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# Linearization of the Lorenz system

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ABSTRACT

A partial and complete piecewise linearized version of the Lorenz system is proposed. The linearized versions have an independent total amplitude control parameter. Additional further linearization leads naturally to a piecewise linear version of the diffusionless Lorenz system. A chaotic circuit with a single amplitude controller is then implemented using a new switch element, producing a chaotic oscillation that agrees with the numerical calculation for the piecewise linear diffusionless Lorenz system.


## 1. Introduction

The Lorenz model [1] describes the motion of a fluid under the conditions of Rayleigh–Bénard flow [2], and it has become a paradigm for chaotic dynamics. Furthermore, recent publications [3–10] show that the Lorenz system is still being actively researched. There is an inherent mechanism for the motion of the convective flow, which is governed by the stream function and the temperature deviation function. When the goal is to find the factors that lead to chaotic dynamics, it is necessary to consider the nonlinearity in the Lorenz model that represents a coupling between the fluid motion and the temperature deviation. The Lorenz equations provide a useful physical model of the dynamics assuming the actual fluid motion has only one spatial mode in the $x$ direction and the temperature difference between top and bottom boundaries is not too large. Therefore, the Lorenz model has inherent limitations, and it is instructive to study diversified forms of it that could have physical implications. A natural question to ask is how the Lorenz system is modified when the amplitude information in the nonlinearity is removed by using a signum function, which leads to a piecewise linearization of the Lorenz model, which to our knowledge has not previously been done.

Furthermore, the piecewise linearity can be simply implemented electronically using diodes and operational amplifiers [11–16], whereas the usual quadratic nonlinearities require multipliers [17–19]. For some dynamical systems, this substitution preserves the chaotic dynamics. Another reason for doing this is that the resulting equations can be solved exactly in the linear regions with boundary conditions where the discontinuities occur. The method is analogous to Lozi's piecewise linearization of the Hénon map, where the quadratic term is replaced by an absolute-value term [20], or to the piecewise linearization of a jerk system by Linz and Sprott [21]. In addition, the piecewise linearization may allow a single amplitude control parameter [17,22,23], which is helpful for circuit implementation in radar or communication engineering to reduce the circuit complexity and avoid saturation of the amplifiers, which can be a problem because of the broadband frequency spectrum of a chaotic signal.

In this paper, linearization of the Lorenz system is achieved by ignoring the amplitude of one variable in the quadratic terms. What we are doing is not the same as the common linearization of a nonlinear system about an equilibrium point, but rather a piecewise linearization of a nonlinear system that retains the chaotic dynamics. In Section 2, one of the two quadratic terms is transformed into a non-smooth term with a signum operation, and a partially linearized version of the Lorenz system is derived. In Section 3, both of the quadratic terms are linearized by the signum operation, and a corresponding completely linearized version of the Lorenz system is obtained. Both cases have a total amplitude control parameter. In Section 4, a piecewise linear diffusionless

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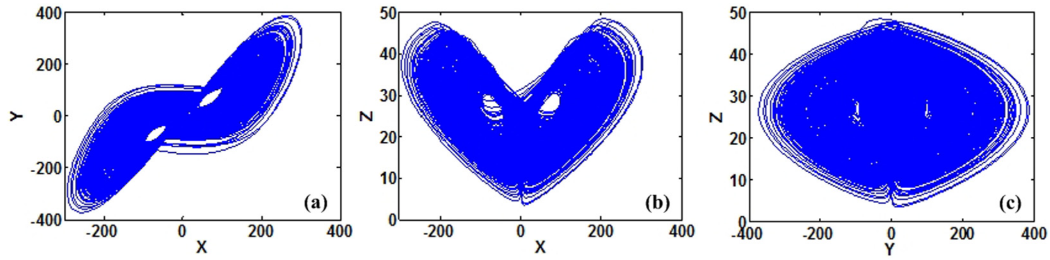



**Fig. 1.** Strange attractor from system (2) with $\sigma = 10$, $r = 28$, $\beta = 8/3$ for initial conditions $(0, 1, 0)$ with LEs $= (0.40_{56}, 0, -14.07_{23})$ (a) $x$–$y$ plane, (b) $x$–$z$ plane, (c) $y$–$z$ plane.

Lorenz system is obtained by further simplification, which has the same structure as the quadratic case but with coexisting strange attractors for some values of the parameters. In Section 5, the bifurcation and multistability of the piecewise linear diffusionless Lorenz system is analyzed. The circuit implementation is presented in Section 6. Conclusions and discussions are given in the last section.

