19 April 1999

PHYSICS LETTERS A

ELSEVIER

Physics Letters A 254 (1999) 275–278

Controlling chaos in a high dimensional system with periodic parametric perturbations

K.A. Mirus, J.C. Sprott

Department of Physics, University of Wisconsin, Madison, WI 53706, USA

Received 19 December 1998; revised manuscript received 14 January 1999; accepted for publication 14 January 1999
Communicated by C.R. Doering

Abstract

The effect of applying a periodic perturbation to an accessible parameter of a high-dimensional (coupled-Lorenz) chaotic system is examined. Numerical results indicate that perturbation frequencies near the natural frequencies of the unstable periodic orbits of the chaotic system can result in limit cycles or significantly reduced dimension for relatively small perturbations. © 1999 Elsevier Science B.V.

PACS: 05.45.+b; 47.52.+j
Keywords: Chaos; Control; Lorenz; Periodic orbit

Control of low-dimensional nonlinear systems is a well-established art [1–11]. However, the control of high-dimensional systems remains more elusive [12,13]. Controlling high-dimensional systems with traditional closed-loop mechanisms often requires detailed knowledge of the system state, as well as waiting for the system to approach closely the unstable periodic orbit (UPO) to be stabilized, or taking the additional step of steering the trajectory toward the desired UPO [14]. The very simple method of an open-loop control applied as a simple periodic perturbation to a system parameter has been overlooked. Here it is shown for a 96-dimensional polynomial flow described by a system of 32 coupled Lorenz equations [15] that small periodic perturbations to an accessible parameter at the UPO frequencies can produce limit cycles, a reduced Lyapunov dimension (D_L) [16], or a decreased leading Lyapunov exponent (LLE).

Previous work on controlling low-dimensional systems with periodic perturbations showed successful perturbation frequencies to be rational multiples of the periodic drive frequencies that initiated the chaos [17–23], the natural frequencies in a period-doubling route to chaos [24], or frequencies corresponding to peaks in the power spectrum [25,26]. Here we show that these predictors are not always reliable. Rather, the optimum perturbation frequencies correspond to low-order rational multiples of the frequencies of UPOs embedded in the attractor. These UPOs can be extracted directly from the time series of any state-space variable, and thus no model for the system dynamics is required to determine optimum perturbation frequencies.

The example shown here is a coupled Lorenz model consisting of a one-dimensional lattice with each site occupied by a set of Lorenz equations. This model has been used to study the coupling of two lasers [27], and

PII S0375-9601(99)00068-7

it also exactly describes the dynamics of coupled thermal convection loops, or thermosyphons [28]. The model used here is taken from Jackson and Kodogeorgiou [15], which viscously couples N Lorenz equations,

$$\dot{x}_i = \sigma(y_i - x_i) - \mu(x_{i+1} + x_{i-1} + 2x_i)\,,$$
$$\dot{y}_i = -y_i - x_i z_i + r x_i\,,$$
$$\dot{z}_i = x_i y_i - b z_i\,. \tag{1}$$

Here, i denotes the lattice site ($i = 0, 1, \ldots, N-1$), and μ is the viscous coupling constant. Each lattice site is coupled to one neighbor on each side. This system is taken to have periodic boundary conditions: $x_N = x_0$, $y_N = y_0$, $z_N = z_0$. The parameters used were $\sigma = 10$, $\mu = 3$, $r = 45$, $b = 1$, and $N = 32$. In the thermosyphon paradigm, x corresponds to the average fluid velocity around the loop, y corresponds to the temperature difference between points at "12 o'clock" and "6 o'clock", and z corresponds to the horizontal temperature difference. It was found that the system exhibits chaos for r down to about 23.

Calculation of the Lyapunov exponent spectrum is computationally intensive for high-dimensional chaotic systems since the memory and time required both scale as N^2. For this reason, a lattice with only $N = 32$ sites (and thus $3 \times 32 = 96$ variables) was considered, and a first-order "leapfrog" integration scheme was used. In the absence of a perturbation, this 96-dimensional system has an attractor with a Lyapunov dimension of 65.8. The uncoupled Lorenz equations for these parameters was measured to have $D_L = 2.06 \pm 0.005$, which suggests a dimension for the coupled system of $32 \times 2.06 = 65.9$. The agreement between the actual and expected values of D_L is remarkable, although it would not have been surprising if the coupling had significantly reduced the dimension. Although D_L is apparently an extensive property of the system size, the behavior reported here appears general as evidenced by similar behavior in other high-dimensional systems [29]. The parameter r was chosen to be perturbed according to $r = r_0 + r_1 \sin(\omega t)$, because it corresponds to the experimentally accessible temperature difference applied across the thermosyphon.

Fig. 1 shows a recurrence plot histogram of the periods of UPOs found for this coupled Lorenz system.

