcation and Chaos* **25**, 1530036-1–12.

Nazarimehr, F., Jafari, S., Golpayegani, S. M. R. H. & Sprott, J. C. [2017a] "Can Lyapunov exponent predict critical transitions in biological systems?" *Nonlin. Dyn.* **88**, 1493–1500.

Nazarimehr, F., Jafari, S., Golpayegani, S. M. R. H. & Sprott, J. C. [2017b] "Categorizing chaotic flows from the viewpoint of fixed points and perpetual points," *Int. J. Bifurcation and Chaos* **27**, 1750023-1–8.

Nazarimehr, F., Saedi, B., Jafari, S. & Sprott, J. C. [2017c] "Are perpetual points sufficient for locating hidden attractors?" *Int. J. Bifurcation and Chaos* **27**, 1750037-1–7.

Patra, P. K., Sprott, J. C., Hoover, W. G. & Hoover, C. G. [2015] "Deterministic time-reversible thermostats: Chaos, ergodicity, and the zeroth law of thermodynamics," *Mol. Phys.* **113**, 2863–2872.

Patra, P. K., Hoover, W. G., Hoover, C. G. & Sprott, J. C. [2016] "The equivalence of dissipation from Gibbs entropy production with phase-volume loss in ergodic heat-conducting oscillators," *Int. J. Bifurcation and Chaos* **26**, 1650089-1–11.

Piper, J. R. & Sprott, J. C. [2010] "Simple autonomous chaotic circuits," *IEEE Trans. Circuits Syst.-II* **57**, 730–734.

Prasad, A. [2015] "Existence of perpetual points in nonlinear dynamical systems and its applications," *Int. J. Bifurcation and Chaos* **25**, 1530005-1–10.

Rowlands, G. & Sprott, J. C. [1992] "Extraction of dynamical equations from chaotic data," *Physica D* **58**, 251–259.

Rowlands, G. & Sprott, J. C. [2008] "A simple diffusion model showing anomalous scaling," *Phys. Plasmas* **15**, 082308.

Shekofteh, Y., Jafari, S., Sprott, J. C., Golpayegani, S. M. R. H. & Almasganj, F. [2015] "A Gaussian mixture model based cost function for parameter estimation of chaotic biological systems," *Commun. Nonlin. Sci. Numer. Simul.* **20**, 469–481.

Sprott, J. C. [1981] *Introduction to Modern Electronics* (John Wiley & Sons).

Sprott, J. C. [1992] "Simple programs create 3D images," *Comp. Phys.* **6**, 132–138.

Sprott, J. C. [1993a] "Automatic generation of strange attractors," *Comp. Graph.* **17**, 325–332.

Sprott, J. C. [1993b] "How common is chaos?" *Phys. Lett. A* **173**, 21–24.

Sprott, J. C. [1993c] *Strange Attractors*: *Creating Patterns in Chaos* (M&T Books, NY).

Sprott, J. C. [1994a] "Predicting the dimension of strange attractors," *Phys. Lett. A* **192**, 355–360.

Sprott, J. C. [1994b] "Some simple chaotic flows," *Phys. Rev. E* **50**, R647.

Sprott, J. C. & Pickover, C. A. [1995] "Automatic generation of general quadratic map basins," *Comp. Graph.* **19**, 309–313.

Sprott, J. C. [1996] "Strange attractor symmetric icons," *Comp. Graph.* **20**, 325–332.

Sprott, J. C. & Pickover, C. A. [1997] "Scientific visualization in mathematics and physics," *Interactions* **4**, 78–79.

Sprott, J. C. [1997a] "Simplest dissipative chaotic flow," *Phys. Lett. A* **228**, 271–274.

Sprott, J. C. [1997b] "Some simple chaotic jerk functions," *Am. J. Phys.* **65**, 537–543.

Sprott, J. C. [1998] "Artificial neural net attractors," *Comp. Graph.* **22**, 143–149.

Sprott, J. C. [2000a] "A new class of chaotic circuit," *Phys. Lett. A* **266**, 19–23.

Sprott, J. C. [2000b] "Simple chaotic systems and circuits," *Am. J. Phys.* **68**, 758–763.

Sprott, J. C. [2001] "Can a computer produce and critique art?" *Leonardo* **34**, 369–369.

Sprott, J. C. & Rowlands, G. [2001] "Improved correlation dimension calculation," *Int. J. Bifurcation and Chaos* **11**, 1865–1880.

Sprott, J. C., Bolliger, J. & Mladenoff, D. J. [2002] "Self-organized criticality in forest-landscape evolution," *Phys. Lett. A* **297**, 267–271.

Sprott, J. C. [2003] *Chaos and Time-Series Analysis*, Vol. 69 (Oxford University Press, Oxford).

Sprott, J. C. [2004a] "Can a monkey with a computer create art," *Nonlin. Dyn. Psychol. Life Sci.* **8**, 103–114.

