



Pergamon

Available online at www.sciencedirect.com

SCIENCE @ DIRECT®

**ADDICTIVE
BEHAVIORS**

Addictive Behaviors 28 (2003) 369–374

Short Communication

Substance abuse as a dynamical disease: Evidence and clinical implications of nonlinearity in a time series of daily alcohol consumption

Keith Warren^{a,*}, Raymond C. Hawkins^b, Julien C. Sprott^c

^a*College of Social Work, The Ohio State University, Stillman Hall, 1947 College Road,
Columbus, OH 43210, USA*

^b*Department of Psychology, The University of Texas, Austin, TX, USA*

^c*Physics Department, University of Wisconsin, Madison, WI, USA*

Abstract

Several authors have suggested that chaos theory, the study of nonlinear dynamics and the application of the knowledge gained to natural and social phenomena, might yield insight into substance-related disorders. In this article, we examine the dynamics of substance abuse by fitting a nonlinear model to a time series of the amount of alcohol, which an adult male with a diagnosis of substance abuse consumed on a daily basis. The nonlinear model shows a statistically superior fit when compared to a linear model. We then use the model to explore a question that is pertinent to the treatment of substance abuse, whether controlled drinking or abstinence is a preferred strategy for maintaining sobriety.

© 2002 Elsevier Science Ltd. All rights reserved.

Keywords: Alcohol abuse; Alcohol drinking patterns; Chaos theory; Dynamics; Time series

1. Introduction

Several authors have suggested that chaos theory, an interdisciplinary branch of science that studies the dynamics of nonlinear mathematical systems and applies the knowledge gained to phenomena in nature, might be of value in understanding substance-related disorders (Ehlers, 1992; Hawkins & Hawkins, 1998; Skinner, 1989). In this report, we will

* Corresponding author. Tel.: +1-614-292-9187.

E-mail address: warren.193@osu.edu (K. Warren).

apply a nonlinear mathematical model of substance abuse dynamics to the alcohol intake patterns of an adult male in treatment for substance abuse.

2. A nonlinear model of substance abuse

It would not be difficult to create a linear model of substance abuse dynamics. This model would be a straightforward linear regression, using successive points in time as variables, also known as a linear autoregression (Hamilton, 1994):

$$\Delta I_t = a + b\Delta I_{t-1} + e \quad (1)$$

Eq. (1) says that any change in intake will tend to lead to a further change in the same direction. In this model, increases will lead to further increases, a view consonant with much popular and clinical wisdom (Alcoholics Anonymous, 1976; Doweiko, 1999).

However, this model treats all changes in intake as being equivalent in their influence on the next time period. This seems to be an unreasonable assumption. For example, most individuals who abuse alcohol have to fulfill the usual social responsibilities that come with employment and family life (Doweiko, 1999). A slow increase in the intake of alcohol, say 1 oz/day, might continue for some time without noticeably affecting these responsibilities. On the other hand, an increase of 16 oz/day is likely to immediately interfere with these responsibilities and may also lead to an overdose and possible death if repeated (Doweiko, 1999). Thus, large increases in intake should be less sustainable than small increases.

A nonlinear model allows such a variation in response. The particular nonlinear model we will propose is known as a self-exciting threshold autoregression model or SETAR (Hansen, 1997, 1999; Tong, 1990). A SETAR includes multiple regression lines with thresholds between them. A SETAR with two regimes, such as we are proposing, is known as a SETAR(2). The functional form of the SETAR(2) model that we will test is:

$$\begin{aligned} \Delta I_t &= a \pm b\Delta I_{t-1} + e, \text{ if } \Delta I_{t-1} \leq \text{threshold value } t \\ \Delta I_t &= c \pm d\Delta I_{t-1} + e, \text{ if } \Delta I_{t-1} > \text{threshold value } t \end{aligned} \quad (2)$$

This model (Eq. (2)) will allow large increases to be less sustainable than small increases.

3. Methodology

When testing for the presence of nonlinear structure in a time series, the null hypothesis is linear structure, in contrast to the typical null of no structure at all (Hansen, 1997, 1999). The SETAR(2) model is therefore tested against a linear autocorrelation model with the same number of time lags (Hansen, 1999).

However, since the threshold is not identified under the null hypothesis of linearity, the asymptotic distribution of F under the null is unknown (Hansen, 1997). Because of this, a parametric bootstrap procedure is used to construct the null distribution of F . In this

procedure, residuals under both the null (linearity) hypothesis and alternative (SETAR(2)) hypothesis are simulated and used to construct a bootstrap F distribution (Hansen, 1997). This, in turn, means that the relationship between F and P values will differ from that in a standard F distribution table.¹

4. Data

The data for this study were drawn from records of daily alcohol intake kept by a 40-year-old, single, European–American male alcohol abuser and binge drinker with a family history of alcoholism, over a period of 2041 days. The client's alcohol intake pattern over the decade preceding therapy was $\frac{1}{2}$ –1 pint of bourbon three to four times per week, and the time series was a record of ounces of bourbon drunk per day.

