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Chaos in Easter Island Ecology

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Abstract: *This paper demonstrates that a recently proposed dynamical model for the ecology of Easter Island admits periodic and chaotic attractors, not previously reported. Such behavior may more realistically depict the population dynamics of general ecosystems and illustrates the power of simple models to produce the kind of complex behavior that is ubiquitous in such systems.*

Key Words: chaos, Easter Island, ecology, population dynamics

INTRODUCTION

The Easter Island, called "Rapa Nui" by its natives, has an area of about 170 km^2 and is located about 27 degrees south of the Equator in the Pacific Ocean and over 2000 km east of its nearest inhabited neighbor of Pitcairn Island. Thus it is one of the most remote inhabited spots in the World and an ideal location for studying an isolated and relatively simple ecology. Humans arrived on Easter Island sometime between 400 and 1200 AD, presumably from the Polynesian islands to the west despite Thor Heyerdahl's Kon-Tiki expedition in 1947 demonstrating that South Americans could have sailed the 3500 km from the east (Heyerdahl, 1950). The human population may have reached about 10,000 by the year 1680 but declined to a mere 110 in the year 1877 (Cohen, 1995), and it now stands at about 5000. The discovery of the island by Europeans in 1722 led to the spread of disease, slave trade, and eventually to a thriving tourist industry.

Pollen records (Dransfield, Flenley, Harkness, & Rapu, 1984) show that the island was once forested with large but slowly growing Jubaea palm trees that were used to construct and transport the hundreds of stone statues for which the island is famous as well as to make dwellings and fishing boats and to provide nesting sites for birds, which were also a source of food. The common assumption is that the inhabitants deforested the island, leading to starvation, war, possibly cannibalism, and a general decline of their once thriving civilization. It has been cited as a prime example of the dangers of over-consumption on a global scale (Diamond, 2005; Flenley & Bahn, 2003). However, recent evidence (Hunt, 2006, 2007) indicates that the demise of the trees might be partly a result of a large population of Pacific rats brought to the island by the early settlers either as stowaways or as a source of food and who consumed most of the seeds produced by the trees.

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BRANDER-TAYLOR MODEL

One of the earliest attempts to model the population dynamics of Easter Island involved a two-component economic model (Brander & Taylor, 1998) in which P is the labor productivity (people) and T is the resource stock (trees), with a dynamic given by

$$\frac{dP}{dt} = P\left(b - d + \phi\alpha\beta T\right)$$

$$\frac{dT}{dt} = rT\left(1 - \frac{T}{K}\right) - \alpha\beta PT$$

where the seven parameters are assumed constant with estimated values of $d-b = 0.1$, $\phi = 4$, $\alpha = 10^{-5}$, $\beta = 0.4$, $r = 0.04$, and $K = 12{,}000$ with time in units of decades. This system is a conventional Lotka-Volterra (predator-prey) model in which the people are the predators and the trees are the prey.

Without loss of generality, we can linearly rescale $P \rightarrow Pr/\alpha\beta$, $T \rightarrow KT$, and $t \rightarrow t/r$ to obtain an equivalent but simpler two-parameter model given by

$$\frac{dP}{dt} = P\left(\eta T - \gamma\right)$$

$$\frac{dT}{dt} = T\left(1 - T - P\right)$$

where $\eta = \phi\alpha\beta K/r$ can be thought of as the effort per person expended in harvesting trees and $\gamma = (d-b)/r$ is the relative rate at which the human population dies in the absence of resources ($T = 0$). This system has three equilibria given by $(P, T) = (0, 0)$, $(0, 1)$, and $(1-\gamma/\eta, \gamma/\eta)$. The third (coexisting) equilibrium exists only if $\eta > \gamma$, and it is stable whenever it exists. Coexistence requires a minimum harvesting effort, and that effort increases inversely with the decreasing number of trees ($\eta = \gamma/T$). With constant parameters, the model cannot produce a rise and fall of the population to zero, but it can produce a rise and fall to a much smaller stable value.

