Effect of magnetic field errors on confinement in bumpy tori

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(Received 14 August 1972; final manuscript received 30 January 1973)

Numerical calculations of field line trajectories and drift orbits show that small field perturbations can destroy the single particle confinement in bumpy tori. Criteria are derived for tolerable field errors assuming neoclassical diffusion.

The proposal to build a high-beta, electron cyclotron heated bumpy torus at Oak Ridge (ELMO Bumpy Torus) has prompted us to consider the effect of magnetic field errors on single-particle confinement and on collisional diffusion in bumpy tori. Bumpy tori (without a poloidal field) share with toroidal multipoles a high sensitivity to field errors because of the requirement of field line closure. The nonaxisymmetry in bumpy tori leads to drift orbits very different from those in multipoles, however, and the effect of field errors on these orbits is different from the effect of errors on the field lines themselves. Confined field lines do not necessarily lead to confined drift surfaces and, conversely, confined drift surfaces can exist even when all the field lines intersect the wall.

The drift trajectory in a bumpy torus consists of a vertical drift due to the toroidal curvature and a precessional drift due to the mirror curvature. If the mirror curvature dominates the toroidal curvature (large mirror ratio and/or large aspect ratio), the drift orbits lie on toroidal surfaces that are displaced toward the major axis. The mirror precession thus produces single-particle confinement and equilibrium in a manner analogous to a rotational transform. There always exist regions of velocity space where the mirror drifts vanish, however, and the drift orbits intersect the wall. This region resembles the loss cone in a mirror except that the particles are lost on the slower drift time scale.

The prime vehicle for investigating the effect of field errors in a bumpy torus has been a computer code that calculates the trajectory of a field line and of a typical, nonrelativistic, drift orbit in an idealized, low-beta, perturbed bumpy torus with 24 bumps and a 2:1 mirror ratio. The unperturbed magnetic field is given by

$$B_\theta=\tfrac{3}{2}-(\tfrac{1}{2}+\tfrac{1}{8}r^2)\cos 24\theta,$$

$$B_R=[R_0(R_0-R)/R](\tfrac{1}{4}+\tfrac{1}{32}r^2)\sin 24\theta,$$

$$B_Z=-(R_0Z/R)(\tfrac{1}{4}+\tfrac{1}{32}r^2)\sin 24\theta,$$

where the coordinate system is defined in Fig. 1. This field satisfies $\nabla\cdot\mathbf{B}=0$ exactly and $\nabla\times\mathbf{B}=0$ to within a few percent in the region of interest.

The field line trajectory is determined by numerically integrating the three-dimensional differential equation for a field line

$$d\mathbf{l}/dl=\mathbf{B}/B,$$

where l is an element of length parallel to $\mathbf{B}$ and $B=|\mathbf{B}|$. The integration scheme is a third-order Runge–Kutta and the step size is $\Delta l=1.67\times10^{-3}\ R_0$, which causes an unperturbed field line to close on itself to within a distance of $<10^{-5}\ R_0$ after one transit around the major axis.

The drift orbit of a particle with no gyrational velocity was calculated in the same way by adding a curvature drift to the above equation

$$\frac{d\mathbf{l}}{dl}=\mathbf{B}/B+\mathbf{B}\times(\mathbf{B}\cdot\nabla)\mathbf{B}v_{||}/\omega_cB^3,$$

where $\omega_c=eB/m$, and $v_{||}$ is the component of the thermal velocity ($\mathbf{v}$) parallel to $\mathbf{B}$. The case chosen represents a particle that is more strongly affected by field errors than most, but less affected than an almost trapped particle. Trapped particles are relatively insensitive to field errors. A 250-keV electron (or 6-keV proton) was chosen so as to execute one precession in about 17 transits around the major axis for a midplane field of 5 kG. The field line and drift trajectory were followed for 40 transits around the major axis, during which time the particle precesses about three orbits in the minor direction. The coordinates of the field line and particle are graphically plotted at every crossing of a midplane of a mirror section. For a 24-bump torus, this gives $\sim$1000 points along the trajectory of the particle.

A plot of the two trajectories is shown in Fig. 2. The plot shows a minor cross section of the torus in a mirror midplane with distances normalized to $R_0=24$. The major axis is on the left at $r=-24$. The field line closes and appears on the graph only as a square data point at the initial position. The drift orbit maps out a closed surface that is displaced toward the inside wall as expected.[1] Each datum point indicates one complete transit around the major axis. The code also calculates $\oint dl$ between midplane crossings, and the particle is observed to drift on a surface of constant $\oint dl$ to within the step size of the iteration, implying that the second adiabatic invariant, $J(=\oint v_{||}\,dl=v\oint dl)$, is conserved to better than 1%.