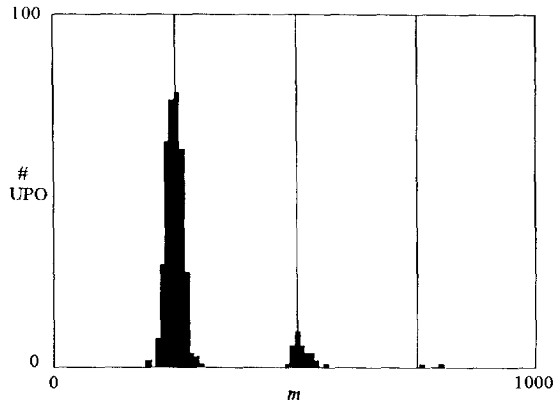

Fig. 1. Recurrence plot histogram for the coupled Lorenz system. The period-one peak occurs at $m = 252.5$ time steps, or $f = 1.014$.

The method identified a period of m time steps ($\Delta t = 1/256$) as a UPO whenever the trajectory passed within some distance ε of an arbitrary starting point [30]. A value of ε roughly twice the average step size in the time series was used. All 96 state-space variables were used to identify UPOs instead of the common technique of using a single time series with time-delayed embedding. The frequency of the period-one UPO was found to be 1.014 ± 0.050, which is roughly half the frequency of the period-one UPO for a single Lorenz system. The power spectral peak for a single variable in the coupled system was roughly 2.0, which corresponds to the period-one UPO for a single Lorenz attractor. Thus, the global dynamics of the coupled system seem to evolve more slowly than the fluctuations in any state-space variable, which means that this is a system where the power spectral peak frequency for a single variable is a misleading indicator of the optimum perturbation frequency. There is no necessity that the UPO with the lowest frequency corresponds to the peak in the power spectrum, since its vicinity may be only rarely visited by the dynamics.

The perturbation frequencies were chosen to be the frequencies of the UPOs for the coupled system. Fig. 2 shows the Lyapunov dimension (D_L) and largest Lyapunov exponent (LLE) as a function of perturbation amplitude for seven different perturbation frequencies. Remarkably, a small perturbation amplitude of $r_1 = 4$, which is only 9% of the unperturbed value, resulted in limit cycle behavior for a perturbation frequency equal to the frequency of a period-one UPO ($f = 1.014$).

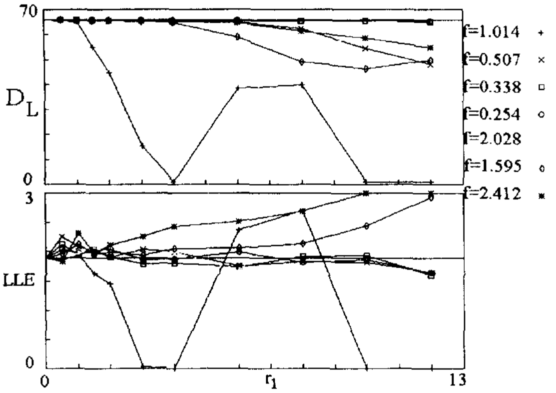

Fig. 2. Results of perturbing a coupled Lorenz system as evidenced by the Lyapunov dimension (D_L) and the leading Lyapunov exponent (LLE). The solid horizontal lines indicate values for the unperturbed system. Note that a perturbation frequency $f = 1.014$ produces a limit cycle for $r_1 = 4$, 10, and 12.

This perturbation produced the best overall performance. Perturbations of frequency $f = 2.028$ (twice the period-one UPO frequency) and $f = 1.595$ (about three times the period-two UPO frequency) significantly decreased the dimension of the system. Even if a perturbation does not produce limit cycles, drastically decreasing the dimension of a high-dimensional system is a significant step toward controlling it. Perturbations of frequency corresponding to a period-two ($f = 0.507$), a period-three ($f = 0.338$), and a period-four ($f = 0.254$) UPO decrease the dimension of the system, but require respectively larger perturbations. A perturbation frequency of $f = 2.412$, which was shown to be successful at eliminating chaos for the single Lorenz system [29], also requires a large amplitude to decrease the dimension of the system. None of the frequencies increased the dimension by more than one, as has been seen when perturbing coupled logistic equations [29]. Frequencies that are further mismatched from UPO frequencies may excite additional, latent degrees of freedom. This case could have important applications in biological and other high-dimensional systems, where it is sometimes desirous to "uncontrol" chaos [31].

Fig. 3 shows a spatiotemporal plot of the value of x_i, which corresponds to the average fluid velocity at each site. The perturbation at the optimal period-one frequency ($f = 1.014$) was turned on at $t = 20$ without any regard for the location of the trajectory with respect to the period-one UPO. The system then settled into a period-two ($f = 0.507$) limit cycle at $t > 35$, and remained in that limit cycle until the perturbation was turned off at $t = 180$. The onset of chaos is immediate after the perturbation is removed. Numerous trials starting from different random initial conditions showed that the chaos was suppressed on average after a time of $t = 33.4 \pm 16.8$. Thus, this method is useful since one need not wait for the system to approach a UPO. Also, a limit cycle is always achieved, although the time taken to obtain it is variable.

