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QUANTIFYING AESTHETIC PREFERENCE FOR CHAOTIC PATTERNS

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ABSTRACT

Art and nature provide much of their aesthetic appeal from a balance of simplicity and complexity, and order and unpredictability. Recently, complex natural patterns have been produced by simple mathematical equations whose solutions appear unpredictable (chaotic). Yet the simplicity and determinism of the equations ensure a degree of order in the resulting patterns. The first experiment shows how aesthetic preferences correlate with the fractal dimension (F) and the Lyapunov exponent (L) of the patterns. F reflects the extent that space is filled and L represents the unpredictability of the dynamical process that produced the pattern. Results showed that preferred patterns had an average $F = 1.26$ and an average $L = 0.37$ bits per iteration, corresponding to many natural objects. The second experiment is a preliminary test of individual differences in preferences. Results suggest that self-reported creative individuals have a marginally greater preference for high F patterns and self-reported scientific individuals preferred high L patterns. Objective tests suggest that creative individuals had a slightly greater preference for patterns with a low F.

INTRODUCTION

Surrounding us is an ever-changing world that has evolved from natural dynamical forces and chaotic events. The result has been a landscape consisting of branches, folds, fractures, and detail embedded within detail. Yet when we evaluate properties of the environment, we typically do so with Euclidean geometry without paying much attention to the dynamics and sources of environmental change. Historically studies of perception and aesthetics have overlooked subtleties of nature and have focused instead on simplified geometric forms that consist of straight lines, angles, and circles [1, 2]. No unifying theory exists to describe what information or organization of information leads to our aesthetic preference. Theories have only spoken of some optimal amount or balance between order and disorder [1, 3, 4], simplicity and complexity [5-8], and predictability and unpredictability [9, 10].

In recent years, complex shapes resembling those found in nature (i.e., mountain ranges, clouds, and coastlines) have been produced by simple mathematical equations with complicated solutions. These solutions are unpredictable over the long term, yet the simplicity and determinism of the equations ensure a degree of order in the resulting patterns. Such equations are said to exhibit chaos [11]. A single equation with different coefficients can produce an almost endless variety of chaotic patterns (see Figure 1).

This capability of generating images from simple equations, and quantifying their important attributes is unique and contrasts with conventional approaches to understanding our perception and preference for particular visual patterns. A typical approach from the field of perception might consider the ease of coding properties of symmetry [8, 12], simplicity [3, 5], and proximity [13]. The classic Gestalt approach argues that many of these properties determine "figural goodness" or the degree to which a pattern is organized into coherent objects [13, 14]. A more recent approach involves an analysis of figural complexity based on information theory where information is defined as the logarithm of the number of possible alternatives contained within a given figure [5, 12, 15]. Yet another approach is to enumerate a pattern's features where the amount of information is defined by the number of lines and corners present in a picture [16, 17]. These theories predict that low information patterns (i.e., symmetric and simpler) are easier to code. An extrapolation of these findings might suggest that ease of coding may relate to aesthetic preferences [2, 5, 18].

In the present study, we quantify two unique attributes of patterns that may elicit aesthetic preferences: dimension and unpredictability. These attributes are captured by two measures used in the analysis of dynamical systems: 1) the fractal dimension (F) which is a generalization of the usual concept of dimension that admits non-integer values and reflects the extent that space is filled. Examples include the British coastline with a fractal dimension of about 1.25 and a crumpled piece of paper with a dimension of about 2.5. The actual dimension used here is the correlation dimension [19], and was chosen because it can be calculated quickly and accurately. It is technically a lower bound on the fractal dimension, but in practice it is a close approximation. 2) The Lyapunov experiment (L) quantifies the unpredictability of the dynamical process that produced the pattern. This measure captures a critical aspect of chaos—the sensitivity to initial conditions. If we iterate a system of equations with two nearby initial conditions and the