Sprott, J. C. [2004b] "Competition with evolution in ecology and finance," *Phys. Lett. A* **325**, 329–333.

Sprott, J. C. [2004c] "Dynamical models of love," *Nonlin. Dyn. Psychol. Life Sci.* **8**, 303–314.

Sprott, J. C. [2005] "Dynamical models of happiness," *Nonlin. Dyn. Psychol. Life Sci.* **9**, 23–36.

Sprott, J. C., Vano, J. A., Wildenberg, J. C., Anderson, M. B. & Noel, J. K. [2005a] "Coexistence and chaos in complex ecologies," *Phys. Lett. A* **335**, 207–212.

Sprott, J. C., Wildenberg, J. C. & Azizi, Y. [2005b] "A simple spatiotemporal chaotic Lotka–Volterra model," *Chaos Solit. Fract.* **26**, 1035–1043.

Sprott, J. C. [2006a] "Complex behavior of simple systems," *Unifying Themes in Complex Systems*, pp. 3–11.

Sprott, J. C. [2006b] "High-dimensional dynamics in the delayed Hénon map," *Electron. J. Theor. Phys.* **3**, 19–35.

Sprott, J. C. & Chlouverakis, K. E. [2007] "Labyrinth chaos," *Int. J. Bifurcation and Chaos* **17**, 2097–2108.

Sprott, J. C. [2007a] "Maximally complex simple attractors," *Chaos* **17**, 033124.

Sprott, J. C. [2007b] "A simple chaotic delay differential equation," *Phys. Lett. A* **366**, 397–402.

Sprott, J. C. [2008a] "Predator–prey dynamics for rabbits, trees, and romance," *Unifying Themes in Complex Systems IV* (Springer), pp. 231–238.

Sprott, J. C. [2008b] "Simple models of complex chaotic systems," *Am. J. Phys.* **76**, 474–480.

Sprott, J. C. [2009a] "Anti-Newtonian dynamics," *Am. J. Phys.* **77**, 783–787.

Sprott, J. C. [2009b] "Simplifications of the Lorenz attractor," *Nonlin. Dyn. Psychol. Life Sci.* **13**, 271.

Sprott, J. C. [2010] *Elegant Chaos: Algebraically Simple Chaotic Flows* (World Scientific, Singapore).

Sprott, J. C. [2011a] "Chaos in easter island ecology," *Nonlin. Dyn. Psychol. Life Sci.* **15**, 445–454.

Sprott, J. C. [2011b] "A new chaotic jerk circuit," *IEEE Trans. Circuits Syst.-II* **58**, 240–243.

Sprott, J. C. [2011c] "A proposed standard for the publication of new chaotic systems," *Int. J. Bifurcation and Chaos* **21**, 2391–2394.

Sprott, J. C. [2012] "Spatiotemporal chaos in easter island ecology," *Nonlin. Dyn. Psychol. Life Sci.* **16**, 387–395.

Sprott, J. C. [2013] "Is chaos good for learning," *Nonlin. Dyn. Psychol. Life Sci.* **17**, 223–232.

Sprott, J. C., Wang, X. & Chen, G. [2013] "Coexistence of point, periodic and strange attractors," *Int. J. Bifurcation and Chaos* **23**, 1350093-1–5.

Sprott, J. C. & Li, C. [2014] "Comment on how to obtain extreme multistability in coupled dynamical systems," *Phys. Rev. E* **89**, 066901.

Sprott, J. C., Hoover, W. G. & Hoover, C. G. [2014] "Heat conduction, and the lack thereof, in time-reversible dynamical systems: Generalized Nosé–Hoover oscillators with a temperature gradient," *Phys. Rev. E* **89**, 042914.

Sprott, J. C. [2014a] "A dynamical system with a strange attractor and invariant tori," *Phys. Lett. A* **378**, 1361–1363.

Sprott, J. C. [2014b] "Simplest chaotic flows with involutional symmetries," *Int. J. Bifurcation and Chaos* **24**, 1450009-1–9.

Sprott, J. C. & Xiong, A. [2015] "Classifying and quantifying basins of attraction," *Chaos* **25**, 083101.

Sprott, J. C., Jafari, S., Pham, V.-T. & Hosseini, Z. S. [2015] "A chaotic system with a single unstable node," *Phys. Lett. A* **379**, 2030–2036.

Sprott, J. C. [2015a] "New chaotic regimes in the Lorenz and Chen systems," *Int. J. Bifurcation and Chaos* **25**, 1550033-1–7.

Sprott, J. C. [2015b] "Strange attractors with various equilibrium types," *Eur. Phys. J.* **224**, 1409–1419.

Sprott, J. C. [2015c] "Symmetric time-reversible flows with a strange attractor," *Int. J. Bifurcation and Chaos* **25**, 1550078-1–7.