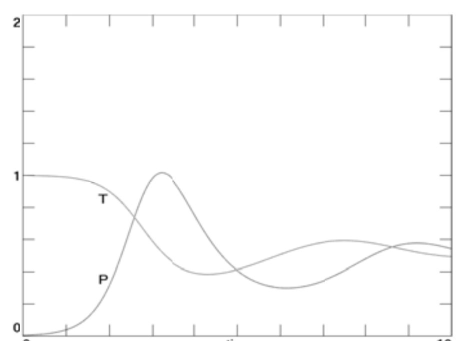

Fig. 1. History of the people (*P*) and trees (*T*) predicted by the Brander-Taylor Model. Time is in units of 250 years, and the human population peaks at a value of about 10,100 people.

The parameters suggested by the authors give $\eta = 4.8$ and $\gamma = 2.5$. A numerical solution with initial conditions of $P = 0.004$ (corresponding to 40 people) and $T = 1$ at $t = 0$ produces the transient shown in Fig. 1 and replicates a graph in the original paper. It shows a rise and fall approaching a steady-state value about 51% of its peak value with decaying oscillations about the equilibrium. The model has a plausible population growth, but the crash is rather too slow, and incomplete.

BASENER-ROSS MODEL

To counter the limitations of the Brander-Taylor Model, Basener and Ross (2005) proposed an alternate model in which the carrying capacity of the population is equal to the number of trees in units of the number required to sustain one person and governed by a logistic growth equation with a harvesting term ($-hP$) proportional to the number of people as given by

$$\frac{dP}{dt} = aP\left(1 - \frac{P}{T}\right)$$

$$\frac{dT}{dt} = cT\left(1 - \frac{T}{K}\right) - hP$$

where the four parameters are chosen to fit population estimates from Easter Island data as given by $a = 0.0044$, $c = 0.001$, $h = 0.025$, and $K = 70{,}000$ with time in units of years.

As before, we can linearly rescale $P \rightarrow PcK/h$, $T \rightarrow KT$, and $t \rightarrow t/c$ to obtain an equivalent two-parameter model given by

$$\frac{dP}{dt} = \gamma P\left(1 - \frac{P}{\eta T}\right)$$

$$\frac{dT}{dt} = T(1 - T) - P$$

where $\gamma = a/c$ is the growth rate of the population in the presence of unlimited resources ($T \rightarrow \infty$) and $\eta = h/c$ is the rate at which trees are harvested, both in units of the initial growth rate of the undisturbed forest. The coexisting equilibrium at $(P, T) = (\eta - \eta^2, 1 - \eta)$ exists only if $\eta < 1$, and in its absence, the human population either days to zero asymptotically or crashes abruptly after a prolonged period of slow growth.

The parameters suggested by the authors give $\gamma = 4.4$ and $\eta = 25$. A numerical solution with initial conditions of $P = 0.018$ (corresponding to about 50 people) and $T = 1$ is shown in Fig. 2 and replicates a graph in the original paper. The crash is much more abrupt and takes the population of both people and trees to zero. The truth probably lies somewhere between the two models, the former being too gentle, and the latter being too drastic.

Basener and Ross (2005) also point out the existence of periodic solutions for a rather different but limited range of parameters corresponding to

$0.5 < \eta < 1$ and $\gamma = 2\eta - 1$. One example of such a solution for $\eta = 0.8$ and $\gamma = 0.6$ is shown in Fig. 3. However, this is a conservative system and structurally unstable, with even the slightest change in parameters causing attraction to the coexisting equilibrium or with growing oscillations leading to extinction of both species. Thus it is not biologically realistic even if the rather extreme parameters could be justified. In a subsequent article, Basener, Brooks, Radin, & Wiandt (2008a) showed that attracting periodic and chaotic solutions exist for a discrete-time version of their model, but the assumptions are questionable for a subtropical island with limited seasonality.

