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A GENERAL APPROACH FOR HYPERCHAOTIFYING N-DIMENSIONAL CONTINUOUS-TIME SYSTEMS

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Abstract:

This paper is concerned with the rigorous construction of hyperchaotic attractors in n-dimensional continuous-time systems. The method of analysis is based on the construction of a matrix controller and by using the standard dentition of Lyapunov exponents for an asymptotically stable limit set of the original system. The relevance of this approach is to show mathematically the possibility of controlling asymptotically stable limit sets of n-dimensional continuous-time systems to hyperchaos. This general approach is valid for all *n*-dimensional continuous-time systems with at least one asymptotically stable limit set.

Keywords: Hyper chaotification, matrix controller, n-dimensional continuous-time systems.

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Introduction:

The anti control of chaos or chaotification is the reverse of suppressing chaos in a dynamical system. The aim of this process is to create or enhance the system complexity for some special application, in particular, some novel, time or energy-critical, interdisciplinary application such as high-performance circuits and devices, liquid mixing, chemical reactions, biological systems, crisis management, secure information processing, critical decision-making in politics, economics, as well as military applications, and so forth. In other words, anticontrolling chaos produces chaotic behavior in a system that would not otherwise be chaotic [1-5]. Many chaotification methods have been proposed to generate chaos in n-dimensional continuous-time systems, including defferential geometry control [2-3], time-delayed feedback [2], and switching piecewise-linear control [3]. An effective strategy for anticontrolling chaos in
continuous-time systems has been discussed [4] using a homogeneity-based approach with the *p*-normal forms of nonlinear systems. However, a general approach to hyperchaotifying these systems is not available in the current literature. To make a link between chaotification methods and what is currently done, we note that generally, the main role of the media is to inform people about the reality of events. However, the media often uses its reporting of important events to influence its own interests. This action can be regarded as an intervention in the nature of events (or the creation of events) to control them for a desired objective. Generally, the result is in opposition to the interests of its enemies. The resulting behavior can vary from nothing to catastrophes. Examples of this situation can be seen in the case of Iraq, Libya, and Syria, resulting in civil war. Mathematically, if the event under consideration is modeled by a differential equation, then any action of the media on it can considered as a controller. This paper presents a new general approach based on a matrix controller, which creates hyperchaos in n-dimensional continuous-time systems. The rigorous mathematical justification is given by using the standard definition of Lyapunov exponents. Finally, some concluding remarks are given.

A general approach for hyperchaotifying n-dimensional continuous-time systems:

Consider the uncontrolled n-dimensional continuous-time system

$$\begin{cases} \frac{dx}{dt} = f(x,t),\ (x,t) \in (0,+\infty) \subset \Re^n \times (0,+\infty) \\ x_0 \in \Omega - given \end{cases} \quad (1)$$

where $n \geq 2$ if system (1) is non-autonomous and $n \geq 3$ if system (1) is autonomous. Here $x(t) = (x_1(t),...,x_n(t))^T \in \Re^n$ is the state variable, and $f(x,t) = (f_1(t),...,f_n(t))^T \in \Re^n$ is assumed to be a nonlinear continuous function with respect to x in Ω for any $t \in (t_0,+\infty)$, and the Jacobian $f_x(x,t)$ is bounded. Here f_x is given by $f_x(x,t) = \frac{\partial f(x,t)}{\partial x}$. Assume that system (1) has at least one asymptotically stable limit set $x(t)$. Thus the fundamental solution matrix of system (1) solves the initial-value problem $\frac{d\psi(t)}{dt} = f_x(x,t)\psi(t)$ with $\psi(0) = I_n$, where I_n, is the unit matrix. Now consider the controlled system

$$\begin{cases} \frac{dy}{dt} = f(y,t) + g(t)y \\ y_0 \in \Omega - given \end{cases} \quad (2)$$

where $g(t) = (g_{ij}(t))_{1\leq i,j\leq n}$ is the matrix controller to be determined in which system (2) becomes hyperchaotic and the vector $y(t) = (y_1(t),...,y_n(t))^T \in \Re^n$ is the state variable. The fundamental solution matrix of the controlled system (2) solves the initial-value problem $\frac{d\Phi(t)}{dt} = B(t)\Phi(t)$ with $\Phi(0) = I_n$, where $B(t) = f_y(t,y) + g'(t)$ is the Jacobian matrix of system (2).

Assume that $\Phi(t) = h(\Psi(t))$, where $h : \Re^{+} \to \Re^{+}$ is any differentiable, invertible and positive function. Assume that h can be extended to square matrices. The case of polynomials of degree d and some exponential functions are special choices for the function *h*. The equation $\Phi(t) = h(\Psi(t))$ implies that $\Phi'(t) = h'(\Psi(t))\Psi'(t)$.