Sprott, J. C. & Li, C. [2017] "Asymmetric bistability in the Rössler system," *Acta Phys. Polon. B* **48**, 97–107.

Sprott, J. C., Jafari, S., Khalaf, A. J. M. & Kapitaniak, T. [2017] "Megastability: Coexistence of a countable infinity of nested attractors in a periodically-forced oscillator with spatially-periodic damping," *Eur. Phys. J.* **226**, 1979–1985.

Sun, K. & Sprott, J. C. [2009a] "Dynamics of a simplified Lorenz system," *Int. J. Bifurcation and Chaos* **19**, 1357–1366.

Sun, K. & Sprott, J. C. [2009b] "A simple jerk system with piecewise exponential nonlinearity," *Int. J. Nonlin. Sci. Numer. Simul.* **10**, 1443–1450.

Sun, K. & Sprott, J. C. [2010] "Periodically forced chaotic system with signum nonlinearity," *Int. J. Bifurcation and Chaos* **20**, 1499–1507.

Sun, K., Wang, X. & Sprott, J. C. [2010] "Bifurcations and chaos in fractional-order simplified Lorenz system," *Int. J. Bifurcation and Chaos* **20**, 1209–1219.

Sun, K., Liu, X., Zhu, C. & Sprott, J. C. [2012] "Hyperchaos and hyperchaos control of the sinusoidally forced simplified Lorenz system," *Nonlin. Dyn.* **69**, 1383–1391.

Suwannasen, O., Allen, M. A. & Sprott, J. C. [2016] "The speed of reaction–diffusion fronts on fractals: Testing the Campos–Méndez–Fort formula," *ScienceAsia* **42**, 33–39.

Tabatabaei, S. S., Yazdanpanah, M. J., Jafari, S. & Sprott, J. C. [2014] "Extensions in dynamic models of happiness: Effect of memory," *Int. J Happiness Dev.* **1**, 344–356.

Taylor, R. P. & Sprott, J. C. [2008] "Biophilic fractals and the visual journey of organic screen-savers," *Nonlin. Dyn. Psychol. Life Sci.* **12**, 117–129.

Vano, J. A., Wildenberg, J. C., Anderson, M. B., Noel, J. K. & Sprott, J. C. [2006] "Chaos in low-dimensional Lotka–Volterra models of competition," *Nonlinearity* **19**, 2391.

Warren, K., Sprott, J. C. & Hawkins, R. C. [2002] "The spirit is willing: Nonlinearity, bifurcations, and mental control," *Nonlin. Dyn. Psychol. Life Sci.* **6**, 55–70.

Warren, K., Hawkins, R. C. & Sprott, J. C. [2003] "Substance abuse as a dynamical disease: Evidence and clinical implications of nonlinearity in a time series of daily alcohol consumption," *Addict. Behav.* **28**, 369–374.

Watts, C., Newman, D. E. & Sprott, J. C. [1994] "Chaos in reversed-field-pinch plasma simulation and experiment," *Phys. Rev. E* **49**, 2291.

Wei, Z., Sprott, J. C. & Chen, H. [2015] "Elementary quadratic chaotic flows with a single non-hyperbolic equilibrium," *Phys. Lett. A* **379**, 2184–2187.

Wei, Z., Moroz, I., Wang, Z., Sprott, J. C. & Kapitaniak, T. [2016] "Dynamics at infinity, degenerate Hopf and zero-Hopf bifurcation for Kingni–Jafari system with hidden attractors," *Int. J. Bifurcation and Chaos* **26**, 1650125-1–16.

Wei, Z., Moroz, I., Sprott, J. C., Wang, Z. & Zhang, W. [2017] "Detecting hidden chaotic regions and complex dynamics in the self-exciting homopolar disc dynamo," *Int. J. Bifurcation and Chaos* **27**, 1730008-1–19.

Wildenberg, J. C., Vano, J. A. & Sprott, J. C. [2006] "Complex spatiotemporal dynamics in Lotka–Volterra ring systems," *Ecol. Compl.* **3**, 140–147.

Xiong, A., Sprott, J. C., Lyu, J. & Wang, X. [2017] "3D printing — The basins of tristability in the Lorenz system," *Int. J. Bifurcation and Chaos* **27**, 1750128-1–5.

Zeraoulia, E. & Sprott, J. C. [2011] "Robustification of chaos in 2D maps," *Adv. Compl. Syst.* **14**, 817–827.

Zeraoulia, E. & Sprott, J. C. [2012] "About structural stability of 3D quadratic mappings," *Nonlin. Stud.* **20**, 9–15.

Zhdankin, V. & Sprott, J. C. [2010] "Simple predator–prey swarming model," *Phys. Rev. E* **82**, 056209.

Zhdankin, V. & Sprott, J. C. [2013] "Chaos in the planar two-body coulomb problem with a uniform magnetic field," *Ann. Rev. Chaos Th. Bif. Dyn. Syst.* **3**, 23–33.