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# Experimental Study of Axisymmetric Instability of Inverse-Dee and Square Tokamak Equilibria

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Inverse-dee– and square–shaped equilibria are observed by experimentally mapping the magnetic-flux plot as a function of time in a tokamak with a 4-null poloidal divertor. Inverse-dee equilibria are observed to be unstable to the vertical magnetohydrodynamic axisymmetric instability on a time scale ~1000 poloidal Alfvén times. Square equilibria are stable on the time scale available for observation. Instability growth is apparently slowed by field-shaping hoop and wall passive stabilization.

Tokamaks with noncircular cross sections (such as dee's and ellipses) are advantageous with respect to $q$-limited magnetohydrodynamic (MHD) modes. Noncircularity arises unavoidably in poloidal-divertor configurations. However, elongation introduces instability to axisymmetric displacements (with toroidal-mode number $n=0$) to which circles can be neutrally stable. The poloidally asymmetric placement of external currents necessary to establish a noncircular equilibrium, in turn, creates destabilizing forces on the plasma current. The importance of these modes has given rise to a fairly large amount of linear theory—mostly for idealized displacements of ideal analytic equilibria (see, for example, Ref. 1–4). Recently, nonlinear evolution of the instability has been followed numerically.[5,6] Axisymmetric displacement of dee and elliptical[7-9] plasmas has been deduced in a few previous experiments from magnetic probes external to the plasma. Plasma shapes have been inferred from equilibrium computer codes with use of external experimental signals as input.

Here we present the first direct experimental observation of the stability to axisymmetric modes of square– and inverse-dee–shaped equilibria in a 4-null poloidal-divertor configuration. The equilibria are verified by mapping the magnetic field in the plasma as a function of time. The stability of these equilibria to axisymmetric modes is determined by studying the evolution of these experimental flux plots. We have found that inverse-dee– and square–shaped equilibria experimentally exist. Inverse-dee equilibria are unstable, with growth times on the order of 1000 poloidal Alfvén times. The instability is slowed from the Alfvén MHD time scale by passive stabilization from the hoops and walls. Square equi-

 

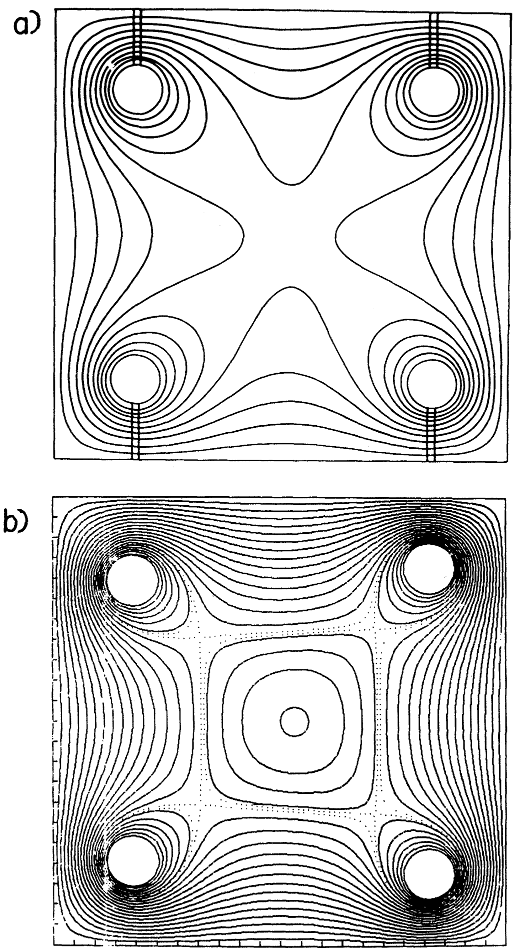


FIG. 1. Numerical flux plots (major axis to left). (a) Without plasma; (b) with plasma (hoops in case-B position). Adjacent tick marks represent a separation of 2 cm.

libria appear stable.

These experiments were performed on the University of Wisconsin Tokapole. The Tokapole has a vacuum-magnetic flux plot of an octupole [Fig. 1(a)] which provides vertical and horizontal fields to center the discharge. When plasma current is driven toroidally through the octupole null, a tokamak with four poloidal divertors is generated [Fig. 1(b)]. By varying the placement of the internal rings, we can change the shape of the tokamak separatrix from outside "dee" to square [Figs. 2(a) and 2(b)].