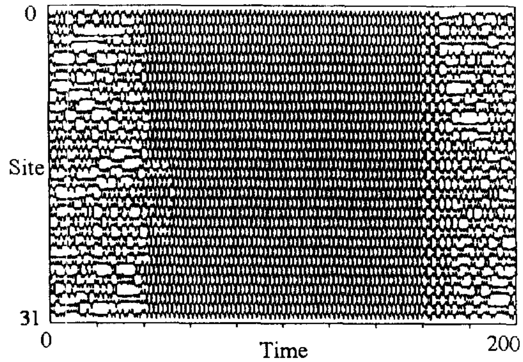

Fig. 3. Spatiotemporal plot of x_i vs time for the coupled Lorenz equations. A perturbation amplitude of $r_1 = 4$ and frequency $f = 1.014$ is turned on at $t = 40$ and off at $t = 160$.

In summary, a small-amplitude perturbation applied to an accessible parameter of a numerical system described by 96 equations with a Lyapunov dimension of 65.8 suppressed the chaos. The optimum frequency to obtain control with a perturbation of small amplitude was the frequency of the UPOs obtained from the dynamical fluctuations of the system. UPOs can have vastly different time scales than the fluctuations in a given state-space variable, which means that the best perturbation frequencies are not always those with the most power. These results appear to be general as evidenced by similar behavior in other chaotic systems of both low and high dimension as will be reported elsewhere [29].

This work was supported by the US Department of Energy.

References

[1] E. Ott, C. Grebogi, J.A. Yorke, Phys. Rev. Lett. 64 (1990) 1196.

[2] W.L. Ditto, S.N. Rauseo, M.L. Spano, Phys. Rev. Lett. 65 (1990) 3211.
[3] P.M. Alsing, A. Gavrielides, V. Kovanis, Phys. Rev. E 50 (1994) 1968.
[4] U. Dressler, G. Nitsche, Phys. Rev. Lett. 68 (1992) 1.
[5] B. Hubinger, R. Doerner, W. Martienssen, M. Herdering, R. Pitka, U. Dressler, Phys. Rev. E 50 (1994) 932.
[6] E.R. Hunt, Phys. Rev. Lett. 67 (1991) 1953.
[7] K. Pyragas, Phys. Lett. A 170 (1992) 421.
[8] M.A. Matias, J. Guemez, Phys. Rev. Lett. 72 (1994) 1455.
[9] Y.H. Chen, M.Y. Chou, Phys. Rev. E 50 (1994) 2331.
[10] C.-C. Fuh, P.-C. Tung, Phys. Rev. Lett. 75 (1995) 2952.
[11] J.L. Breeden, Phys. Lett. A 190 (1994) 264.
[12] D. Auerbach, C. Grebogi, E. Ott, J.A. Yorke, Phys. Rev. Lett. 69 (1992) 3479.
[13] L.R. Keefe, Phys. Fluids A 5 (1993) 931.
[14] T. Shinbrot, E. Ott, C. Grebogi, J.A. Yorke, Phys. Rev. Lett. 69 (1992) 3479.
[15] E.A. Jackson, A. Kodogeorgiou, Phys. Lett. A 168 (1992) 270.
[16] A. Wolf, J.B. Swift, H.L. Swinney, J.A. Vastano, Physica D 16 (1985) 285.
[17] R. Lima, M. Pettini, Phys. Rev. A 41 (1990) 726.
[18] L. Fronzoni, M. Giocondo, M. Pettini, Phys. Rev. A 43 (1991) 6483.
[19] Y. Braimin, I. Goldhirsch, Phys. Rev. Lett. 66 (1991) 2545.
[20] R. Meucci, W. Gadomski, M. Ciofini, F.T. Arecchi, Phys. Rev. E 49 (1994) 2528.
[21] J. Simonet, M. Warden, E. Brun, Phys. Rev. E 50 (1994) 3383.
[22] Y. Liu, N. Kikuchi, J. Ohtsubo, Phys. Rev. E 51 (1995) 2697.
[23] S.T. Vorha, L. Fabiny, F. Bucholtz, Phys. Rev. Lett. 75 (1995) 65.
[24] A. Azevedo, S.M. Rezende, Phys. Rev. Lett. 66 (1991) 1342.
[25] H.-E. Sätherblom, Phys. Plasmas 4 (1997) 174.
[26] W.X. Ding, H.Q. She, W. Huang, C.X. Yu, Phys. Rev. Lett. 72 (1994) 96.
[27] N.M. Lawandy, D.V. Plant, K. Lee, Phys. Rev. A 34 (1986) 1247.
[28] S.H. Davis, M.N. Roppo, Physica D 24 (1987) 226.
[29] K.A. Mirus, J.C. Sprott, Phys. Rev E, submitted for publication.
[30] D.P. Lathrop, E.J. Kostelich, Phys. Rev. A 40 (1989) 4028.
[31] W. Yang, M. Ding, A.J. Mandell, E. Ott, Phys. Rev. E 51 (1994) 102.