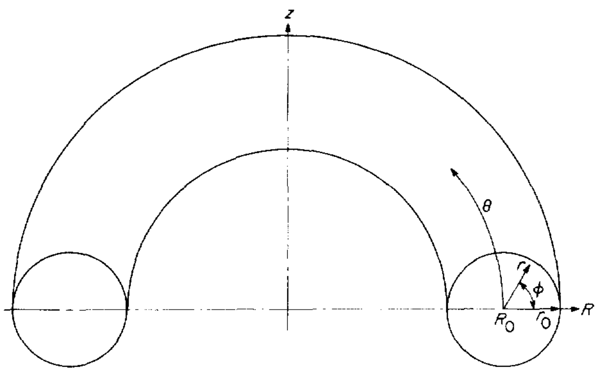

FIG. 1. Coordinate systems used for describing bumpy torus fields.

Nearly a hundred different perturbations have been examined, but only two will be described here. Figure 3 shows the effect of a uniform vertical magnetic field with a magnitude 0.04% of the midplane field. The field line spirals downward with a pitch of

$$\Delta Z = \oint \Delta B_z \, dl/B,$$

but the drift orbit remains closed and is shifted horizontally by an amount

$$\Delta x \cong (r_0 R_0 \omega_c / v)(\Delta B_z / \langle B \rangle),$$

where v is the thermal velocity. If we require that Δx be much less than the minor radius r_0, then such a field error is tolerable if

$$\Delta B_z / \langle B \rangle \ll v / R_0 \omega_c.$$

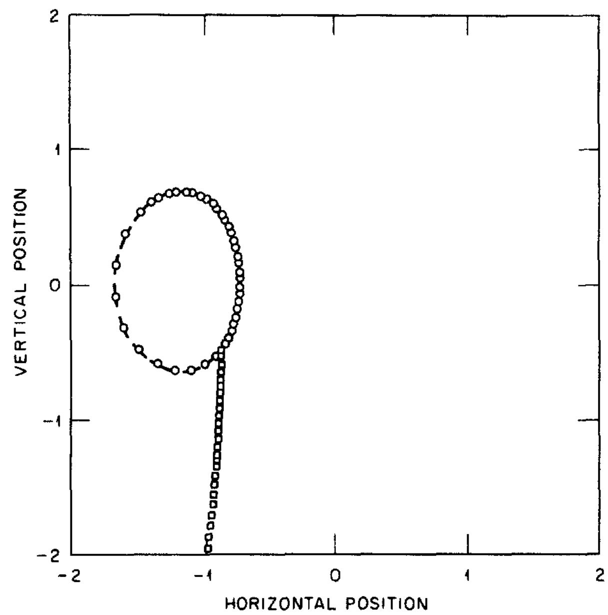

FIG. 3. Bumpy torus with a uniform vertical perturbation of 0.04%. Drift surface (○) is confined but the field lines (□) are unconfined.

A contrasting example arises when a rotational transform is added:

$$B_\phi \propto \ln(R_0/R) \cos\phi - (Z/R) \sin\phi.$$

Figure 4 shows that the field line is confined on a flux surface, but if the poloidal field is adjusted so that the transform causes a rotation opposite and nearly equal to the precession, the drift trajectory is unconfined.

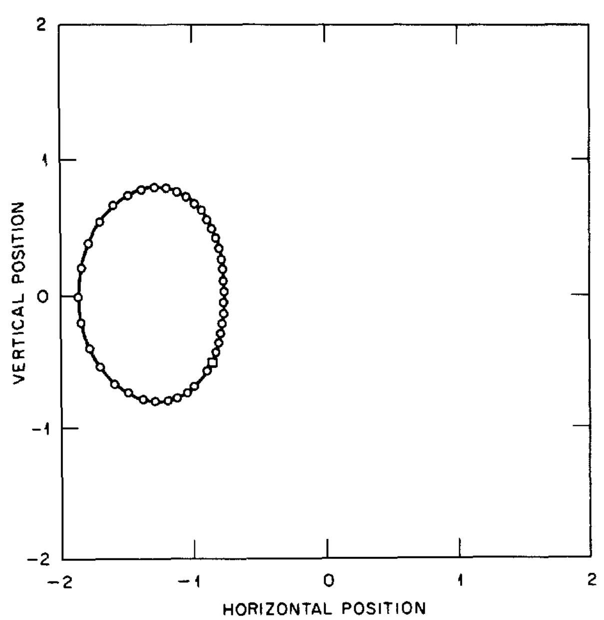

FIG. 2. Field line trajectory (□) and drift trajectory for $v_\perp = 0$ particle (○) in a minor cross section of an unperturbed bumpy torus. The major axis is at −24.