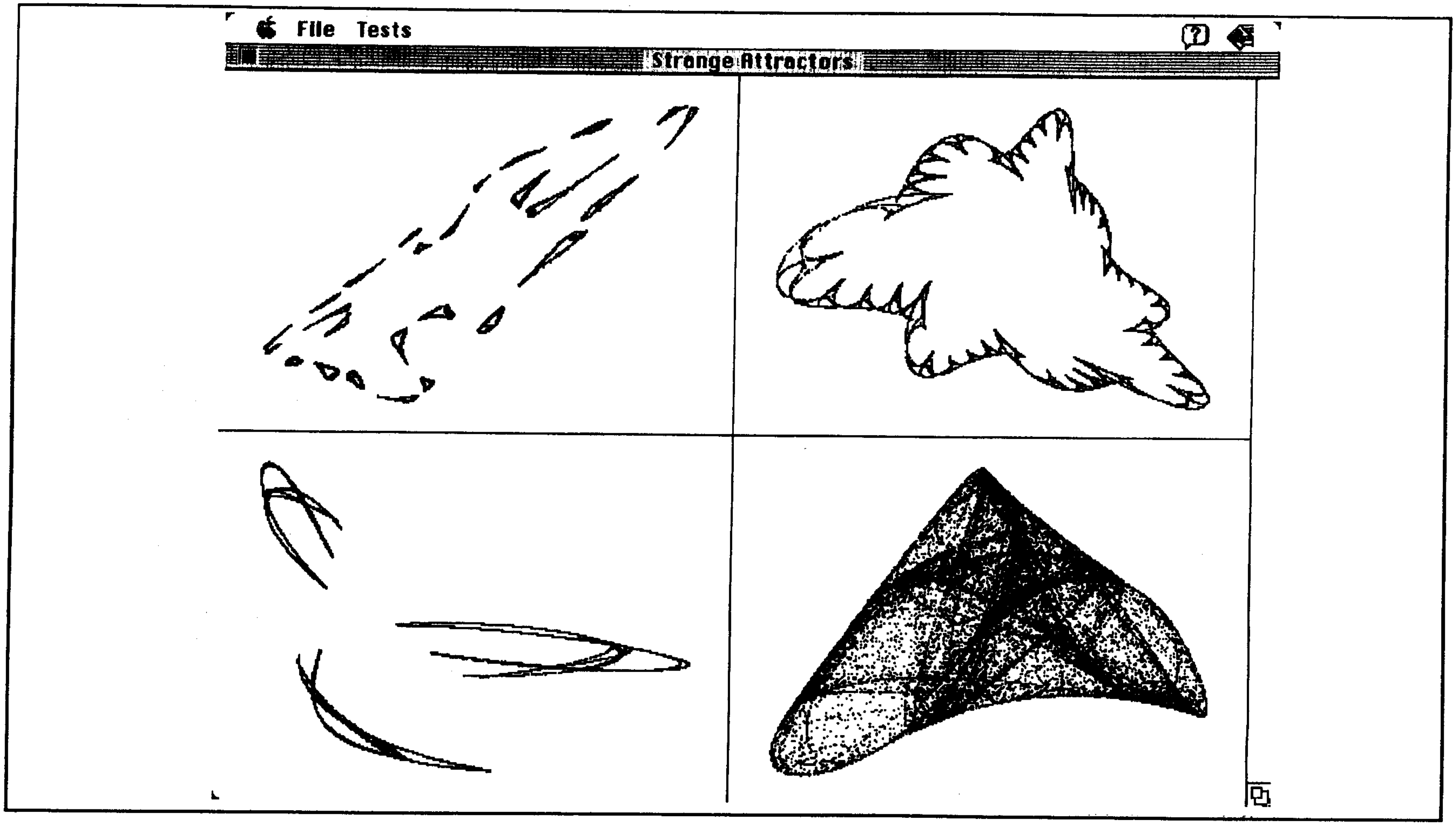

Figure 1. Sample screen display as exhibited to subjects showing four typical attractors. Top row has L near zero; bottom row has L near 0.2. Left column has F about 1.1; right column has F about 1.3.

solutions separate exponentially with each iteration, the system is considered chaotic [20]. In other words, the Lyapunov exponent reflects the rate of divergence [21] and can be considered as the rate at which information about the initial condition is lost or, equivalently, the rate at which the accuracy of a prediction declines as one projects further into the future [22]. If the Lyapunov exponent is positive and the solution is bounded, the system is considered chaotic, and the solution usually asymptotically approaches an invariant set called a strange attractor.

