First Name: SOLUTIONS *Last Name:* ________________ *Section:* ________

December 21, 1998

Physics 207

Final Exam

Print your name and section clearly on all nine pages. (If you do not know your section number, write your TA's name.) Show all work in the space immediately below each problem. Your final answer must be placed in the box provided. Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units wherever necessary, and the direction of vectors. Each problem is worth 25 points. In doing the problems, try to be neat. Check your answers to see that they have the correct dimensions (units) and are the right order of magnitudes. You are allowed two 8½ x 11" sheets of notes and no other references. The exam lasts exactly 2 hours.

(Do not write below)

SCORE:

Problem 1: JC

Problem 2: AD

Problem 3: PK

Problem 4: CZ

Problem 5: PK

Problem 6: MS

Problem 7: CZ

Problem 8: AD

TOTAL: __________

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1. A target is located 30 meters horizontally and 10 meters above you. You throw a ball toward the target at an angle of 45° from the horizontal.

   a. With what speed must you throw the ball in order to hit the target? *(15 pts.)*

$$x = v_{0x} t \Rightarrow t = \frac{x}{v_{0x}}$$

$$y = v_{0y} t - \frac{1}{2} g t^2$$

$$= v_{0y} \frac{x}{v_{0x}} - \frac{1}{2} g \frac{x^2}{v_{0x}^2}$$

$$= x - \frac{g x^2}{v_0^2}$$

$$v_0^2 (x - y) = g x^2$$

$$v_0 = x \sqrt{\frac{g}{x-y}} = 30 \sqrt{\frac{9.8}{30-10}} = 21$$

21 m/s

   b. For how much time is the ball in flight before it hits the target? *(10 pts.)*

$$t = \frac{x}{v_{0x}} = \frac{\sqrt{2}\, x}{v_0}$$

$$= \frac{\sqrt{2} \times 30}{21} = 2.02$$

2.02 s

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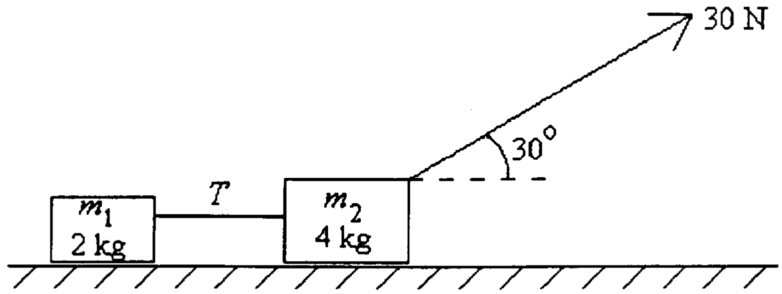
2. Two blocks connected by a massless string slide across a horizontal floor with a coefficient of kinetic friction $\mu = 0.4$ in response to a 30-N force as shown.



a. What is the acceleration of the blocks *(15 pts.)*

$$(m_1+m_2)a = F\cos\theta - \mu m_1 g - \mu(m_2 g - F\cos\theta)$$

$$a = \frac{F\cos\theta - \mu(m_1+m_2)g + \mu F\sin\theta}{m_1+m_2}$$

$$= \frac{30\cos30^\circ - 0.4(2+4)9.8 + 0.4\times30\sin30^\circ}{2+4}$$

$$= 1.41 \text{ to the right}$$

$1.41\ m/s^2$

b. What is the tension *T* in the string connecting the blocks? *(10 pts.)*

$$m_1 a = T - \mu m_1 g$$

$$T = m_1(a + \mu g)$$

$$= 2(1.41 + 0.4\times 9.8)$$

$$= 10.7$$

$10.7\ N$

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3. A bicyclist rides at a speed of 10 km/hr up a hill that rises 60 m in a distance of 1 km. Assume the bicycle plus person has a mass of 80 kg. Neglect energy losses due to friction.

a. How much work does the bicyclist do? *(5 pts.)*

$W = mgh = 80 \times 9.8 \times 60$

$= 47040$

47.0 kJ

b. What power does the bicyclist produce? *(5 pts.)*

$P = \frac{W}{t} = \frac{Wv}{L}$

$= \frac{47040}{1000} \frac{10^4}{3600} = 131$

131 W

c. If the pedals rotate once per second on a diameter of 36 cm, what average tangential force must the bicyclist produce on the pedals? *(5 pts.)*

$P = \bar{F} v = \bar{F} \frac{2\pi r}{T}$

$\bar{F} = \frac{PT}{2\pi r} = \frac{131}{2\pi \times 0.18} = 115$

115 N

d. For the conditions above, what average torque is applied to the pedals? *(5 pts.)*