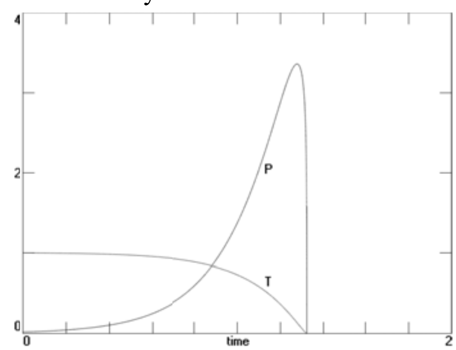

Fig. 2. History of the people (*P*) and trees (*T*) predicted by the Basener-Ross Model. Time is in units of millennia, and the population peaks at a value of about 9400 people.

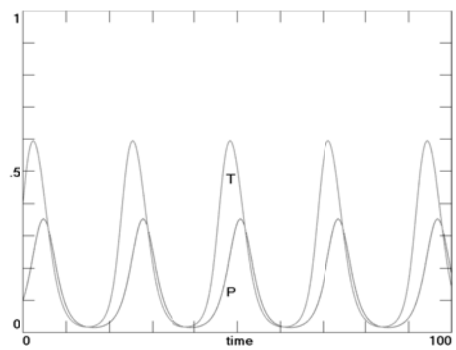

Fig. 3. A periodic solution predicted by the Basener-Ross Model. The unit of time is about 7 years, and the human population oscillates between about 1300 and 30,000 people.

INVASIVE SPECIES MODEL

To include Hunt's claim that rats were in part responsible for the deforestation, Basener, Brooks, Radin, and Wiandt (2008b) advanced a three-component generalization of their model with people (*P*), rats (*R*), and trees (*T*) given by

$$\frac{dP}{dt} = aP\left(1 - \frac{P}{T}\right)$$
$$\frac{dR}{dt} = cR\left(1 - \frac{R}{T}\right)$$
$$\frac{dT}{dt} = \frac{b}{1 + fR}T\left(1 - \frac{T}{M}\right) - hP$$

where T is in units of the number of trees required to support one human and R is in units of the number of rats that can be supported by one tree. The six parameters are typically taken as $a = 0.03$, $c = 10$, $b = 1$, $f = 0.001$, $M = 12{,}000$, and $h = 0.25$ with time in units of years. Note that the people and the rats do not interact directly but they compete for the trees, in the former case through harvesting ($-hP$) and in the latter case through a reduction in the growth rate of the forest (fR).

As before, we can linearly rescale $P \rightarrow PbM/h$, $R \rightarrow MR$, $T \rightarrow MT$, and $t \rightarrow t/b$ to obtain an equivalent four-parameter model given by

$$\frac{dP}{dt} = \gamma_P P\left(1 - \frac{P}{\eta_P T}\right)$$
$$\frac{dR}{dt} = \gamma_R R\left(1 - \frac{R}{T}\right)$$
$$\frac{dT}{dt} = \frac{T}{1 + \eta_R R}(1 - T) - P$$

where $\gamma_P = a/b$ and $\gamma_R = c/b$ are the growth rates of the people and rats, respectively, in the presence of unlimited resources ($T \rightarrow \infty$), and $\eta_P = h/b$ and $\eta_R = fM$ are the rates at which people and rats consume the trees and their seeds, respectively. This system has four equilibria with only the one at $R = P/\eta_P = T = (1- \eta_P)/(1+ \eta_P\, \eta_R)$ corresponding to coexistence of all four species, and it exists only if $\eta_P < 1$.

The parameters suggested by the authors give $\gamma_P = 0.03$, $\gamma_R = 10$, $\eta_P = 0.25$, and $\eta_R = 12$, but with considerable uncertainty. Not surprisingly, they find solutions resembling their two-component model since the models are identical in the limit of $\eta_R = 0$, as well as solutions that resemble those for the Brander-Taylor Model for $\eta_R = 7.2$. They do not report periodic or chaotic attractors but only show attraction to one of the stable equilibria. The main new result reported here is the existence of such attractors in their three-component continuous-time model.