The parameters F and L are only two of an infinite number of quantitative measures of a strange attractor. Whenever the density of points is not uniform on the attractor, the resulting object is a multifractal with a continuous spectrum of generalized dimensions [22]. Furthermore, two-dimensional dissipative chaotic maps have two Lyapunov exponents, a positive one describing the exponential separation of nearby orbits, and a negative one describing the contraction of orbits onto the attractor. However, it has been conjectured [23] that the ratio of the positive to the negative exponent is $1\text{-}F$. Thus the negative exponent is not an independent parameter.

For these reasons, it is unrealistic to expect that aesthetic preferences are dependent only on the values of F and L. The space of parameters dictating aesthetics is most certainly very high dimensional. In focusing on F and L, we are in effect projecting this high dimensional space onto a plane and averaging over many other relevant variables. F and L relate to the *probability* that a given pattern will be aesthetically appealing. It is in this sense that the present results should be understood.

Whereas F is a geometrical property of the pattern that can be easily discerned, L is a dynamical property that describes the sequence in which points that make up the pattern were generated, and thus is not necessarily visually apparent in the static pattern. Why then might we expect human responsiveness to F and L? Two studies from the field of perception have demonstrated visual sensitivity to statistical properties of fractal dimensions [24], particularly those corresponding to natural contours [25]. In addition, preliminary research investigating aesthetic preferences has suggested that patterns within a particular range of the fractal dimension and Lyapunov exponent are deemed aesthetically appealing to viewers. Pickover found that subjects preferred lattice patterns with a mean fractal dimension of 1.8 [26]. Sprott showed that preferences to particular patterns generated by iterated function systems and containing only negative Lyapunov exponents were stronger for the largest negative exponents [27]. With strange attractors, subjects showed a preference for those with an average correlation dimension of $F = 1.30 \pm 0.20$ and an average Lyapunov exponent of $L = 0.21 \pm 0.13$ bits per iteration [28]. The present study extends this work and provides a more rigorous test of F and L as useful predictors of aesthetic preferences for strange attractor patterns.

EXPERIMENT 1: QUANTIFYING AESTHETIC PREFERENCE

Method

Subjects

Twenty-four (6 male and 18 female, age range = 17-31 years) members of the university community participated in the assessment of preference for strange attractors. All reported normal or corrected-to-normal vision.

Apparatus

Display presentation and data collection were controlled by a Macintosh computer using software that generates patterns from general nonlinear quadratic difference equations:

$$x_{n+1} = a_1 + a_2x_n + a_3x^2_n + a_4 x_ny_n + a_5y_n + a_6y^2_n$$

$$y_{n+1} = a_7 + a_8x_n + a_9x^2_n + a_{10}x_ny_n + a_{11}y_n + a_{12}y^2_n$$

The twelve coefficients a_1 though a_{12} were assigned randomly, and the equations were iterated, yielding a series of x and y values that were used to produce the plots [27]. A subset of 324 chaotic patterns was selected that spanned a wide region of the *F-L* plane as suggested by Sprott's preliminary study. These attractors were selected from a collection of 7500 that had been previously evaluated by eight subjects. They were chosen to span uniformly a wide range of *F* and *L*, and to have aesthetic evaluations close to the average of similar *F* and *L* cases. This procedure ensured that the examples used in the present study were not somehow atypical or redundant. The fractal dimension of the patterns used in the present study ranged between $F = 0.49$ and 1.78, and the Lyapunov exponent ranged between $L = 0.01$ and 0.84 bits per iteration.

Design and Procedure

Subjects free-viewed the series of 324 strange attractors each of which was presented in a quadrant of the display monitor on one or more of the 231 trials. Four of these images were presented simultaneously while subjects selected with the mouse the image that appeared "most visually appealing." The four cases represented two with the same *F* and different *L* and two with the same *L* and different *F*, positioned on the screen randomly. The sequence of cases was also chosen randomly and was unique for each subject. Subjects completed the experiment within twenty to forty-five minutes.