## 2. Partially linearized Lorenz system

The familiar Lorenz system is given by

$$\begin{aligned}\dot{x} &= \sigma(y - x)\\ \dot{y} &= -xz + rx - y\\ \dot{z} &= xy - \beta z\end{aligned} \tag{1}$$

with chaotic solutions for $\sigma = 10$, $r = 28$, $\beta = 8/3$. The system has rotational symmetry with respect to the $z$-axis as evidenced by its invariance under the coordinate transformation $(x, y, z) \to (-x, -y, z)$, and it has a partial amplitude control parameter hidden in the coefficient of the $xy$ term, which controls the amplitude of $x$ and $y$, but not $z$. To obtain total amplitude control, it is necessary to introduce an equal control factor into the $xz$ term [22]. To obtain total amplitude control with a single parameter, it is necessary to make all the terms have the same order except for the one whose coefficient provides the amplitude control [17,23]. Since a signum operation will retain the polarity information while removing the amplitude information, applying it to one of the factors in a quadratic term reduces the order of that term from 2 to 1. Then the coefficient of the remaining quadratic term gives total amplitude control because it is the only term with an order different from unity. This idea leads to the partial linearization

$$\begin{aligned}\dot{x} &= \sigma(y - x)\\ \dot{y} &= -xz + rx - y\\ \dot{z} &= x\,\mathrm{sgn}(y) - \beta z\end{aligned} \tag{2}$$

With $(\sigma, r, \beta)$ the same as for the quadratic system, system (2) gives the strange attractor shown in Fig. 1, which resembles the familiar Lorenz attractor, but with considerably larger $x$ and $y$ values. The Lyapunov exponents (LEs) given in the figure caption imply a Kaplan–Yorke dimension of $2.02_{88}$ and provide the main evidence that the system is chaotic. Note that the variable $x$ appears in four of the seven terms, and thus it is especially important in determining the dynamic behavior. Although there are two ways to linearize $xy$, namely $x\,\mathrm{sgn}(y)$ and $y\,\mathrm{sgn}(x)$, it is reasonable that $x\,\mathrm{sgn}(y)$ works better for retaining the chaos. It is tempting to linearize the $xz$ term in system (2) by replacing it with $x\,\mathrm{sgn}(z)$, but that destroys the coupling among the variables since $z$ is always positive, and consequently, the first two dimensions will be independent of the third dimension.

Systems (1) and (2) both have three equilibrium points. The equilibrium points of system (1) are $(x, y, z) = (0, 0, 0)$ and $(\pm 8.4853, \pm 8.4853, 27)$, whose eigenvalues are $(11.8277, -2.6667, -22.8277)$ and $(-13.8546, 0.0940 \pm 10.1945i)$, respectively. The origin equilibrium point is a saddle-node, and the symmetric pair of equilibrium points are saddle-foci with identical eigenvalues. The equilibrium points of system (2) are $(0, 0, 0)$ and $(\pm 72, \pm 72, 27)$, whose eigenvalues are $(11.8277, -2.6667, -22.8277)$ and $(-14.9316, 0.6324 \pm 6.9152i)$, indicating the same stability as for system (1). For both systems, the rate of volume expansion is $-(\sigma + \beta + 1)$, and thus the systems are dissipative when the parameters are positive with solutions as time goes to infinity that contract onto an attractor of zero measure in their state space. However, the bifurcations for the parameters $\sigma$ or $\beta$ in the original system (1) and the revised system (2) are totally different. The revised system (2) shows relatively robust chaos over a range of both parameters. Specifically, there is a wide range of the parameter $\sigma$ for system (2) to give a symmetric pair of coexisting strange attractors, while the original system (1) shows global attraction and bifurcations with different dynamics.

There is a well-known difficulty when calculating Lyapunov exponents for systems that involve discontinuous functions such as the signum. This problem arises because of the abrupt change in the direction of the flow vector at the discontinuity and the difficulty of maintaining the correct orientation of the Lyapunov vectors. Although there is a proper procedure for correcting this problem [24], we use here a simpler method in which $\mathrm{sgn}(y)$ is replaced by a smooth approximation given by $\tanh(Ny)$ with $N$ sufficiently large that the calculated Lyapunov exponents are independent of its value [25]. For the case of system (2), a value of $N = 10$ is sufficient to give three-digit accuracy because of the large values of $y$. It is important with this method to use an integrator with an adaptive time step and error control to resolve the rapid change in the vicinity of $y = 0$ and to repeat the calculation with slightly perturbed initial conditions to verify the number of significant digits. Out of an abundance of caution, we quote only two significant digits in the largest Lyapunov exponents and include the remaining questionable digits as subscripts.