The search for chaotic solutions involved numerical calculation of the largest Lyapunov exponent (Sprott, 2003), for thousands of combinations of the four parameters (γ_P, γ_R, η_P, η_R) in a Gaussian neighborhood of the origin with all parameters positive. The only concession to biological reality was to constrain γ_R to be greater than γ_P since rats reproduce faster than people in given abundant resources. One such solution with (γ_P, γ_R, η_P, η_R) = (0.1, 0.3, 0.47, 0.7) and (P_0, R_0,

T_0) = (0.01, 0.01, 1) is shown in Fig. 4. There is a dominant 80-year cycle, but the behavior is not perfectly periodic, and it is not a transient. A calculation extending the time to 20 million years shows continued chaotic oscillations with Lyapunov exponents of (0.0094, 0, −0.2650) and a Kaplan-Yorke dimension of 2.035. Thus the system is weakly chaotic with information lost on a time scale of about 106 years.

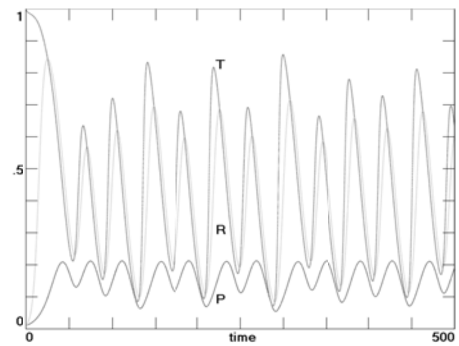

Fig. 4. A chaotic solution predicted by the Invasive Species Model. The unit of time is years, and the human population averages about 7200 people.

Figure 5 shows a return map of the successive minima in the human population. It is unimodal and essentially one-dimensional, although a zoom by a factor of 1000 into a portion of it as shown in the insert reveals the fractal structure expected for a chaotic attractor. What appears to be a single line at low resolution is actually a pair of lines, which in turn presumably consist of pairs of lines, and so forth ad infinitum. A 45-degree diagonal line on such a plot would intersect the curve at the position of the unstable coexistent equilibrium and is approximately the location shown in the zoom.

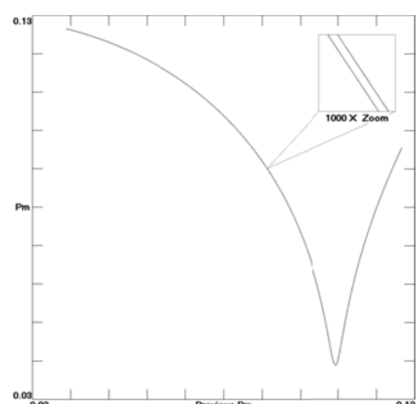

Fig. 5. A return map showing the minimum human population versus the previous minimum for the chaotic Invasive Species Model.

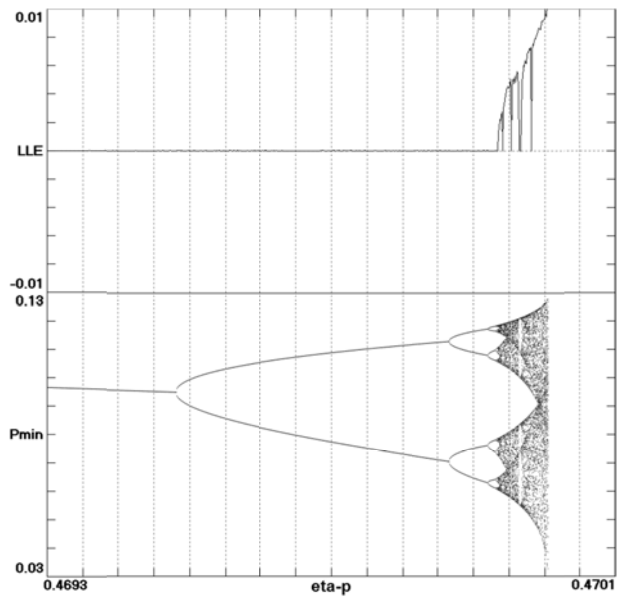

Fig. 6. Largest Lyapunov exponent and minimum value of the human population as a function of the tree-harvest rate for the Invasive Species Model shows a period-doubling route to chaos followed by mass extinction.