The machine has an iron-cored, square–cross-section (44 cm×44 cm), 50-cm–major-radius, aluminum chamber. Four octupole hoops, made of a chromium-copper alloy, are placed near each corner (Fig. 1). Typical electrical charac-

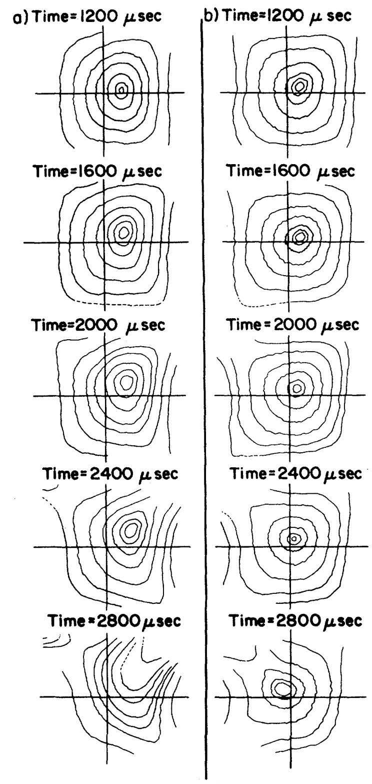


FIG. 2. Electrical characteristics. (a) Loop voltage at machine center with and without plasma. (b) Plasma current. Toroidal magnetic field ≈ 3.5 kG on axis.

teristics of the discharge are shown in Fig. 3. A normal discharge lasts ~ 4 msec with toroidal currents ≈ 40 kA, the peak electron temperature ~ 100 eV (surmised, with ~ 30% accuracy, from modeling of the time evolution of different sets of impurity lines, e.g., O I-O VI), electron density ~ $10^{13}$ $cm^{-3}$ (microwave interferometry) and ion temperature ≈ 20 eV (Doppler broadening of He II). Temperatures are sufficiently low that probes

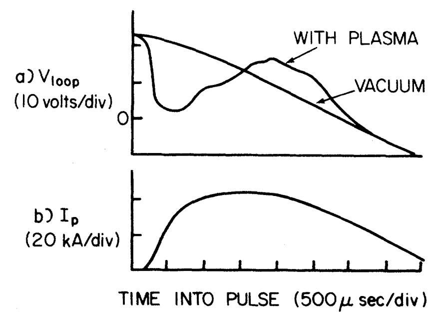


FIG. 3. Time evolution of experimental flux plots mapped out with magnetic probes. Only the area inside the initial separatrix is shown. (a) case A; (b) case B.

can be inserted into the plasma with little observable effect, and parameters such as plasma current, electric field, and magnetic field can be plotted as a function of space and time.

Extremely important to this study is the special ability to produce experimental flux plots. The magnetic probe (0.5 cm$\times$0.4 cm) was inserted at the midplane at several random azimuths to verify axisymmetry. We are able to adjust the vertical positions of the internal hoops by $\pm$5 mm enabling us to study a range of poloidal-divertor equilibria.

Axisymmetric instability was observed in the form of a nonrigid mostly vertical shift of the plasma. The deeness is varied by moving our hoops, which exert attractive forces on the plasma. The hoops although disconnected from each other, appear electrically in parallel, with constant voltage. Two cases are illustrated. Case A [Fig. 2(a)] has the outside hoops moved closer together yielding an inverse-dee equilibrium. Case B [Fig. 2(b)] with the inside hoops moved closer together gives roughly a square equilibrium. It is seen that the inverse dee is unstable while the square is stable on the time scale of the experiment. Figure 4 illustrates the motion of the central magnetic axis for the unstable inverse dee. The square exhibits a small horizontal displacement, probably due to the changing magnitude of the current.

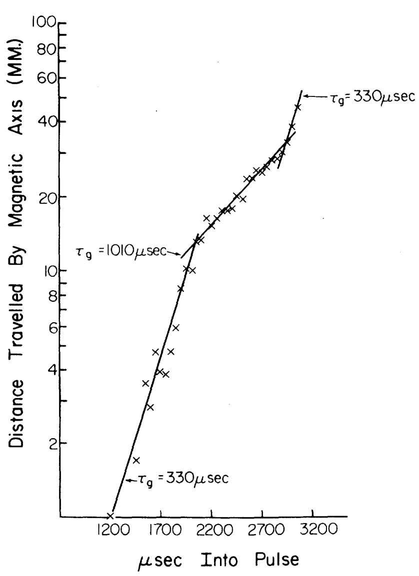


FIG. 4. Distance traveled by magnetic axis as a function of time. Note that case B moves only horizontally. Uncertainty in distance traveled is 1–2 mm. 100 $\mu$sec = 500 poloidal Alfvén times.