The linearized system (2) has two amplitude parameters, unlike system (1), which has only one. A new introduced coefficient $h$ in the remaining quadratic term is a total amplitude controller,

$$\begin{aligned}\dot{x} &= \sigma(y - x)\\ \dot{y} &= -hxz + rx - y\\ \dot{z} &= x\,\mathrm{sgn}(y) - \beta z\end{aligned} \tag{3}$$

To show this, let $x = u/h$, $y = v/h$, $z = w/h$ to obtain new equations in the variables $u$, $v$, $w$ that are identical to system (2). Therefore, the coefficient $h$ controls the amplitude of all variables according to $1/h$. Otherwise, simply note that $xz$ is the only term not of first order.

As with the quadratic system (1), a coefficient $m$ in the signum term will realize partial amplitude control,

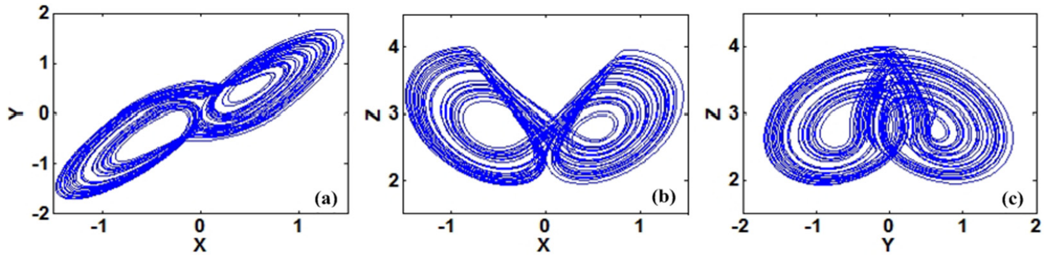



**Fig. 2.** Strange attractor from system (6) with $a = 0.1$, $b = 0.2$, $c = 2.8$ for initial conditions $(0, 1, 0)$ with LEs $= (0.02_{15}, 0, -1.32_{15})$ (a) $x$–$y$ plane, (b) $x$–$z$ plane, (c) $y$–$z$ plane.

$$\begin{aligned}\dot{x} &= \sigma(y - x)\\ \dot{y} &= -xz + rx - y\\ \dot{z} &= mx\,\mathrm{sgn}(y) - \beta z\end{aligned} \tag{4}$$

To show this, let $x = u/m$, $y = v/m$, $z = w$ to obtain new equations in the variables $u$, $v$, $w$ that are also identical to system (2). Therefore, the coefficient $m$ controls the amplitude of the variables $x$ and $y$ according to $1/m$, while the amplitude of $z$ is unchanged. Note that this control has a different scaling from the quadratic case in system (1) where the amplitude of $x$ and $y$ varies as $1/\sqrt{m}$ rather than $1/m$. Finally, note that $m$ and $h$ must be positive to retain the form of the respective equations. The two different kinds of amplitude parameters are independent. If they both exist in the second and third dimension of the system, the amplitude of the variables $x$ and $y$ will change according to $1/mh$, while the amplitude of $z$ will change according to $1/h$. To show this, let $x = u/mh$, $y = v/mh$, $z = w/h$ to obtain new equations in the variables $u$, $v$, $w$ that are identical to system (2).

## 3. Completely linearized Lorenz system

As mentioned above, we selected the piecewise-linearity of $x\,\mathrm{sgn}(y)$ in the third dimension for retaining the chaos since the variable $x$ appears repeatedly in equations determining the dynamic behavior. Furthermore, in order to obtain a completely linearized Lorenz system it is also necessary to linearize the $xz$ term in equation (2) by replacing it with $z\,\mathrm{sgn}(x)$ for positive $z$. However, that would make all the terms of first order, and there would be nothing to determine the size of the attractor. Thus to retain the chaos, it is necessary either to increase the order by imposing a quadratic nonlinearity or reduce the order by applying a further signum operation to one of the linear terms.

To scale the variables to more convenient values and move the parameters to different terms, we make a linear transformation of system (1) by taking $x \to \sigma x$, $y \to \sigma y$, $z \to \sigma z$, $t \to t/\sigma$ to obtain

$$\begin{aligned}\dot{x} &= y - x\\ \dot{y} &= -xz + cx - ay\\ \dot{z} &= xy - bz\end{aligned} \tag{5}$$

where the new parameters are $a = 1/\sigma = 0.1$, $b = \beta/\sigma = 4/15$, and $c = r/\sigma = 2.8$. The new system (5) is the same as the original Lorenz system (1) except for the amplitude and frequency, and so the physics is preserved. Linearization of system (5) will not only provide amplitude control, but will provide an explanation of the sensitive dependence on initial conditions in the Lorenz model.