Data analysis used an iterative procedure that ordered all pictures based on their relative appeal. The patterns initially were randomly assigned a ranking between

1 and 324. For each trial, the ranking of the preferred pattern of the four was swapped with the ranking of the one with the highest value. After each pass through the data, the ratings of those cases that were never chosen as most preferred were randomly shuffled. In addition, a modest amount of smoothing was applied to coerce each ranking to a value close to the average of its neighbors. This procedure was repeated thirty times until a stable ordering was obtained. Each F and L value was arbitrarily weighted according to the fourth power of its ranking, and the weighted averages of the values were calculated. The power of four was chosen to give strong emphasis to those cases that were most highly ranked. The standard deviation is the square root of the variance of the weighted values from the mean.

Results

Figure 2 shows typical evaluations on a gray scale plot in which the fractal dimension and the Lyapunov exponent are the axes and the dark regions indicate greater preference. The white regions represent combinations of F and L for which no attractors were available, either because the equations did not generate them or because there were too few cases from which to choose. All evaluators preferred attractors with a fractal dimension between 1.17 and 1.38 and a Lyapunov exponent between 0.30 and 0.46 bits per iteration. The weighted average for all subjects had a mean fractal dimension of 1.26 ± 0.06 and a mean Lyapunov exponent of 0.37 ± 0.05. Errors represent the standard deviation from the mean (*SD*). The average within subject *SD* was 0.233 ± 0.006 for the fractal dimension and 0.186 ± 0.019 for the Lyapunov exponent. There was a strong inverse relationship between the preferred F and L ($r(22) = -.43$, $p < 0.05$), despite the positive correlation ($r(322) = .10$, $p < 0.001$) in the entire distribution.

Discussion

The mean and range of preferred fractal dimension closely matches previous work [29], and corresponds to those of the natural environment (i.e., coastlines). However, subjects in the present study preferred higher Lyapunov exponents (0.16 bits per iteration higher). There are a number of possible reasons to account for this difference. 1) Different methods were used. The rating procedure in the present study was a relative measure across patterns in that direct comparisons were made between patterns. 2) Subjects may represent different populations with distinct aesthetic preferences. Perhaps younger (undergraduate) students inherently prefer indeterminacy, or perhaps there were differences for individuals specializing in different disciplines. The subjects used by Sprott were adult professionals and graduate students in the arts and sciences [29]. The next experiment addresses whether individual differences in aesthetic preferences can be predicted using the present technique.

