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OFF-RESONANCE HEATING OF MIRROR CONFINED PLASMAS

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Abstract—Experiments are described in which microwave power above or below the electron-cyclotron frequency is applied to mirror confined plasmas. Above resonance, a finite heating rate is measured. Below resonance, the plasmas are expelled. The upper off-resonance heating rate and the lower off-resonance power required to expel the plasma agree with the predictions of a proposed theoretical model in which the spatial variation of the microwave electric field of the multimode cavity randomizes the electron motion.

EXPERIMENTS by DANDL *et al.* (1969) at Oak Ridge first showed that microwave heating above the electron-cyclotron frequency can be very effective, while heating below the cyclotron frequency causes enhanced axial diffusion. The upper off-resonance heating (UORH) is in qualitative agreement with a theory of GRAWE (1968) that predicts strong off-resonance heating at relativistic (~1 MeV) energies. The lower off-resonance heating (LORH) results have remained unexplained. UORH has also been observed by SPROTT (1971) in a levitated toroidal octupole at Wisconsin, but LORH produces no enhanced loss in an octupole with a superimposed toroidal magnetic field. Off-resonance heating is a useful technique because, (1) it provides a means of heating dense plasmas in which waves near the cyclotron frequency do not propagate, (2) the heating is more uniform, rather than localized in regions of cyclotron resonance, and (3) the anisotropy can be controlled, thereby suppressing a variety of anisotropy driven instabilities.

We present here the results of experiments using a small mirror ratio ($R = 1{\cdot}02$ or $1{\cdot}19$) device described elsewhere by SHOHET (1968). The plasmas are relatively cold ($kT_e \sim 10$–100 eV) and tenuous ($\sim 10^9$ cm^{-3}). Microwave sources are 3·0 GHz, 50 W, *cw* and 8·54 GHz, 100 kW, 2 μsec pulses at 360 pulses/sec. The magnetic field strength can be adjusted for resonance at either frequency. The plasma can thus be produced by a low power, *cw* source at resonance and heated with a high power, pulsed, UORH source, or produced by a high power, pulsed source at resonance and expelled with a low power, *cw*, LORH source. The vacuum (and microwave) cavity is 15 cm in dia. and 60 cm long, and is therefore excited in a high order mode by the microwaves.

Observed UORH rates are much larger than predicted by GRAWE (1968) and agree well with a proposed theoretical model in which the inverse transit time of an electron through a microwave cavity mode plays the role of a collision frequency. The expulsion of the plasma with LORH is explained by enhanced axial diffusion resulting from the parallel heating which dominates the perpendicular heating at sufficiently low frequencies in a multimode cavity.

The heating rate for a cold, tenuous, uniform plasma in a uniform magnetic

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field can be calculated from Ohm's law and is given by

$$\frac{\mathrm{d}\overline{W}}{\mathrm{d}t} = \frac{e^2 E_\perp{}^2 \nu(\omega^2 + \omega_c{}^2 + \nu^2)}{m[(\omega^2 - \omega_c{}^2 - \nu^2)^2 + 4\omega^2\nu^2]} + \frac{e^2 E_\parallel{}^2 \nu}{m(\omega^2 + \nu^2)}, \tag{1}$$

where $E_\perp (E_\parallel)$ is the component of the *rf* electric field perpendicular (parallel) to **B**, ω is the applied *rf* frequency, ω_c is the electron-cyclotron frequency, and ν is the collision frequency. For $\omega = \omega_c$ and $\nu \ll \omega$, electron cyclotron resonance heating (ECRH) occurs, and for a non-uniform field, the heating rate was determined by SPROTT (1971) by integrating equation (1) over the resonance to get the result

$$\frac{\mathrm{d}\overline{W}}{\mathrm{d}t} = \frac{e^2 E_\perp{}^2}{m\omega} G, \tag{2}$$

where G is a factor of order unity that depends on the geometry of the field and on the density distribution. For $\omega^2 \gg \omega_c{}^2$ (UORH), equation (1) becomes

$$\frac{\mathrm{d}\overline{W}}{\mathrm{d}t} = \frac{e^2 E^2 \nu}{m(\omega^2 + \nu^2)}, \tag{3}$$

which is the usual result for collisional heating in the absence of a magnetic field. The effective collision frequency ν is the inverse correlation time between the electron gyration and the *rf* electric field, and in general consists of terms resulting from real collisions, the phase incoherence of the *rf* field, and the incoherence of the electron motion relative to the *rf* field.