It is possible to linearize both quadratic terms (making them first order) and remove the amplitude dependence in the $cx$ term (making it zeroth order) by replacing it with $c\,\mathrm{sgn}(x)$ to obtain a completely linearized Lorenz system given by

$$\begin{aligned}\dot{x} &= y - x\\ \dot{y} &= -z\,\mathrm{sgn}(x) + c\,\mathrm{sgn}(x) - ay\\ \dot{z} &= x\,\mathrm{sgn}(y) - bz\end{aligned} \tag{6}$$

There is no direct chaotic solution in the completely linearized Lorenz system if it comes from system (1) because the common coefficient $\sigma$ in the first dimension implies different time scales in the evolution of the variables. The system (5) gives birth to chaotic solutions with small amplitude and low frequency. Therefore the loss of amplitude information in the variables $x$ and $y$ will not greatly influence the dynamics. Although system (6) is no longer chaotic for the given parameters, a small change of $b$ from $b = 4/15$ to $b = 3/15 = 0.2$ restores the chaos and gives trajectories as shown in Fig. 2 that resemble the familiar Lorenz attractor except for discontinuities in the direction of the flow. The corresponding Lyapunov exponents (LEs) are calculated by replacing $\mathrm{sgn}(x)$ and $\mathrm{sgn}(y)$ by $\tanh(100x)$ and $\tanh(100y)$, respectively, giving the values in the caption and a corresponding Kaplan–Yorke dimension of $2.01_{63}$.

Since $c\,\mathrm{sgn}(x)$ in system (6) is now the only term that is not first order, the parameter $c$ is an amplitude parameter, and it gives total amplitude control. Even though the linearization does not change the rotational symmetry, partial amplitude control for variables $x$ and $y$ is now destroyed since the amplitude modification of the variable $x$ is shielded by the signum function, and thus the second dimension cannot give a common coefficient for the reduction by a fraction. However, the coefficient of the term $x\,\mathrm{sgn}(y)$ in the third dimension can change the amplitude of the variables $x$ and $y$ and give robust chaotic solutions over a large range, while the amplitude of the variable $z$ remains nearly constant.

The piecewise linearization of the Lorenz system (6) retains the equilibrium at the origin and also has the other two equilibrium points, which occur at $(\frac{bc}{1+ab}, \frac{bc}{1+ab}, \frac{c}{1+ab})$ and $(\frac{-bc}{1+ab}, \frac{-bc}{1+ab}, \frac{c}{1+ab})$, like the original system (1) except that here the position of both symmetric equilibrium points is proportional to the parameter $c$, as expected, since $c$ is an amplitude parameter. The equilibrium at the origin has eigenvalues $(-1, -a, -b)$, showing it is a stable focus for positive $a$ and $b$. When $a = 0.1$, $b = 0.2$, and $c = 2.8$, the other two equilibrium points are at $(x, y, z) = (\pm 0.5490, \pm 0.5490, 2.7451)$ with eigenvalues given by $(-1.5276, 0.1138 \pm 0.8092i)$, indicating that the equilibrium points are saddle-foci.

## 4. Linearized diffusionless Lorenz system

It happens that system (6) is chaotic even when $a = 0$, by analogy with a similar result for the ordinary Lorenz system [26]. Furthermore, the system remains chaotic if the amplitude information in the variable $z$ in $bz$ is removed by replacing $bz$ with $b\,\mathrm{sgn}(z)$ to obtain

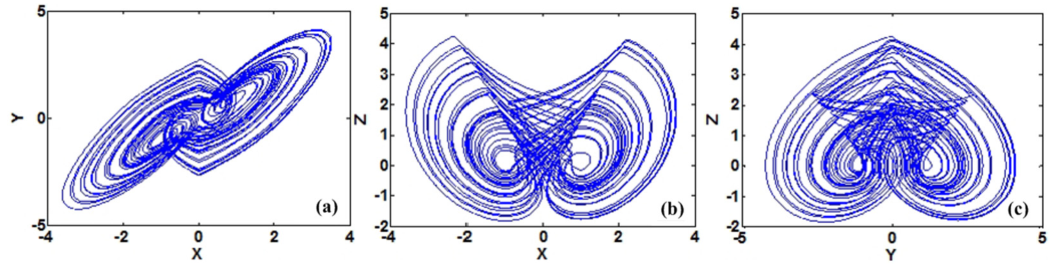



**Fig. 3.** Strange attractor from system (8) with $b = 1$ for initial conditions $(0, 1, 0)$ with LEs $= (0.12_{23}, 0, -1.12_{23})$ (a) $x$–$y$ plane, (b) $x$–$z$ plane, (c) $y$–$z$ plane.

