COMMENTS

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Comment on "A new class of exact solutions of the Vlasov equation" [Phys. Plasmas 8, 5081 (2001)]

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A recent paper obtained a class of exact one-dimensional solutions of the Vlasov–Poisson equations for a one-component plasma in a parabolic electrostatic potential well. Here it is shown that a significant class of these solutions has already been studied and shown to be chaotic. © *2002 American Institute of Physics.* [DOI: 10.1063/1.1498837]

In a recent paper, Yu, Chen, and Stenflo[1] obtained a class of exact one-dimensional solutions of the Vlasov–Poisson equations describing a one-component plasma in a parabolic electrostatic potential well. In particular, the electric field at any time is given by $E=E_0(t)x$ so that the plasma density is given by $A_0=E_0+n_0$, where n_0 is the constant background ion density. The Vlasov–Poisson system is then reduced exactly to an infinite set of coupled ordinary nonlinear differential equations of the form

$$\frac{dA_m}{dt}-mE_0A_{m-1}+(m+1)A_{m+1}=0. \tag{1}$$

The A_m's essentially specify the velocity moments of the distribution function.

The authors then introduce various closure schemes, such as $A_4=0.03A_0$, and show that bounded chaotic solutions exist for all the other moments for *suitable* initial conditions. As they themselves state, these solutions were found by a trial-and-error search in which the initial conditions of A_m were varied.

This note points out that this search procedure has already been carried out for a significant class of solutions which can result from a particular closure procedure applied to Eq. (1).

If one introduces the closure condition that

$$A_3=f(A_0,\ A_1,\ A_2), \tag{2}$$

where as yet f is an arbitrary function, then the dynamics reduce to a single third-order equation of "jerk" form,[2,3] namely

$$\frac{d^3A_0}{dt^3}=(5n_0-6A_0)\frac{dA_0}{dt}-6f \tag{3}$$

with

$$A_1=-\frac{dA_0}{dt} \tag{4}$$

and

$$A_2=\frac{1}{2}\frac{d^2A_0}{dt^2}+\frac{1}{2}(A_0-n_0)A_0. \tag{5}$$

If one now limits $f(A)$ to be a polynomial of a most quadratic nonlinearity, then following from numerical work by Sprott,[4] Eichhorn, Linz, and Hänggi[5] identified seven distinct forms for $f(A)$ that lead to chaotic dynamics. For example, the choice

$$A_3\equiv f(A)=-\tfrac{1}{6}[2k_1A_2+(5n_0-7A_0)A_1+(k_1n_0-k_1A_0+k_2)A_0+k_3] \tag{6}$$

leads to the equation

$$\frac{d^3A_0}{dt^3}=k_1\frac{d^2A_0}{dt^2}+A_0\frac{dA_0}{dt}+k_2A_0+k_3, \tag{7}$$

where k_1, k_2, and k_3 are arbitrary parameters with chaotic solutions for values such as $k_1=-1.8$, $k_2=-2$, and $k_3=-1$. Equation (7) is an example of the JD_1 model classified by Eichhorn *et al.*

More recently, Sprott and Linz[6] have identified a number of jerk equations giving chaotic solutions in which the nonlinearity is not necessarily polynomial. These systems are of the form

$$\frac{d^3A_0}{dt^3}+a\frac{d^2A_0}{dt^2}+\frac{dA_0}{dt}=g(A_0), \tag{8}$$

where $g(A_0)$ is nonlinear but not necessarily quadratic in A_0. Equation (8) corresponds to the closure condition

$$A_3 \equiv f(A) = \tfrac{1}{3} a A_2 - \tfrac{1}{6}[(5n_0 - 6A_0 + 1)A_1 + a(A_0 - n_0)A_0 + g]. \tag{9}$$

For example, the case

$$g(A_0) = \frac{1}{c}\{1 - b[1 + \tanh(cA_0)]\} \tag{10}$$

turned up in a model of the interaction of the solar wind with the magnetosphere (Horton *et al.*[7]) with chaotic solutions for $a = 0.6$, $b = 13$, and c arbitrary.

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