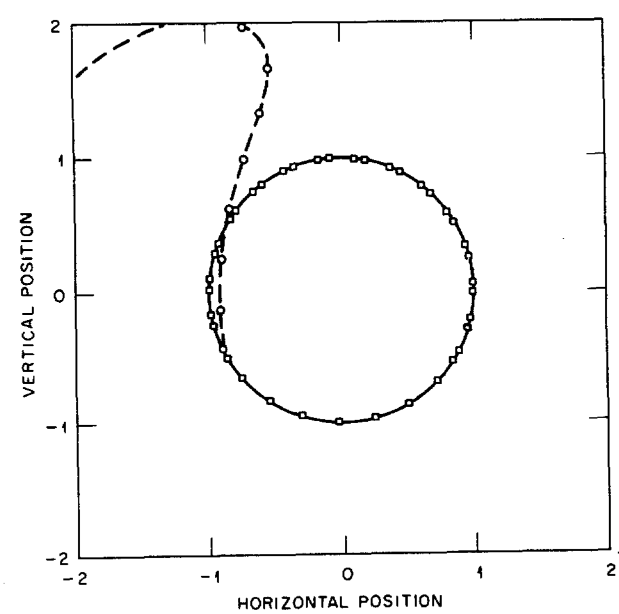

FIG. 4. Bumpy torus with a poloidal field produced by a weak $1/R$ toroidal plasma current. Field lines (□) are confined but the drift surface (○) is unconfined.

The neoclassical diffusion in an unperturbed bumpy torus has a different form in the three regimes[2]:

Collisional regime $(\nu > \rho\omega_c/2R_0)$:

$$D \cong \tfrac{1}{3}(\rho^2\nu),$$

Transition regime $(\rho^2\omega_c/3r_0^2 < \nu < \rho\omega_c/2R_0)$:

$$D \cong \rho^4\omega_c^2/6R_0^2\nu,$$

Collisionless regime $(\nu < \rho^2\omega_c/3r_0^2)$:

$$D \cong 3r_0^4\nu/2R_0^2,$$

where ν is the collision frequency and ρ is the gyroradius. In all three regimes, the diffusion coefficient can be written as the product of a mean-square effective step size times the collision rate. In the presence of an average field error $\langle \Delta B \rangle$, an untrapped particle will move in a collision time a distance

$$\xi = v_{||}\langle \Delta B \rangle / \langle B \rangle \nu$$

perpendicular to the unperturbed field. In the collisional regime, the net mean-square step size is $\rho^2 + \xi^2$ and the modified diffusion coefficient is

$$D \cong \frac{1}{3}\left(\rho^2 + \frac{v_{||}^2 \langle \Delta B \rangle^2}{\nu^2 \langle B \rangle^2}\right)\nu, \qquad (1)$$

a result previously obtained by Jernigan *et al.*[3] In this regime, a field error is tolerable if

$$\langle \Delta B \rangle / \langle B \rangle \ll \rho/\lambda,$$

where $\lambda (= v_{||}/\nu)$ is the mean free path. Actually, the enhanced diffusion is somewhat overestimated because only the untrapped particles contribute to the enhancement. Meade[4] has suggested that in the large mirror ratio limit, the diffusion due to field errors is reduced by a factor $1/M^2$, where M is the mirror ratio. One factor of M comes from the reduction of untrapped particles and the other from the reduction in scattering angle required to trap a particle.

In the transition regime, the same sort of calculation gives

$$D \cong (\rho^4\omega_c^2/6R_0^2\nu) + (v_{||}^2/3\nu)(\langle \Delta B \rangle^2/\langle B \rangle^2), \qquad (2)$$

so that a field error is tolerable if

$$\langle \Delta B \rangle / \langle B \rangle \ll \rho/R_0.$$

This result has a simple physical interpretation since it requires that in one transit around the torus the field lines close on themselves to within a gyroradius.

The collisionless regime is more difficult to calculate since the particle drift orbits are closed but displaced. However, as previously shown, a field error is tolerable if the drift surface is displaced by an amount much less than the minor radius of the torus

$$\langle \Delta B \rangle / \langle B \rangle \ll v/R_0\omega_c.$$

The effect of field errors is thus minimized at low energies by collisions [Eq. (1)] and at high energies by precession [Eq. (2)]. The worst case occurs at energies where the mean free path is the order of the major radius of the torus. In the ELMO Bumpy Torus with $R_0 = 153$ cm and $n = 10^{12}$ cm^{-3}, this condition occurs at about 40 eV for electrons and requires that average field errors be $\lesssim 10^{-5}\langle B \rangle$. This criterion is probably too conservative for several reasons (1) pessimistic values were assumed for the density and temperature; (2) ambipolar electric fields have been neglected; (3) only the untrapped particles are considered, and (4) the losses are assumed to be neoclassical. The steps that are being taken to minimize the errors in the ELMO Bumpy Torus will be described elsewhere.[5]

ACKNOWLEDGMENT

Research sponsored by the U.S. Atomic Energy Commission under Contract with the Union Carbide Corporation.

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[4]D. M. Meade (private communication).

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