$$\dot{x} = y - x$$
$$\dot{y} = -z\,\mathrm{sgn}(x) + c\,\mathrm{sgn}(x)$$
$$\dot{z} = x\,\mathrm{sgn}(y) - b\,\mathrm{sgn}(z) \tag{7}$$

System (7) retains the three equilibrium points at $(0, 0, 0)$, $(b, b, c)$ and $(-b, -b, c)$.

Since $z$ is always positive for the attractor of system (7), the term $b\,\mathrm{sgn}(z)$ is just $b$. In addition, the term $c\,\mathrm{sgn}(x)$ is symmetric about the origin and hence averages to zero, leading to the further reduced system given by

$$\dot{x} = y - x$$
$$\dot{y} = -z\,\mathrm{sgn}(x)$$
$$\dot{z} = x\,\mathrm{sgn}(y) - b \tag{8}$$

System (8) is exactly a piecewise linearized version of the quadratic diffusionless Lorenz system [18,27],

$$\dot{x} = y - x$$
$$\dot{y} = -zx$$
$$\dot{z} = xy - R \tag{9}$$

System (8) has two saddle-focus equilibria at $(b, b, 0)$ and $(-b, -b, 0)$ with eigenvalues $(-1.4656, 0.2328 \pm 0.7926i)$, and thus it satisfies the Shilnikov condition for the existence of chaos [28].

Note that the parameter $b$ in system (8) is the only term not of first order, and hence it is an amplitude parameter and can be set to unity without loss of generality, unlike the case of system (9) where $R$ is a bifurcation parameter. The strange attractor for system (8) shown in Fig. 3 resembles the diffusionless Lorenz system but with discontinuities in the direction of the flow. The Lyapunov exponents were calculated by replacing the signum functions with hyperbolic tangents using $N = 100$ and imply a Kaplan–Yorke dimension of $2.10_{90}$.

The coefficient of $x\,\mathrm{sgn}(y)$ cannot give partial amplitude control unless the amplitude information of variable $x$ in the second dimension is restored by replacing $z\,\mathrm{sgn}(x)$ with $zx$. The strange attractor in system (8) attracts nearly all initial conditions except those along the $z$-axis that attract to $z = -\infty$. Unlike system (9) where the parameter $R$ can be used to observe the merging of a symmetric pair of strange attractors [29], the parameter $b$ in system (8) cannot be used in that way since it only controls the amplitude of the variables.

## 5. Bifurcation analysis

Since system (8) has five terms, four of which can be scaled to $\pm 1$ by a linear rescaling of the variables $x$, $y$, $z$, and $t$, it should have one bifurcation parameter, just like system (9). In fact, a linear transformation of system (9) with $x \to \sqrt{R}x$, $y \to \sqrt{R}y$, $z \to Rz$, $t \to t$ moves the parameter $R$ from the $\dot{z}$ equation to the $\dot{y}$ equation whose linearized form is then