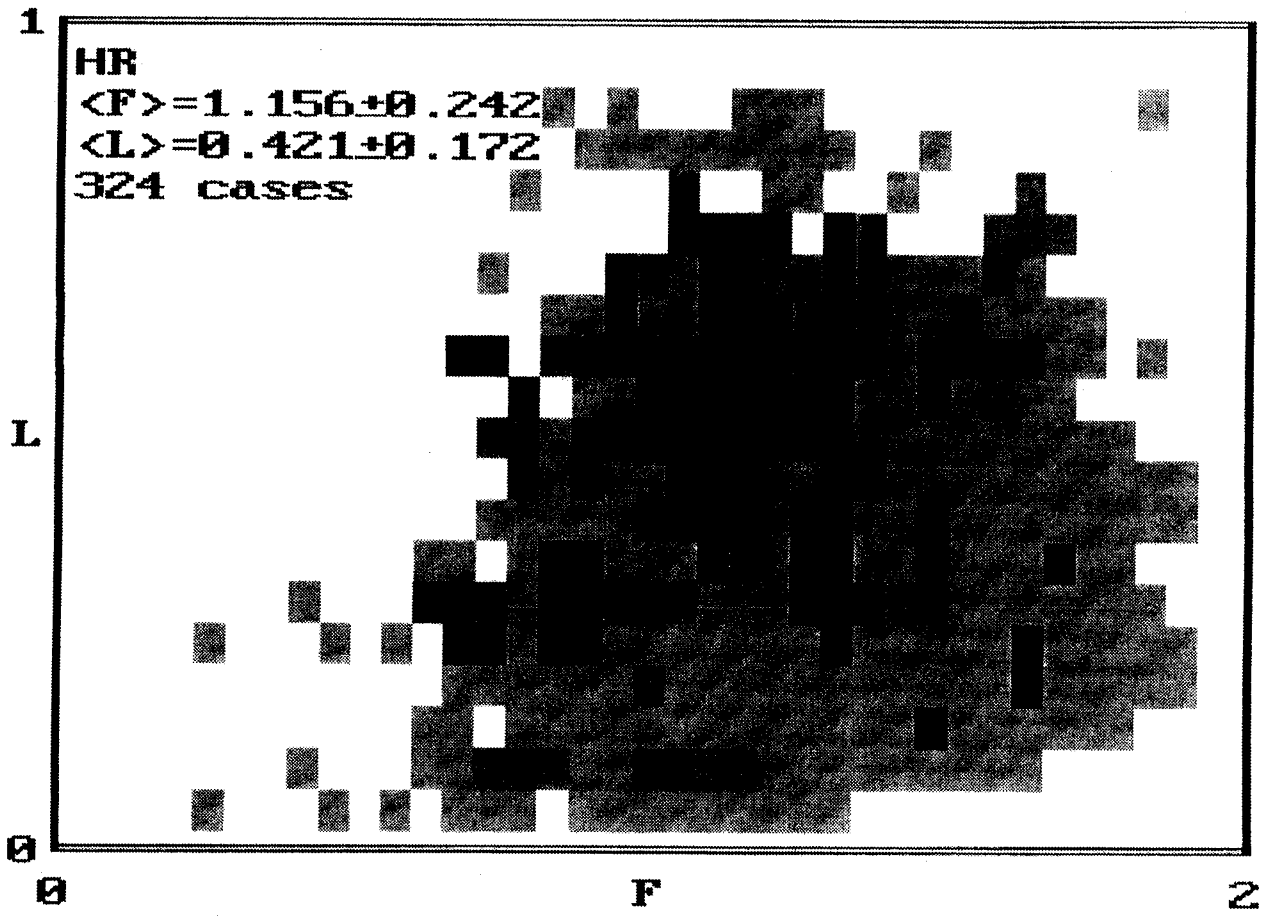

Figure 2. Summary of preferences in the *F-L* plane for two typical subjects; (a) HR prefers low *F* and high *L*; (b) DA prefers high *F* and lower *L*.