Improvement in the client's condition led to a visually evident nonstationarity in the time series around the 800th recorded day. Around this time, the client's alcohol consumption lessened noticeably. Because we were interested in the most active period of the client's substance use, we analyzed the first 796 days of the intake record.

Analysis of this record showed a strong 7-day periodicity. Such a periodicity can obscure underlying nonlinear patterns (Williams, 1997). We removed the periodicity by taking seven means, one for all Sundays, a second for all Mondays, and so on, and then subtracted the seven means from their corresponding days for the entire time series. We then took seven standard deviations and divided each day by its standard deviation. Finally, we took a first difference of the time series to create a record of daily changes.

5. Results

The results of the nonlinear time series analysis are detailed in Table 1.

A SETAR(2) model over two time lags showed a statistically superior fit when compared to the null hypothesis of a linear autoregression model ($r^2=.24$, $F=65.4$, $P=.00$). Since the model applies to a time series of changes in standard deviation units, the threshold between regimes occurs at an increase of about 0.82 standard deviations; the confidence interval around this threshold reaches from 0.54 to 1.11 standard deviations. One standard deviation in this data set is approximately 7.5 oz, so the threshold occurs at an increase in intake level of between 4.05 and 8.33 oz/day of whiskey; the best least-squares fit occurs at an increase of 6.15 oz/day of whiskey.

As predicted, the two regimes show very different autocorrelation structures. Below the threshold, the 2 days previous to the current day are weakly negatively correlated with the current day. Above the threshold, the day previous to the current day is strongly negatively correlated with the current day. The implication is that either an increase or a decrease in

¹ The program used to fit the models in this paper, along with several of the referenced papers and other work on threshold autoregression models, can be found at <http://www.ssc.wisc.edu/~bhansen/>.

Table 1

Results of threshold autoregression estimated on first difference of daily intake of alcohol

Parameter	Regime 1, $I_{t-1} \leq 0.82$		Regime 2, $I_{t-1} > 0.82$	
	Value	Standard error	Value	Standard error
Constant	0.05	0.03	0.77	0.28
Slope I_{t-1}	-0.29	0.05	-0.92	0.16
Slope I_{t-2}	-0.25	0.04	0.40	0.11

Threshold estimate: 0.82. 95% Confidence interval: 0.54, 1.11. Joint $r^2=0.24$. Bootstrap $F=65.4$. $P=0.00$.

consumption will tend to lead to a rebound effect; an increase in consumption will typically lead to a decrease the next day, and vice versa. The strength of the rebound depends on the strength and direction of the initial change.

6. Discussion

Single subject time series analysis is necessary for the investigation of nonlinear dynamics (Kaplan & Glass, 1995; Williams, 1997) since combining different nonlinear time series will yield a data set that is indistinguishable from random noise (Peak & Frame, 1994). That having been said, it would be useful to obtain more single subject time series for nonlinear analysis, and it is naturally unclear how far one can generalize the results of any single case study.

However, even an exploratory study such as this may yield insight into the dynamics of substance abuse. In turn, these dynamics may play a role in the maintenance of abuse. Glass and Mackey (1988, pp. 172–181) have referred to diseases in which dynamics play a fundamental role as “dynamical diseases.” If substance abuse is a dynamical disease, time series studies such as this one are likely to be of value in both understanding and treating the problem.

To most social scientists and clinicians, it may not be immediately obvious how an understanding of dynamics might affect our understanding of substance abuse. To demonstrate that a dynamical view of alcoholism can be of interest, we will ask a question that is pertinent to the treatment of alcohol abuse: Is controlled drinking or abstinence a preferable strategy for maintaining sobriety?

At the core of the debate on controlled drinking vs. abstinence lies the question of whether controlled drinking is likely to stay controlled (Doweiko, 1999). A dynamical model allows us to address this by asking what will happen after an increase in intake of alcohol. If increases tend to last then controlled drinking strategies are likely to fail, as successive lasting increases raise the intake of alcohol to higher and higher levels.