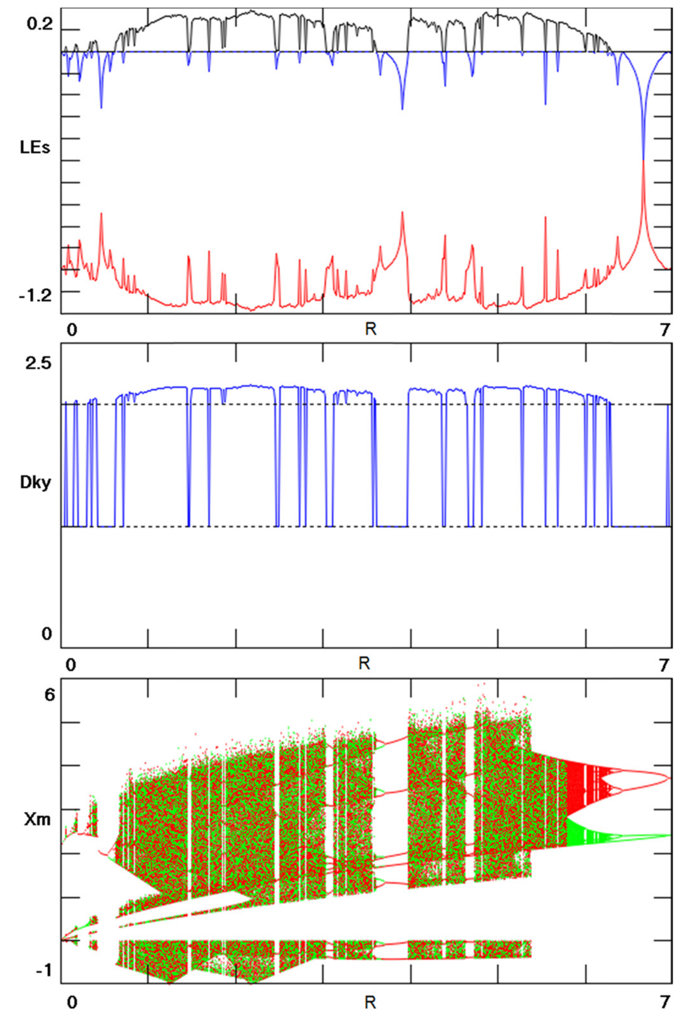



**Fig. 4.** Lyapunov exponents (LEs), Kaplan–Yorke dimension (Dky) and maximum values of $x$ (Xm) as a function of the parameter $R$ for system (10). (For interpretation of the colors in this figure, the reader is referred to the web version of this article.)

$$\dot{x} = y - x$$
$$\dot{y} = -Rz\,\mathrm{sgn}(x)$$
$$\dot{z} = x\,\mathrm{sgn}(y) - 1 \tag{10}$$

The amplitude parameter $b$ can be inserted back into system (10) if desired, but there is no loss of generality by taking $b = 1$. Now the bifurcation parameter $R$ can be used to illustrate multistability and attractor merging as shown in Fig. 4.

In this figure, the plot labeled Xm shows the local maxima of $x$ in red and the negative of the local minima of $x$ in green. Since the equations are symmetric under the transformation $(x, y) \to (-x, -y)$, any attractor must either share that symmetry, or there

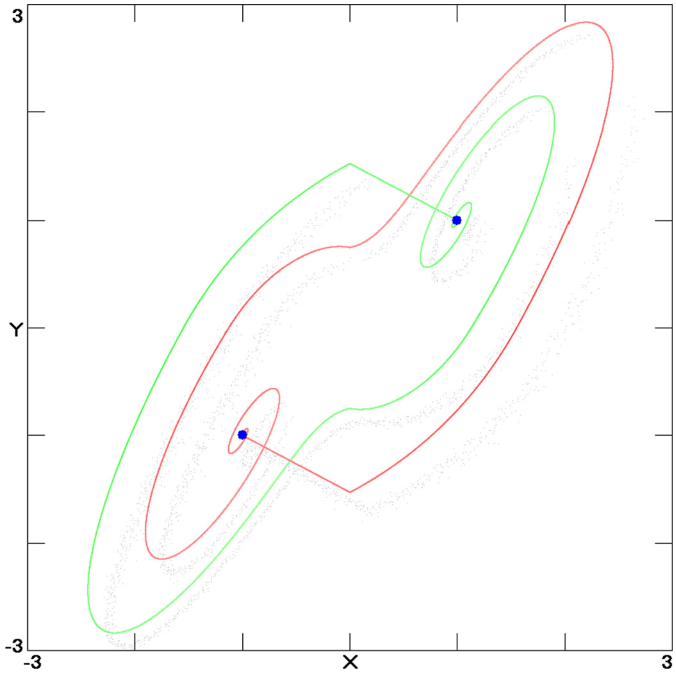



**Fig. 5.** Projection onto the $x$–$y$ plane for a symmetric pair of homoclinic orbits for system (10) with $R = 1.2353$. The equilibrium points are shown as blue dots.

must be a symmetric pair of them. In the former case, the two colors are intermingled, and in the latter case, they are separated.

The two equilibrium points have eigenvalues that satisfy $\lambda^3 + \lambda^2 + R = 0$ and hence are unstable for all values of $R$. For $R = 7$, there is a symmetric pair of limit cycles that undergo period doubling as $R$ decreases, forming a pair of strange attractors that merge into a single symmetric attractor at approximately $R = 5.81$ that persists with periodic windows down to approximately $R = 0.63$. The behavior at small $R$ is similar except there is a presumably fractal succession of ever smaller pairs of limit cycles that period-double into a pair of strange attractors that merge into ever smaller symmetric strange attractors. Similar behavior has been observed in system (9) [27]. There is no evidence for hysteresis or the coexistence of attractors other than the symmetric pair shown in red and green in Fig. 4.