In the present experiment, electron-neutral collisions dominate above $\sim 10^{-3}$ torr, and in this range the measured heating rate is proportional to neutral pressure and is within a factor of two of the value calculated from published collision cross section data for helium. At lower pressures, the transit of an electron through a cavity mode is expected to dominate ($\nu = \bar{v}_\parallel \omega/c$), provided the turning point of the electron or the cavity mode structure fluctuates. In a multimode cavity, the electric field has a complicated spatial variation with an average standing wavenumber of $\sim\omega/c$. If the particle's bounce motion were perfectly periodic and if the mode structure remained fixed in time, the resulting energy change would be periodic and no heating would result. The *rf* field, however, changes $v_\perp/v_\parallel$ in a complicated way that depends on the phase of the particle's gyration and causes a fluctuation in the turning point which destroys the simple periodic sampling of the mode structure and leads to a stochastic heating. Furthermore, density changes alter the mode pattern and further destroy the correlation. The calculated heating rate (generalized to relativistic energies) is

$$\frac{\mathrm{d}\overline{W}}{\mathrm{d}t} \simeq \frac{e^2 E^2}{\gamma^2 m\omega} \frac{\bar{v}_\parallel}{c} \simeq \frac{e^2 E^2}{m\omega} \left(\frac{R_T - 1}{2}\right)^{1/2} \frac{(\gamma^2 - 1)^{1/2}}{\gamma^3}, \tag{4}$$

where $\gamma = 1 + W_\perp(0)/mc^2$, $W_\perp(0)$ is the perpendicular electron energy at the midplane, and R_T is the ratio of the magnetic field strength at the turning point to the field at the midplane. The UORH rate is thus comparable to the ECRH rate for relativistic ($v \simeq c$) plasmas.

Absorption at harmonics of the cyclotron frequency can occur with hot plasmas, but the relative heating rate of the Nth harmonic is of order $(\bar{v}_\perp{}^2/c^2)^{N-1}$. Since the proposed heating rate scales like $\bar{v}_{\|}/c$, the harmonic effects are unimportant for relatively cold plasmas. Furthermore, for relativistic electrons, the resonances are highly damped since $\nu \simeq \omega$.

The prediction of equation (4), along with the experimental measurements are shown in Fig. 1. The heating rates were determined by dividing the signal from a diamagnetic loop surrounding the plasma by the average density as measured with a Langmuir probe. The probe results were checked against density measurements using microwave cavity perturbation techniques. The magnitude and scaling with energy (as measured with a Langmuir probe) are consistent with the prediction, although a heating rate proportional to energy would also fit the data. Computer simulation calculations of the heating for these conditions are also being done and give results consistent with those presented here. Anomalous heating could occur in the presence of an energetic electron component which would not be observed with the probe. X-ray measurements did not indicate the existence of any such electrons, however.

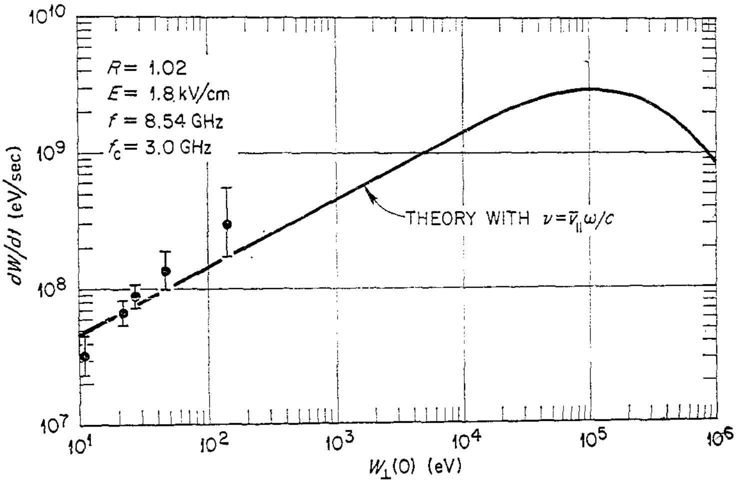

FIG. 1.—Comparison of calculated and measured upper off-resonance heating rates.

GRAWE (1968) assumed loss of correlation after an electron bounce time but derived a heating rate that is strongly energy dependent, such that essentially no heating occurs below ~10 keV. For relativistic energies ($\gtrsim$1 MeV), the predictions are similar, and so both theories are consistent with the Oak Ridge experiments. Grawe's theory, when applied to the small mirror ratio experiment described here, predicts a heating rate many orders of magnitude smaller than either the experimental measurements or the present theory, even at relativistic energies.