For the sake of simplicity, we will assume that the fitted parameters of the model are the true parameters. This gives us the following equation that governs changes in intake of alcohol expressed in standard deviation units:

$$\Delta I_t = 0.05 - 0.29\Delta I_{t-1} - 0.25\Delta I_{t-2} + e \text{ if } I_{t-1} \leq 0.82$$

$$\Delta I_t = 0.77 - 0.92\Delta I_{t-1} + 0.40\Delta I_{t-2} + e \text{ if } I_{t-1} > 0.82 \quad (3)$$

In order to use this equation to model the dynamics of substance abuse, we will assume an initial increase in intake, feed that increase into Eq. (3), then take the output of the equation, and put it back into the equation as input.

Let us assume that we have an initial increase in intake of 0.5 standard deviations, approximately 3.5 oz. The change in intake during the next time period will be :

$$\Delta I_t = 0.05 - (0.29 * 0.5) = -0.095 \quad (4)$$

Thus, there is a small “rebound effect.” The increase is followed by a small decrease.

We now take the output value -0.095 and feed it back into Eq. (4) as input. Our new equation includes the initial increase as an influence that appears in the second time lag ΔI_{t-2} . So, 2 days after the initial increase, the change will be:

$$\Delta I_t = 0.05 - (0.29 * -0.095) - (0.25 * 0.5) = -0.04745 \quad (5)$$

We could continue this process indefinitely, but the implication is already clear. An initial increase will tend to be followed by a decrease, a rebound effect, but the decrease will be much smaller than the initial increase, since the two multipliers in Eq. (5) are 0.29 and 0.25. We can think of this as an effort at control that is only partly successful. If our subject increases his consumption of alcohol, he will lower it a bit over the next few days but not enough to compensate for the initial increase.

We can use the same process to model a larger increase, one that crosses the threshold. Assume an increase of one standard deviation, about 7.5 oz of whiskey. In this case, the change in intake during the next time period will be (Eq. (6)):

$$\Delta I_t = 0.77 - 0.92 * 1 = -0.15 \quad (6)$$

The new value, -0.15 , lies below the threshold; thus, further fluctuations will conform to the dynamics expressed in Eq. (7), with differing values for the variables:

$$\Delta I_t = 0.05 - (0.29 * -0.15) - (0.25 * 1) = -0.1565 \quad (7)$$

Again, the rebound is much smaller than the initial increase. A pattern of stable, controlled drinking will be difficult to maintain.

7. Conclusion

To some extent, this analysis simply confirms conventional practice wisdom and research (Doweiko, 1999). Such confirmation is of value; converging lines of evidence tend to strengthen any scientific position. But, we believe that our analysis adds to the conventional wisdom in at least one way, since it suggests a detailed model of relapse. In this model, relapse includes both increases and decreases in intake, with the increases outweighing the decreases. This is intriguing because such a pattern could be deceptive for both client and practitioner. At any given time, it could look as if the client is making progress in bringing her

or his problem under control, while over a sufficiently long period of time the problem would be slipping out of control.

References

- Alcoholics Anonymous* (1976). New York: Alcoholics Anonymous World Services.
- Doweiko, H. E. (1999). *Concepts of chemical dependency* (4th ed.). Pacific Grove, CA: Brooks/Cole.
- Ehlers, C. L. (1992). The new physics of chaos: can it help us understand the effects of alcohol? *Alcohol Health and Research World*, 16 (4), 169–176.
- Glass, L., & Mackey, M. C. (1988). *From clocks to chaos: the rhythms of life*. Princeton, NJ: Princeton Univ. Press.
- Hamilton, J. D. (1994). *Time series analysis*. Princeton, NJ: Princeton Univ. Press.
- Hansen, B. E. (1997). Inference in TAR models. *Studies in Nonlinear Dynamics and Econometrics*, 2(1), 1–14.
- Hansen, B. E. (1999). *Testing for linearity*. Social Systems Research Institute Working Paper #9907. Madison, WI: Social Systems Research Institute.
- Hawkins, R. C., & Hawkins, C. A. (1998). Dynamics of substance abuse: implications of chaos theory for clinical research. In L. Chamberlain, & M. R. Butz (Eds.), *Clinical chaos: a therapist's guide to nonlinear dynamics and therapeutic change* (pp. 89–101). Philadelphia, PA: Brunner/Mazel.
- Kaplan, D., & Glass, L. (1995). *Understanding nonlinear dynamics*. New York: Springer-Verlag.
- Peak, D., & Frame, M. (1994). *Chaos under control: the art and science of complexity*. New York: W.H. Freeman.
- Skinner, H. A. (1989). Butterfly wings flapping: do we need more 'chaos' in understanding addictions. *British Journal of Addiction*, 84, 353–356.
- Tong, H. (1990). *Non-linear time series: a dynamical system approach*. New York: Clarendon Press.
- Williams, G. P. (1997). *Chaos theory tamed*. Washington, DC: Joseph Henry Press.