$\bar{\tau} = \bar{F} r = 115 \times 0.18$

$= 20.8$

20.8 N·m

e. What minimum coefficient of friction between the wheels and the ground is required to keep the wheels from slipping? *(5 pts.)*

$P = F \cdot v = \mu N v = \mu m g v$

$\mu = \frac{P}{mgv} = \frac{131 \times 3600}{80 \times 9.8 \times 10^4}$

$= 0.06$

0.06

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4. Suppose all 5 x 10^9 human inhabitants of the Earth line up on the equator and begin walking westward at a speed of 2 m/s.

a. If the average mass of a person is 70 kg, what impulse is applied to the Earth? *(5 pts.)*

$$I = \Delta p = N m v$$
$$= 5 \times 10^9 \times 70 \times 2$$
$$= 7 \times 10^{11}$$

$$\boxed{7 \times 10^{11}\ N \cdot s}$$

b. If the Earth is a uniform sphere with mass M = 6 x 10^{24} kg, radius R = 6000 km, and moment of inertia $I = 0.4MR^2$, by what fraction would its rotational velocity change? *(10 pts.)*

$$\Delta L = R\Delta p = I\Delta\omega \Rightarrow \Delta\omega = \frac{R\Delta p}{I}$$
$$f = \frac{\Delta\omega}{\omega} = \frac{R\Delta p T}{I\, 2\pi} = \frac{R\Delta p T}{0.4MR^2\, 2\pi} = \frac{T\Delta p}{0.8\pi M R}$$
$$= \frac{24 \times 3600 \times 7 \times 10^{11}}{0.8\pi \times 6 \times 10^{24} \times 6 \times 10^{6}}$$
$$= 6.68 \times 10^{-16}$$

$$\boxed{6.68 \times 10^{-16}}$$

c. Now imagine that all the inhabitants of the Earth crowded together in a single location and simultaneously jumped upward, reaching an average height of 0.5 m. What velocity is imparted to the Earth? *(10 pts.)*

$$M\Delta v = N m v_0 = N m \sqrt{2gh}$$
$$\Delta v = \frac{Nm}{M}\sqrt{2gh}$$
$$= \frac{5 \times 10^9 \times 70}{6 \times 10^{24}}\sqrt{2 \times 9.8 \times 0.5}$$
$$= 1.83 \times 10^{-13} \quad \text{downward}$$

$$\boxed{1.83 \times 10^{-13}\ m/s}$$

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5. A meter stick with a density of 700 kg/m^3 is lowered end on into a swimming pool filled with water.

a. How much of the meter stick is above the water when the meter stick reaches static equilibrium with the water? *(8 pts.)*

$$\rho_M A L = \rho_W A (L - \Delta L)$$

$$\rho_W L - \rho_M L = \rho_W \Delta L$$

$$\Delta L = \frac{\rho_W - \rho_M}{\rho_W} = \frac{1000 - 700}{1000}$$

$$= 0.3$$

0.3 m

b. If the meter stick is raised 10 cm above its equilibrium position and released, with what period does it oscillate? *(10 pts.)*

$$m \frac{d^2x}{dt^2} = -mg \frac{\rho_W}{\rho_M} \frac{x}{L}$$

$$\frac{d^2x}{dt^2} = -\frac{\rho_W g x}{\rho_M L} = -\omega^2 x$$

$$\omega = \sqrt{\frac{\rho_W g}{\rho_M L}}$$

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{\rho_M L}{\rho_W g}}$$

$$= 2\pi \sqrt{\frac{700}{1000 \times 9.8}} = 1.68$$

1.68 s

c. For the condition above, what is the maximum speed of oscillation of the meter stick? *(7 pts.)*

$$v = \omega x_0 = x_0 \sqrt{\frac{\rho_W g}{\rho_M L}} = 0.1 \sqrt{\frac{1000 \times 9.8}{700}}$$

$$= 0.374$$

0.374 m/s

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6. In a search for planets orbiting distant stars, you observe a star 40 light years away (3.8×10^{17} m) that wobbles with a period of 80 (earth) days and an amplitude of 3×10^{-8} degrees. Assume the star has the same mass as the sun. $(1.99 \times 10^{30}\ kg)$

a. What is the distance of the planet from the star assuming it is in a circular orbit? *(6 pts.)*

$$G\frac{Mm}{r^2} = \frac{mv^2}{r} = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2$$

$$G\frac{M}{r} = \frac{4\pi^2 r^2}{T^2}$$

$$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}}$$

$$= \sqrt[3]{\frac{6.67\times10^{-11} \times 1.99\times10^{30}(80\times24\times3600)^2}{4\pi^2}} =$$