For arbitrary frequency, we define a heating anisotropy

$$A = E_{\|}^2 \frac{\mathrm{d}\bar{W}_{\perp}}{\mathrm{d}t} \Big/ E_{\perp}^2 \frac{\mathrm{d}\bar{W}_{\|}}{\mathrm{d}t} = \frac{(\omega^2 + \omega_c^2 + \nu^2)(\omega^2 + \nu^2)}{(\omega^2 - \omega_c^2 - \nu^2)^2 + 4\omega^2\nu^2}, \tag{5}$$

such that $A = 1$ represents isotropic heating. In a multimode cavity, $E_{\|}^2 \sim E_{\perp}^2$ on the average. A is plotted in Fig. 2 vs. ω/ω_c for various values of ν. The theory requires $\nu < \omega$, and in the present experiments $\nu \lesssim 10^{-3}\ \omega$. For $\omega \gtrsim \omega_c$, the

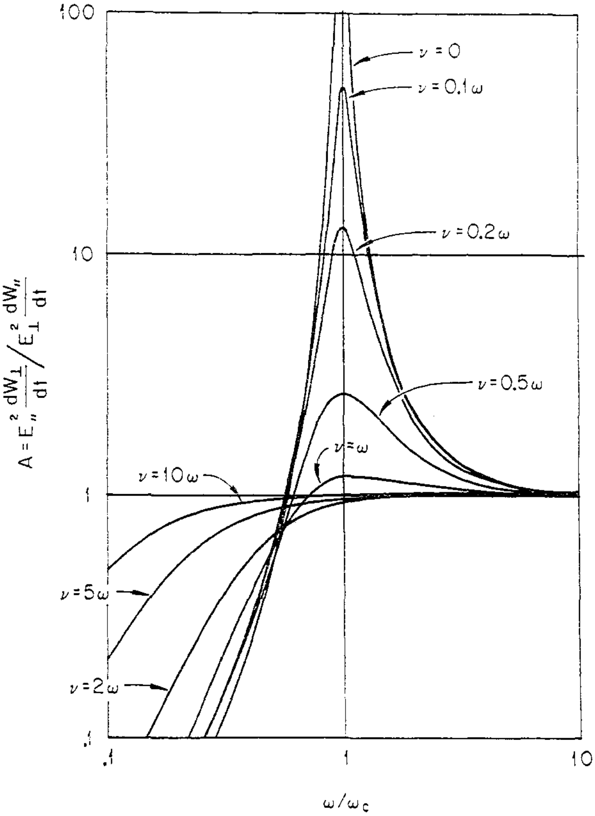

FIG. 2.—Heating anisotropy vs. frequency.

perpendicular heating dominates, but for $\omega < \omega_c/\sqrt{3}$, the parallel heating dominates, producing enhanced axial diffusion.

The experimental results for LORH are summarized in Fig. 3. The axial, integrated X-ray flux increases with increasing off-resonance power until the plasma is finally extinguished, while the density in the afterglow and the power absorbed by the plasma during the resonance heating pulse decrease. The observations are consistent with enhanced axial losses as predicted. The power required to expel the plasma depends on microwave frequency and on position and orientation of the coupling loop, presumably because the ratio of $E_\parallel^2/E_\perp^2$ changes for this relatively low order mode (cavity radius $\sim$ LORH wavelength).

Without LORH, the radial density profile as measured by a Langmuir probe is peaked off axis. The plasma decays smoothly near the axis, but large ($\delta n/n \sim e\delta V_f/kT_e \sim 100\%$), low frequency (1–10 kHz), $k_\parallel = 0$ fluctuations appear at the boundary. As LORH is applied, the radial profile is unchanged, but similar fluctuations appear in the originally quiescent region near the axis. These fluctuations have $\delta E_\parallel \ll \delta E_\perp$, and so appear to be flute instabilities. The fluctuations are accompanied by enhanced radial loss as measured by an electrostatic particle collector and by X-ray detectors. The radial flux is always small compared with the axial flux, and probably results from an instability caused by a change in the