It is interesting to consider the route to chaos by varying one of the parameters, which for this purpose is taken as η_P, since the harvest rate is presumably the quantity over which the humans can exert the most control and thus best provides a possible lesson in ecological stewardship. Figure 6 shows the largest Lyapunov exponent and the local minima of P as a function of this parameter. It is evident that the chaos exists over a narrow range of the parameter just before all species become extinct, and that the route to chaos is the common period-doubling of a limit cycle. The limit cycle is born in a Hopf bifurcation at about $\eta_P = 0.4611$. State space plots in the *PT*-plane for increasing values of η_P in Fig. 7 show how the dynamics evolve from attraction to the stable coexisting equilibrium through a period-doubling route to chaos and finally to extinction of all species.

Figure 8 shows the final values of each of the three variables over a larger range of η_P. Increasing the harvesting rate is good for the people up to a point of diminishing returns well before the onset of the Hopf bifurcation and stable oscillations. It is tempting to conclude from this figure that the extinction is rather sudden, but the horizontal axis is the harvesting rate, not time, and the vertical axis is the range of possible populations. To the extent that the harvesting rate is under human control, one has at least the order of one human lifespan to recognize the instability and reduce the harvesting rate.

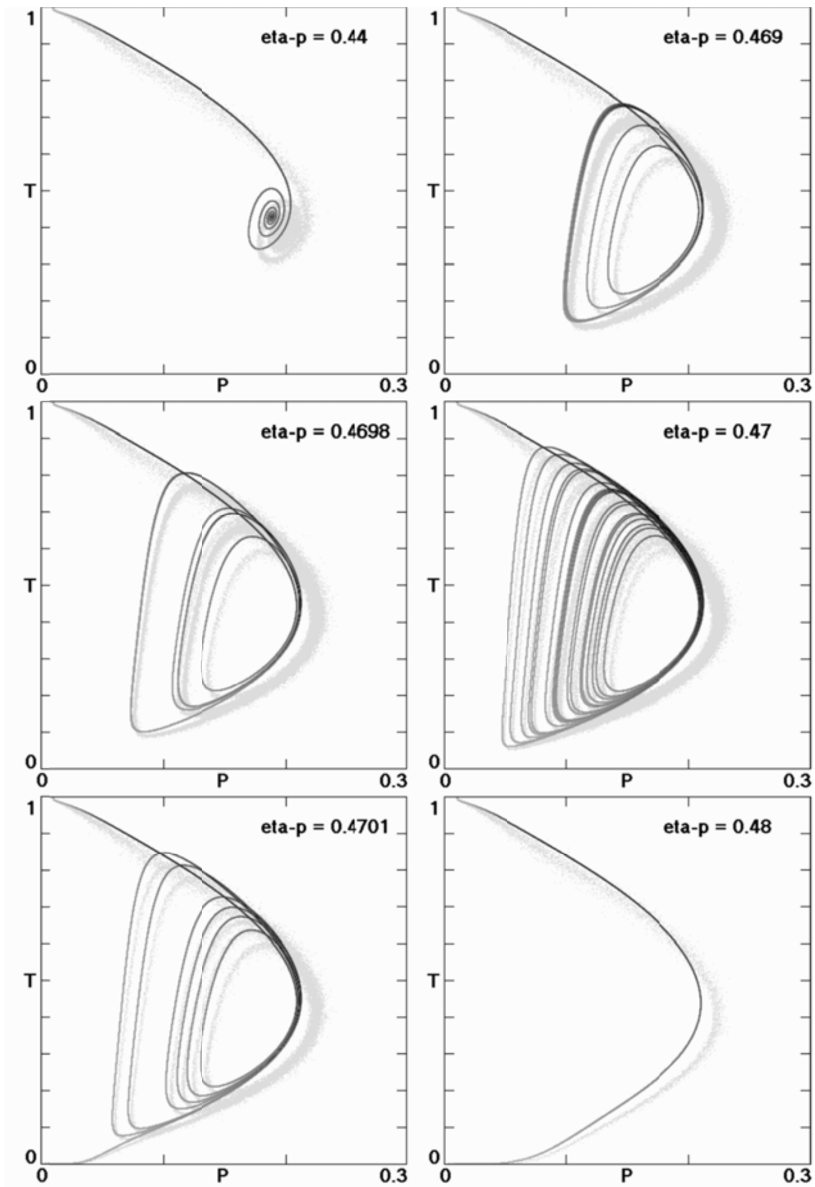

Fig. 7. State space plots for the Invasive Species Model at increasing values of the harvesting rate showing successively attraction to the coexisting equilibrium, a simple limit cycle, a period-doubled limit cycle, a chaotic attractor, transient chaos, and rapid extinction.