System (10) has a symmetric pair of homoclinic orbits for $R = 1.2353$ as shown in Fig. 5, despite the fact that the corresponding strange attractor is symmetric. The figure shows a projection of the orbits onto the $x$–$y$ plane with the equilibrium points shown as blue dots. No obvious bifurcations seem to occur at this value of $R$ in Fig. 4.

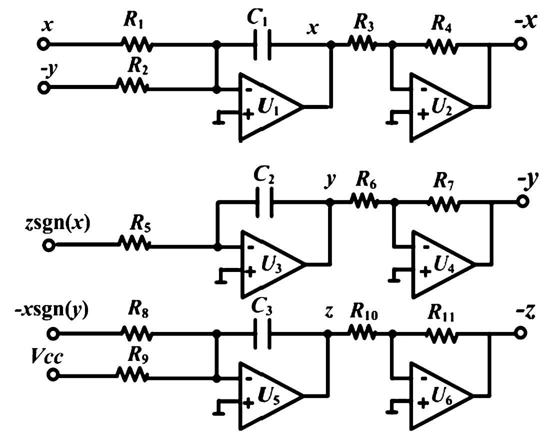
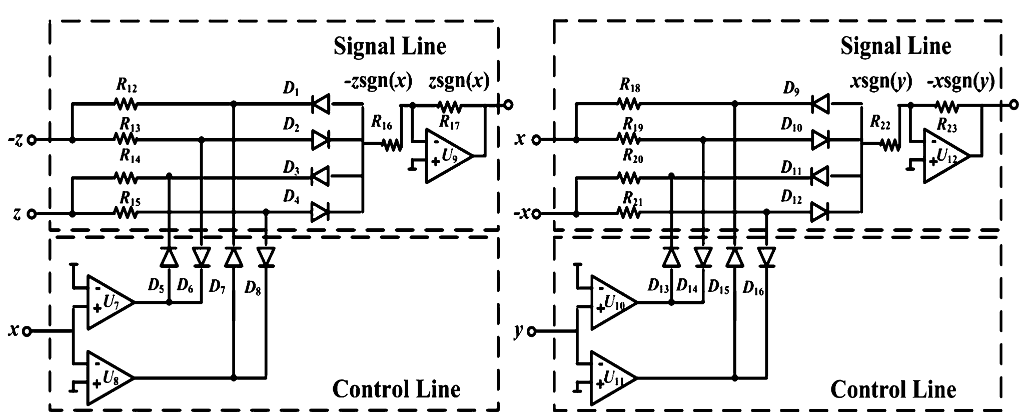



**Fig. 7.** Circuit structure with three integration channels for system (8).

## 6. Electrical circuit realization

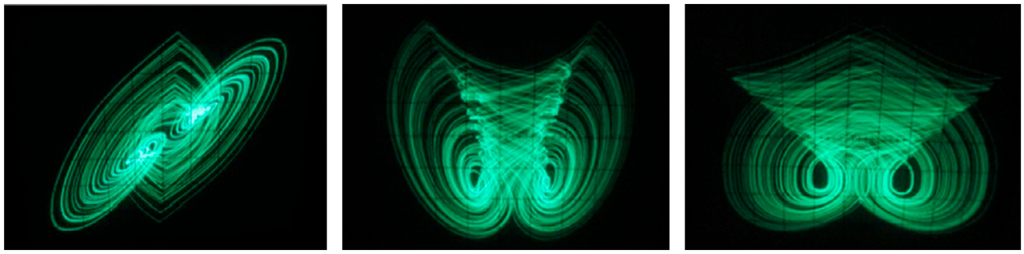
The new switch element, shown in Fig. 6, can be adopted to realize the polarity reversal associated with the signum function, which is the main nonlinearity in the linearized Lorenz systems (6)–(8). Such a circuit was built based on a signal line and a control line with just diodes, resistors, and a few extra operational amplifiers [12]. In Fig. 6(a), signals $z$ and $-z$ pass through a signal line and can be blocked when a voltage source is applied from a control line. Whether the control line will apply this voltage depends on the polarity of the signal $x$. If $x$ is positive, signal $z$ is blocked, and $-z$ will pass through. If it is negative, then $-z$ is blocked, and $z$ passes through. If it is zero, both signals $z$ and $-z$ sum to zero. Two independent signals $z$ and $-z$ are selected by the control line to pass, whereas each signal ($z$ or $-z$) occupies two separate signal lines according to its polarity. Another switch with a signal line and a control line realizes the function $x\,\mathrm{sgn}(y)$. All signal lines have their own independent voltage followers to provide impedance matching.