$$\boxed{5.44\times10^{10}\ m}$$

b. What is the mass of the planet? *(7 pts.)*

$$mr = M\theta d$$

$$m = M\frac{d}{r}\theta = 1.99\times10^{30}\ \frac{3.8\times10^{17}}{5.44\times10^{10}}\ \frac{3\times10^{-8}}{57.3}$$

$$= 7.28\times10^{27}$$

$$\boxed{7.28\times10^{27}\ kg}$$

c. If the intensity of heat radiation from the star reaching the Earth is 2×10^{-10} W/m^2, what is the intensity of heat radiation at the orbit of the planet? *(6 pts.)*

$$I = I_0\left(\frac{d}{r}\right)^2$$

$$= 2\times10^{-10}\left(\frac{3.8\times10^{17}}{5.44\times10^{10}}\right)^2$$

$$= 9760$$

$$\boxed{9760\ W/m^2}$$

d. What is the average surface temperature of the planet in degrees Celsius? *(6 pts.)*

$$eI\pi r^2 = \sigma A e T^4 = \sigma 4\pi r^2 e T^4$$

$$T = \sqrt[4]{\frac{I}{4\sigma}}$$

$$= \sqrt[4]{\frac{9760}{4\times5.67\times10^{-8}}} = 455K = 182°C$$

$$\boxed{182°C}$$

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7. A violin string with a mass of 0.5 g and a length of 35 cm produces a fundamental frequency of 440 Hz (concert A) when bowed.

a. What is the tension in the string? *(7 pts.)*

$$L = \frac{\lambda}{2} = \frac{1}{2}\frac{v}{f} = \frac{1}{2f}\sqrt{\frac{TL}{m}}$$

$$4f^2L^2 = \frac{TL}{m}$$

$$T = 4f^2Lm = 4 \times 440^2 \times 0.35 \times 5\times10^{-4}$$

$$= 136$$

136 N

b. If an observer moves toward the violin at a speed of 20 m/s, what frequency is heard? *(6 pts.)*

$$f = f_o\left(1 + \frac{v_o}{v}\right) = 440\left(1 + \frac{20}{343}\right)$$

$$= 466$$

466 Hz

c. If the tension in the string is increased by 1%, what beat frequency is heard by an observer at rest when the violin is played simultaneously with another violin that is in tune? *(6 pts.)*

$$\Delta f = f - f_o = f_o\frac{\Delta T}{2T}$$

$$= 440\frac{.01}{2} = 2.2$$

2.2 Hz

d. If the sound intensity is 60 dB at a distance of 5 m from the violin, what acoustical power does the violin produce, assuming it radiates equally in all directions? *(6 pts.)*

$$\beta = 10\log\left(\frac{I}{I_o}\right) = 10\log\left(\frac{P}{4\pi r^2 I_o}\right)$$

$$P = 4\pi r^2 I_o\, 10^{\beta/10}$$

$$= 4\pi \times 25 \times 10^{-10} \times 10^{6}$$

$$= 0.0314$$

31.4 mW

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8. A 100-g ice cube at 0° C is mixed with 200 cm^3 of water at 70° C and allowed to reach thermal equilibrium in a thermally insulated container.

a. What is the final temperature of the mixture? *(6 pts.)*

$$80\, m_I + m_I T = m_w (70 - T)$$

$$8000 + 100T = 200(70 - T)$$

$$300T = 14000 - 8000 = 6000$$

$$T = \frac{6000}{300} = 20$$

20° C

b. What is the change in entropy of the mixture? *(7 pts.)*

$$\Delta S = \Sigma \frac{\Delta Q}{T} = \frac{8000}{273} + 100 \int_{273}^{293} \frac{dT}{T} + 100 \int_{343}^{293} \frac{dT}{T}$$

$$= 29.304 + 100 \ln\left(\frac{293}{273}\right) + 200 \ln\left(\frac{293}{343}\right)$$

$$= 4.86 \text{ cal}/K = 20.4 \text{ J}/K$$

20.4 J/K

c. How many joules of energy must be added to the mixture to heat it to 100° C and vaporize it? *(6 pts.)*

$$Q = 300 \times (80 + 540)$$

$$= 1.86 \times 10^5 \text{ cal}$$

$$= 7.79 \times 10^5 \text{ J}$$

779 kJ

d. Assuming the water vapor at 100° C is an ideal gas with a density of 18 g/mole, what volume will it occupy at a pressure of 1 x 10^5 Pa? *(6 pts.)*

$$PV = nRT$$

$$V = \frac{nRT}{P} = \frac{300}{18} \frac{8.31 \times 373}{10^5}$$

$$= 0.517$$

0.517 m^3