Thus we have $g'(t) = \left[h'(\psi(t))f_x(x(t),t)\psi(t) - f_y(y(t),t)h(\psi(t)\right]h^{-1}(\psi(t)$ The formula for the matrix $g(t)$ can be obtained by integrating all the entries of the matrix $g'(t)$ from 0 to t as follows:

$$g(t) = \int_0^t \left[h'(\psi(s))f_z(x(s),s)\psi(s) - f_y(y(s),s)h(\psi(s)\right]h^{-1}(\psi(s)ds \qquad (3)$$

The calculation of the matrix $g(t)$ requires the knowledge of all the matrices in the formula (3). However, this requirement is hard to achieve for nonlinear systems. Thus numerical integrations are the practical way to find the matrix $g(t)$.

Let $\mu_i(t)$ be an eigenvalue of $\Psi(t)$ for an asymptotically stable limit set $x(t)$ of system (1).

We have $\det(\Phi(t) - m_i(t)I_n) = 0$, that is,

$\det(h(\Psi(t)) - m_i(t)I_n) = 0$, which means that

$m_i(t) = h(\mu_i(t))$ is an eigenvalue of $\Phi(t)$ for an asymptotically stable limit set $y(t)$ of the controlled system (2). Now assume that the Lyapunov exponents of system (1) are ordered as follows $\mu_1 \geq \mu_2 \geq ... \geq \mu_n$.

Hence the Lyapunov exponents for system (2) associated with the eigenvalue $m_i(t)$ is defined by

$$\delta_i = \lim_{t\to+\infty} \sup \frac{1}{t}\ln(m_i(t)),\ i = 1,...,n$$

Therefore, all the Lyapunov exponents for the controlled system (2) are given by

$$\delta_i = \lim_{t\to+\infty} \sup \frac{1}{t}\ln(h(\mu_i(t)),\ i = 1,...,n \qquad (4)$$

If there exists a $j \in \{1,...,n\}$ such that

$\lim_{t\to+\infty} \sup \frac{1}{t}\ln(h(\mu_j(t)) > 0,$ then system (2) is chaotic. If $h(\mu_i(t)) > 1,$, for all $i = 1,...,n$ then $\delta_1 \geq 0, \delta_2 \geq 0, ... \geq \delta_n \geq 0.$ Thus system (2) is hyperchaotic. These inequalities are possible if, for example, $h(x) > 1,$ for all $x \in \Re^{+}$. However, this is a strong condition. If one can compute the Lyapunov exponents for the uncontrolled system (1), then it is possible to assume that $h(x) > 1,$ only for all $x \in \{\mu_i, i = 1,...,n\}$.

The Algorithm:

The above method can be summarized in an algorithm as follows:

1. n, f, x_0 given
2. Calculate $f_x(x,t) = \dfrac{\partial f(x,t)}{\partial x}$ and hence $\Psi(t)$ with $\Psi(0) = I_n$.
3. Select a differentiable, invertible, and positive function h that can be extended to square matrices.
4. Calculate $B(t) = f_y(t,y) + g'(t)$ and hence $\Phi(t) = h(\Psi(t))$ with $\Phi(0) = I_n$.
5. Calculate the matrix $g(t)$ by using formula (3).

Discussion:

In this section, we make the following remarks:

1. The result presented here is valid for the hyperchaotification of linear systems if the controller $g(t)$ is nonlinear.
2. The nonlinear controller $g(t)$ transforms any asymptotically stable limit set of system (1) into a hyperchaotic attractor.
3. If the function h satisfies $h(x) > 1$ for all $x \in \Re^{+}$, then the knowledge of the Lyapunov exponents for an asymptotically stable limit set of the uncontrolled system (1) is not necessary for applying the proposed nonlinear control law.
4. Constructing such an example to validate the presented approach is not an easy task and depends on several conditions on the original and the controlled systems. In any case, the correctness of the proposed method through mathematical analysis makes it a valuable analytic result in the domain of differential equations and dynamical systems.

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5. The main importance of the proposed approach is to show the possibility of transforming asymptotically stable limit sets to hyperchaotic attractors in n-dimensional continuous-time systems. This idea has been under investigation for some time. The approach is general and valid for all n-dimensional continuous-time systems with at least one asymptotically stable limit set.

Conclusion:

In this paper, we have constructed a new matrix controller that allows a transition between asymptotically stable limit sets and hyper chaos in n-dimensional continuous-time systems. This new and general approach is valid for all continuous-time systems with at least one asymptotically stable limit set. The usefulness of the method is to present a general framework for generating hyper chaos in dynamical systems.

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