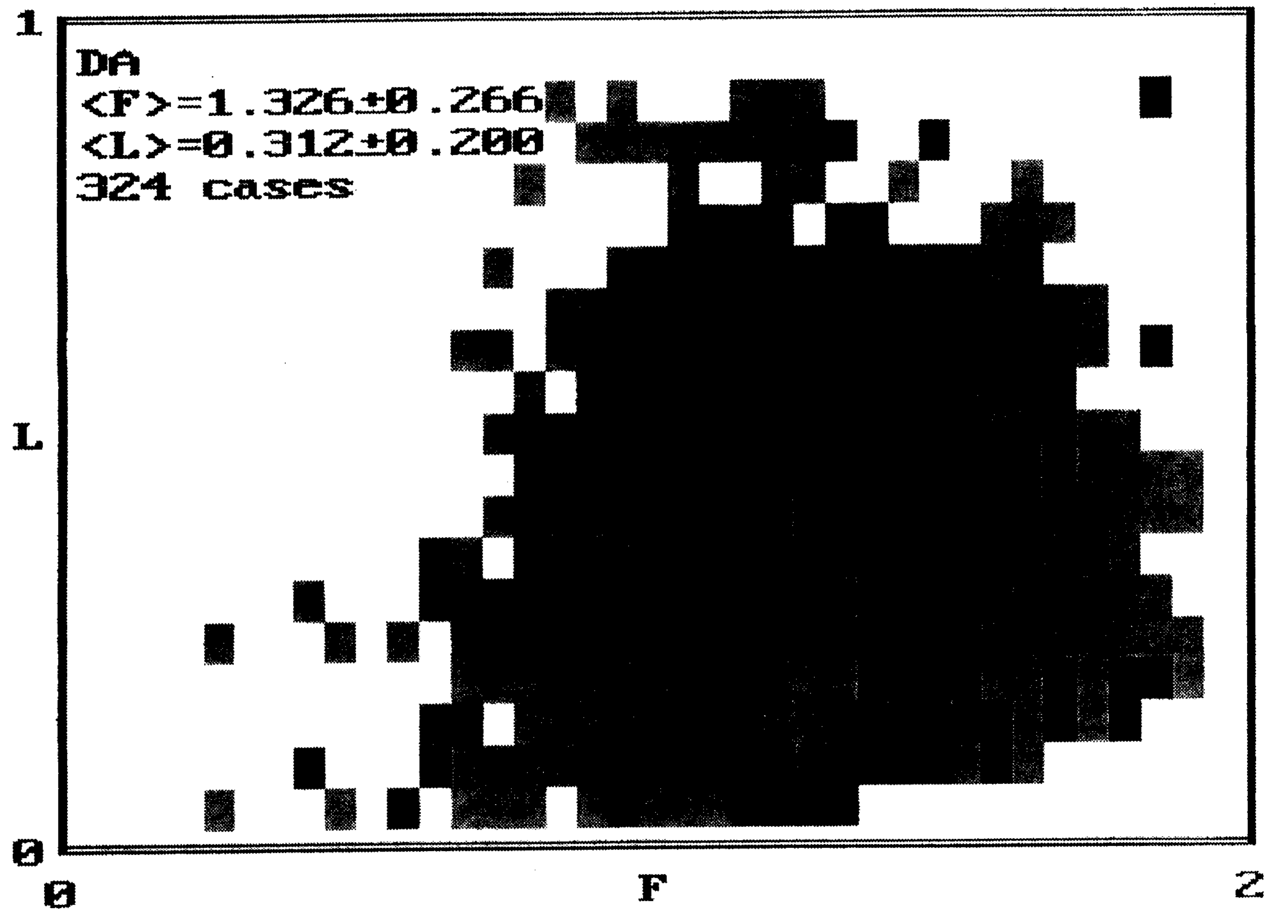

Figure 2. (Cont'd.).

EXPERIMENT 2: INDIVIDUAL DIFFERENCES IN AESTHETIC PREFERENCES

Given the range of preferences found in the previous study as well as in society at large, it is natural to investigate factors that may account for these differences. What are the appropriate individual difference variables? A logical starting point is to explore those traits previously found to predict differences in aesthetic preferences. Ordinary observations suggest that aesthetic dispositions are related to creative potential. In support of this hypothesis, a number of creativity tests have consistently differentiated between more and less creative persons in various scientific, artistic, and literary fields [30, 31]. Additional studies show that more creative people prefer more complex art [32].

Since creativity has been recognized as a multivariate phenomenon [33], we use a battery of tests that capture at least two key aspects: novelty and originality. These imply both an ability and inclination to think fluently and flexibly and are commonly referred to as *creative* ability [34], or *divergent thinking* [33]. In contrast to the single correct answer demanded by convergent tests, divergent tests ask for as many appropriate answers as possible. Some tests emphasize sheer quantity of productions (i.e., see Word Fluency test described in Method) while other tests score for unusual and original responses (i.e., see Consequences test in Method). Factor analyses show that all of these tests form a single factor distinct from general intelligence [35]. Individuals scoring high on these tests are inclined ɔ be creative and prefer unstructured situations. Individuals scoring low on these abilities (and high in intelligence) are characterized as analytical and logical; they prefer structured, explicit, and clearly defined situations.

A number of these distinctions between high and low creativity suggest that there may also be some reliable differences among individuals of different disciplines. For instance, we might expect that more analytical individuals would pursue careers in the sciences, whereas those rated high in creativity or integrative skills might be more inclined to pursue the arts. Thus, individuals from different disciplines (and backgrounds) might reveal differences in aesthetic preferences. Supporting these suppositions are studies showing that creative persons have higher literary and musical interests, are more likely to be artists [36], and have greater aesthetic appreciation and less interest in computational tasks than most people [37]. In addition, these individuals prefer complex art [32] and unstructured activities [30]. Consequently, we might expect these individuals to prefer high F and L patterns (i.e., detailed and unpredictable) as compared to scientific individuals who might be expected to prefer low F and L patterns (i.e., simple and predictable). Scientists may tend to see beauty in simplicity whereas artists may be more tolerant of complexity.