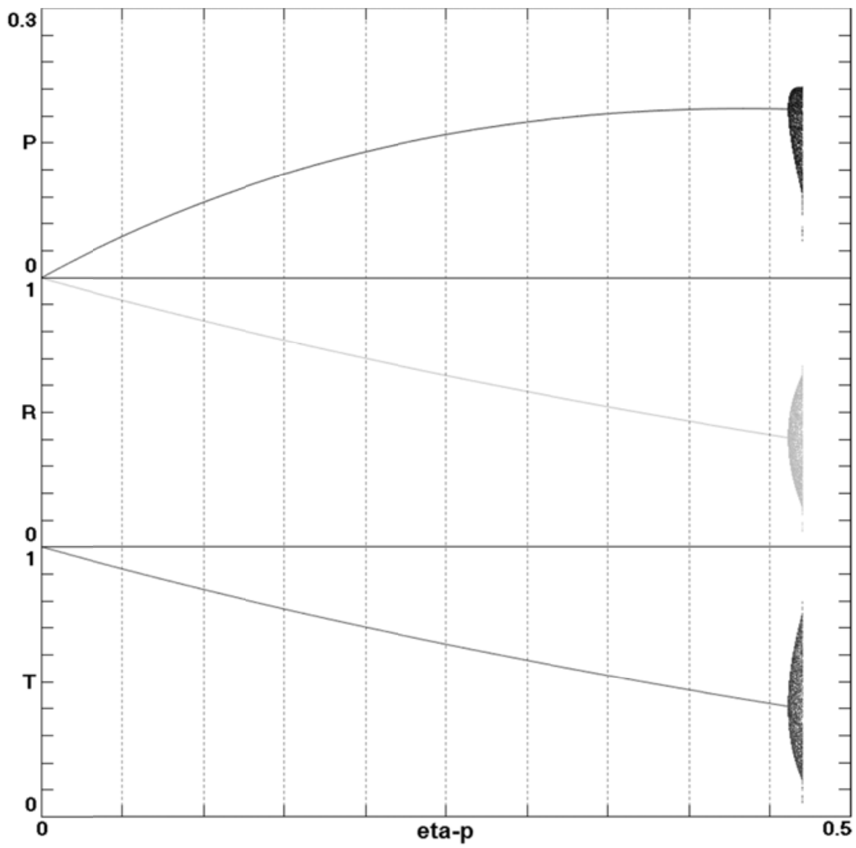

Fig. 8. Population of people (*P*), rats (*R*), and trees (*T*) permitted by the Invasive Species Model as a function of tree harvesting rate, showing the narrow region of periodic and chaotic oscillations preceding the crash.

CONCLUSION

It has been shown here that a simple three-component model of Easter Island ecology admits periodic and chaotic attractors, not previously reported. These solutions exist over a relatively narrow range of parameters, and thus one might argue that they are of limited interest, especially since the dynamics of Easter Island seemingly occurred in a single transient event. However, there are reasons that this result is relevant. Simple models with fixed parameters are unrealistic, and there might be feedback effects not included in the model that alter the parameters to keep the system in a state of weak chaos (sometimes called "the edge of chaos"). Certainly it is human nature to consume at an ever increasing rate until the detrimental effects of that consumption can no longer be ignored. Furthermore, the identification of the route to chaos and eventual extinction might provide a means to recognize impending doom in time to change behavior and avert it. In the case of Easter Island, a decision to reduce the harvesting rate at any time before the last tree was felled might have allowed the island to recover to a stable coexistence. In a society with sufficient technology, other solutions might be preferable such as eradicating the rats. A feature of a chaotic system is that a small change in a parameter can drastically alter the future, just as does a small change in the initial conditions, and we can exploit this sensitivity to produce a more livable world.

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