Although no previous theoretical work exactly applied to the experimental situation at hand, several comparisons between experiment and theory can be made. Figure 4 indicates that the growth of the instability is exponential for excursions of the magnetic axis up to ~0.5$a$ ($a$ is the minor radius ~8 cm). At that time the upper-outer $x$ point joins the $o$ point (magnetic axis), destroying the closed tokamak magnetic surfaces. This change is not discernible from our external diagnostics. Growth times are $\tau_g \cong 300$ $\mu$sec for $t < 2.0$ msec and $\tau_g \cong 1$ msec for $t > 2.0$ msec. Similarly Jardin and co-workers[6,10] found numerically that the displacement is exponential for excursions of at least 0.2$a$ with $\tau_g \sim \tau_A$ (poloidal Alfvén time), without passive stabilization. Including passive stabilization, Jardin and co-workers[6,10] (and Fukiyama[3]) predict that for a perfectly conducting plasma, $\tau_g$ increases to the inductive decay time ($L/R$) of the wall and field-shaping coils. In our experiment $\tau_g \sim 10^3 \tau_A$, where $\tau_A$ is defined with use of a suitably averaged $B_{\text{poloidal}}$. The $L/R$ time of hoops and wall are ~15 msec whereas the plasma $L/R$ time ~100–200 $\mu$sec. The closeness of $\tau_g$ to the plasma $L/R$ time suggests that the passive feedback may be limited by the plasma conductivity. This effect has not yet been explored theoretically. A plate has been installed in the divertor region outside the separatrix to eliminate the plasma in that region. Preliminary observation indicates little difference in plasma stability.

Relative stability of square, dee, and inverse-dee equilibria has been considered in Refs. 2, 10, and 11. Rebhan and Salat[2] evaluate stability boundaries in parameter space for constant-pressure, surface-current equilibria with the outermost flux surface described by

$$e^2(R-1)^2 + 1(+\tau_3^2)z^2 - A\tau_3(R-1)a^2 - A^2\tau_4(R-1)^2 z^2 = e^2/A^2,$$

where $\tau_3$ and $\tau_4$ characterize the trangularity and rectangularity; $e$ and $A$ are the ellipticity and aspect ratio. The inverse-dee and square equilibria of Fig. 3 are well fitted by the above equation with use of $\tau_3 = 0.2$, $\tau_4 = 0.6$, and $e = 1.23$ and with use of $\tau_3 = 0$, $\tau_4 = 0.9$, and $e = 1.1$, respectively. For these parameters Rebhan and Salat predict (ignoring passive stabilization) that our square is stable and our inverse dee appears to be marginally stable to rigid displacements; but both equilibria are unstable to a nonrigid antisymmetric (up-down) perturbation. Jardin,[6,10] with use of a nonlinear numerical code, also found that both our equilibria are unstable without passive feedback, with the inverse dee growing about twice as rapidly as the square. He found passive feedback can stabilize both modes on the time scale of his program (100–200 Alfvén times). Becker and Lackner[11] tested the axisymmetric stability of numerical square and dee equilibria with the toroidal current-density profile input as a variable, $J_t \propto (\psi - \psi_L)^l$ for $\psi > \psi_{\rm lim}$ and zero elsewhere. Numerical solutions for their equilibria best match our experiment for $l = 1$ for both square and dee. (The inverse aspect ratio $\epsilon = 0.12$, the rectangularity $\alpha = 0.15$.) Their stability diagrams predict the square to be marginally unstable and the dee to be unstable (with $\tau_g \sim 1.3\tau_A$) to nonrigid perturbations.

Calculation of Refs. 2 and 5 indicates that the displacements are maximized where the poloidal-field curvature is maximum (toward $x$ points). The flux-plot mappings of Fig. 2 clearly show this to be true. Also, in agreement with analytical Solov'ev equilibria,[12] the inner flux surfaces are elliptical. However, the outer flux surfaces are not well fitted by Solov'ev's model.

In summary, various theoretical models applied to the experiment predict that the square is more stable than the dee and inverse dee, in agreement with experiment. The growth rate of the inverse dee is intermediate between an Alfvén time and the hoop $L/R$ time ($\tau_g \sim 10^3\tau_A \sim 0.01L/R$) and is roughly equal to the plasma $L/R$ time. Thus, passive stabilization from the hoops slows the growth of the instability considerably ($\tau_g \gg \tau_A$), although plasma resistivity may partially defeat the passive feedback ($\tau_g \ll$ hoop $L/R$ time). The wide separation of the various time scales suggests that these conclusions may not be very machine dependent. The importance of inclusion of finite plasma resistivity (ignored in all cited theories) is emphasized. Finally, since the experiment produces flux plots equivalent in detail to that produced by a computer code, it can function as a test case to which the various numerical codes may be applied and compared in detail.

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