We look at two additional variables—rural upbringing and gender—as potential predictors of aesthetic preference. An individual raised in a rural setting might be inclined to prefer more naturalistic patterns as compared to an individual raised in an urban environment. The link between creativity and gender has also been documented, and females score consistently higher on creativity measures [38].

Method

Self-Reports

Fifteen of the subjects from Experiment 1 participated; three were male. Individual differences in perceived creativity, area of skill, and environmental upbringing were obtained using the self-report questionnaire in Appendix A. This self-report provides a measure of perceived skill in the following areas: science, math, art, humanities, and creativity. Also included in the self-report are questions pertaining to the type of environment in which the subject was raised (i.e., urban or rural). Redundant questions were included to check reliability and to make composite scores.

Objective Tests

Eleven of the subjects (2 male) who completed the self-reports participated. Seven tests of divergent thinking were used: Associational Fluency 1 [39] which asks for synonyms; Expressional Fluency [40] which asks subjects to write sentences with words beginning with designated letters; Ideational fluency [41] which asks subjects to name objects in specific classes; Word Fluency [42] which asks subjects to write words containing a designated letter, Consequences [43] which asks subjects to imagine the possible consequences of unusual situations; Alternate Uses [44] which asks subjects to list as many uncommon uses for a common object as possible in the allotted time, and CAB-O [45] which asks subjects to put two common objects together to form a new useful object. A composite score was formed by summing the scores on the five tests. All the tests were timed and administered in two group settings.

Scoring and Reliability

Five trained-scorers were used following procedures recommended in the manuals that document the high reliability of these procedures. Each protocol was repeated at least twice to check for accuracy, and a third time if there was any difference in opinion. Internal consistency of the measures has been found using Spearman-Brown and Retest reliability; values range from 0.63 to 0.85 [43].

Results and Discussion

Self-Reports

Composite summary scores were calculated to isolate four classes of individual difference: *scientific-mathematical skill, artistic-creative, divergent thinking, rural upbringing* (see Appendix A). Items were grouped together in a single class if they were predicted, a priori, to measure the same phenomena, and were significantly correlated (e.g., questions 5 and 11 were predicted as self-reported measures of scientific ability: $r(13) = 0.89$, $p < 0.05$). Alternative groupings and individual items were unable to account for any additional variance.

To determine whether there is a relationship between these categories of individuals and preferences, we simply compared preference ratings of high and low scorers on the self-reported skills. The sample was divided at the mean of each group so that individuals with a combined score above the mean are considered "high" on that self-reported dimension (e.g., scientific-mathematical skill) and those with a score below the mean are considered "low" in that skill.

Overall, self-reports of creativity, scientific or artistic skill, and rural upbringing were not reliable predictors of pattern preferences. The only exception was that individuals who rated themselves high on divergent thinking showed a marginal preference for patterns with greater detail (high F;1.28 ± 0.06) compared to those rated as low in divergent abilities (low F;1.24 ± 0.04; $t(1,13) = 1.6$, $p < 0.12$). Otherwise, there were no differences in preferences for patterns with unique fractal dimensions for any of the remaining categories: artistic-creative ($t(1,13) =$ 0.05, *n.s.*), rural upbringing, ($t(1,13) = 0.71$, *n.s.*), science-math ($t(1,13) = 0.33$, *n.s.*), and gender ($t(1,13) = 0.70$, *n.s.*).

Patterns with marginally higher Lyapunov exponents ($t(1,13) = 1.6$, $p < 0.13$) were preferred by individuals reporting high on Science-Math abilities. Otherwise, there were no significant differences in preferences for particular Lyapunov exponents in the remaining categories: artistic-creativity ($t(1,13) = 0.05$, *n.s.*), rural, upbringing, ($t(1,13) = 0.48$, *n.s.*), divergent skills ($t(1,13) = 0.35$, *n.s.*), and gender ($t(1,13) = 0.76$, *n.s.*).

The low reliability of self-reports may be due to the small sample size or to nonlinear trends that are suggested in the data. Scatter plots indicated that individuals with extreme ratings (i.e., high or low) on artistic-creativity skill showed a weak preference for low dimension ($r^2 = 1.0$) while science-math individuals preferred the more unstructured patterns ($r^2 = .14$). However, these quadratic trends failed to reach significance in both cases perhaps due to the paucity of subjects.