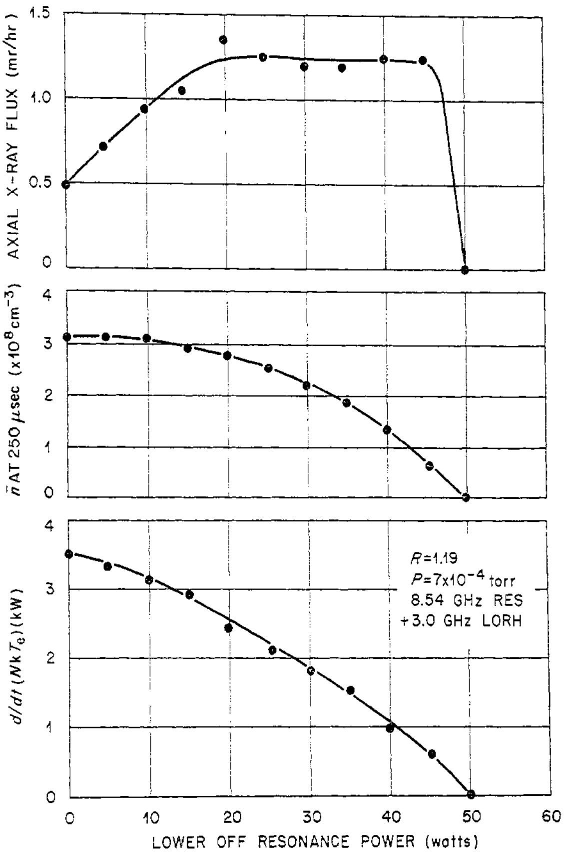

FIG. 3.—Axial X-ray flux, average electron density, and heating rate observed with a 2 μsec, 100 kW, resonance heating pulse with *cw* lower off-resonance heating.

axial density profile. Probe measurements show that the axial density is peaked near the midplane with no LORH, but moves toward the mirror throats with increasing LORH power.

A similar experiment was performed in a different magnetic mirror device ($R = 3{\cdot}8$) using a 3·25 GHz, 10 kW, 10 μsec, 120 pulses/sec, resonant source and a 10 MHz, 1 kW, *cw* LORH source. The plasma is easily expelled when the 10 MHz electric field is applied parallel to **B**, but is not affected when the same field is perpendicular to **B**.

LORH should expel the plasma when the ionization time τ_i exceeds the time required for an electron to diffuse into the loss cone as a result of the parallel heating:

$$\frac{\mathrm{d}\overline{W}_{\parallel}}{\mathrm{d}t} = \frac{W_{\perp}(O)(R-1)}{2\tau_i} \simeq \frac{e^2 E_{\parallel}{}^2 \nu}{m\omega^2} = \frac{e^2 P_{OR} Q \nu}{3\epsilon_0 m \omega^3 V}.$$

Solving for P_{OR} gives

$$P_{OR} \simeq \frac{3\epsilon_0 m\omega^3 V W_{\perp}(O)(R-1)}{2e^2 Q \nu \tau_i}. \tag{6}$$

Taking $Q = 600$, $R = 1{\cdot}19$, $\tau_i = 20\ \mu$sec, $W_\perp(O) = 25$ eV, and $V = 7{\cdot}7 \times 10^3$ cm^3, and assuming that the collision frequency measured for UORH scales as predicted for the LORH case ($\nu \simeq 3{\cdot}1 \times 10^7$ sec^{-1}), the calculated power is ~50 watts in agreement with measurements (Fig. 3).

For the Oak Ridge case, total absorption of both the resonance and LORH power is expected and observed, and so the resulting anisotropy should be

$$\frac{\overline{W}_\perp}{\overline{W}_\parallel} = \frac{P_R + P_{OR}(\perp)}{P_{OR}(\parallel)} = \frac{P_R + AP_{OR}/(A+1)}{P_{OR}/(A+1)}.$$

The plasma is expelled if $\overline{W}_\perp/\overline{W}_\parallel < (R+1)/(R-1)$, and so the required power is

$$P_{OR} = \frac{(R-1)(A+1)}{(R+1) - A(R-1)} P_R \simeq \frac{R-1}{R+1} P_R. \qquad (7)$$

For $R = 2$, the required LORH power is about $\frac{1}{3}$ of the resonance power in agreement with observations.

The model strictly applies only to uniform magnetic fields, although for frequencies well above or below resonance everywhere in a non-uniform field, the results should hold. In a mirror field, the parallel heating should have a resonance at the bounce frequency, causing enhanced diffusion of electrons of a certain energy.

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