Objective Tests

Table 1 shows performance on objective tests of creativity and its relation to F and L. A consistent *inverse* relation emerged between objective measures and the fractal dimension but there was no significant relationship to the Lyapunov exponent. These trends, summarized in the composite scores, show that the inverse relation between creativity and F,($r(9) = -.63$, $p < .05$), is opposite to that based on self-reported creativity, and suggests that creative individuals prefer low F patterns. Therefore, individuals who view themselves as original and open-minded are more likely to prefer complex patterns, whereas individuals that perform in original ways tend to prefer simple patterns.

Table 1. Correlations between Objective Measures of Creativity and *F* and *L*

Divergent Thinking	Fractal Dimension	Lyapunov Exponent
Associational Fluency	–.53**	.15
Ideational Fluency	–.48*	.21
Expressional Fluency	–.41	–.29
Word Fluency	–.44*	–.15
Alternate-Uses	–.28	.18
Consequences	–.09	.31
CAB-O	–.51*	.10
Composite	**–.63****	**.20**

*$p < .10$
**$p < .05$

GENERAL DISCUSSION

Art is present in all things that evoke an aesthetic experience [46]. What exactly evokes such a response? A quantitative approach, based on principles from chaos theory, is used to systematically manipulate and assess preference for abstract images. By generating images with simple equations we were able to isolate important characteristics of the output: 1) the fractal dimension, which represents the density of the image, and 2) the Lyapunov exponent, which captures the pattern's dynamical unpredictability, sensitivity to initial conditions, and the degree of chaoticity.

These measures are distinct from more traditional means for assessing human preference. Theory from the field of perception has focused on how we code properties of symmetry [8, 12], simplicity [1, 3, 7, 16], and proximity [13, 14, 47]. The field of art offers answers to the question of aesthetics that pertain to a suitable balance between the expected and the unexpected [48], novelty and familiarity [49], and regularity and variability [50]. However, both perceptual and art theory, at best, provide a disjointed list of general stimuli deemed to be appealing. The present quantitative approach attempts to clarify a potential source of visual preference.

Our main finding was that preferred patterns congregate around a particular region of the *F-L* plane with an average $F = 1.26$ and an average $L = 0.37$ bits per iteration. These values are of significance because 1) they correspond to many natural objects, and 2) they may point to an abstract form that may be shared by nature and human preference. The range ($F = 1.17$ to 1.38; $L = 0.30$ to 0.46) suggests that extreme values may reflect stable individual differences. Our preliminary tests suggest that creative people prefer less detailed patterns (low *F*), people who judge themselves as high in creativity prefer more detailed patterns (high *F*), and those who rate themselves high in scientific skills prefer unstructured patterns (high L). These findings are only suggestive but indicate a fruitful line of research to demonstrate that the dynamics underlying human preference can be described by the same simple mathematics used to model the natural world.

APPENDIX A

On a scale from 1 to 5 how would you rate yourself on the following:

1 = Least like this
3 = Moderately like this
5 = Most like this

1) I like things to be black and white 1 2 3 4 5
2) I am Creative 1 2 3 4 5
3) I am Artistic 1 2 3 4 5
4) I am good at coming up with lots of new ideas 1 2 3 4 5
5) I am Scientific 1 2 3 4 5
6) I am Mathematical 1 2 3 4 5
7) I don't mind ambiguous situations 1 2 3 4 5
8) I like complex problems 1 2 3 4 5
9) I like open-ended problems 1 2 3 4 5
10) I like problems with a single solution 1 2 3 4 5
11) I tend to get better grades in science courses 1 2 3 4 5
12) I was raised in a rural setting 1 2 3 4 5
13) I am an outdoors person 1 2 3 4 5
14) I tend to get better grades in humanities courses 1 2 3 4 5
15) I was raised in an urban setting 1 2 3 4 5

Composite Scores:

Science-Math = (Q5 + Q6 + Q11)/3
Creativity-Artistic = (Q2 + Q3)/3
Divergent Thinking = (Q4 + Q7 + Q8 + Q9 + (5 – Q1) + (5 – Q10))/6
Rural Background = (Q12 + (5 – Q15)/2

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