By combining this with the main circuit of three integration channels, as shown in Fig. 7, the dynamic behavior of system (8) can be reproduced without any multipliers. The integration channels are designed by general analog computation methods, and the oscilloscope traces from the output of the integration channels are shown in Fig. 8. The circuit parameters are $C_1 = C_2 = C_3 = 1$ nF, $R_1 = R_2 = R_5 = R_8 = R_9 = 100$ kΩ, $R_3 = R_4 = R_6 = R_7 = R_{10} =$




(a) (b)

**Fig. 6.** Switch elements to realize the signum function (a) $z\,\mathrm{sgn}(x)$, (b) $-x\,\mathrm{sgn}(y)$.



(a) (b) (c)

**Fig. 8.** Experimental phase portraits of system (8) (a) $x$–$y$ plane, (b) $x$–$z$ plane, (c) $y$–$z$ plane (1 V/div).

$R_{11} = 10$ kΩ, $R_{16} = R_{17} = R_{22} = R_{23} = 200$ kΩ, $R_{12} = R_{13} = R_{14} = R_{15} = R_{18} = R_{19} = R_{20} = R_{21} = 10$ kΩ, and $V_{cc} = 1$ V. The operational amplifiers are TL084 ICs powered by ±9 volts. Germanium diodes 1n60p were used to reduce the influence of the threshold voltage and crossover distortion. Other diodes such as the 1n4148 will not give an attractor that is in such close agreement with numerical results. Operational amplifiers $U_9$ and $U_{12}$ are required to provide a current sufficient to drive the corresponding resistors $R_5$ and $R_8$. The voltage of $V_{cc}$ or the resistor $R_9$ can provide an easy amplitude control, which corresponds to changing the amplitude parameter $b$ in system (8).

## 7. Discussion and conclusions

A partially linearized Lorenz system and a completely linearized Lorenz system were derived by using the signum function to ignore the amplitude of one of the variables, while retaining its sign. This transformation succeeds in retaining the dynamics in the nonlinear systems because the systems have the potential self-balance for the polarity and amplitude. The method of linearization is widely applicable to other systems including ones with higher order terms if the linearization does not destroy the capacity to self-balance with amplitude and if the resulting equations can be solved in the linear regions.

However, such a linearization changes the order of the original nonlinearity and can thus change a bifurcation parameter into an amplitude parameter or vice versa. If there is only one term in the equations that is not of first order, the coefficient of that term becomes a useful amplitude controller which determines the size of all the variables and thus the scale of the attractor. The rotational symmetry of the Lorenz system is not destroyed in the linearization, but the amplitude extraction of the signum breaks the amplitude balance in the differential equations and destroys the partial amplitude control.

With further signum operations, a concise linearized Lorenz system is obtained, which is exactly a piecewise linear variant of the diffusionless Lorenz system. A unique circuit switch to execute variable selection is designed based on a signal line and control line. The phase portraits from the circuit agree well with those calculated numerically for the piecewise linear diffusionless Lorenz system.

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## References

[1] E.N. Lorenz, J. Atmos. Sci. 20 (1963) 130.
[2] R.C. Hilborn, Chaos and Nonlinear Dynamics: An Introduction for Scientists and Engineers, Oxford University Press, New York, 1994.
[3] Y.J. Sun, Phys. Lett. A 374 (2010) 933.
[4] G.A. Leonov, A.Yu. Pogromsky, K.E. Starkov, Phys. Lett. A 375 (2011) 1179.
[5] Q. Bi, Z. Zhang, Phys. Lett. A 375 (2011) 1183.
[6] G.A. Leonov, Phys. Lett. A 376 (2012) 3045.
[7] A. Algaba, F. Fernández-Sánchez, M. Merino, A.J. Rodríguez-Luis, Phys. Lett. A 377 (2013) 2771.
[8] A. Algaba, F. Fernández-Sánchez, M. Merino, A.J. Rodríguez-Luis, Chaos 23 (2013) 033108.
[9] C. Li, J.C. Sprott, Int. J. Bifurc. Chaos 24 (2014) 1450131.
[10] Q. Yang, Y. Chen, Int. J. Bifurc. Chaos 24 (2014) 1450055.
[11] C. Li, J. Wang, W. Hu, Nonlinear Dyn. 68 (2012) 575.
[12] C. Li, J.C. Sprott, W. Thio, H. Zhu, IEEE Trans. Circuits Syst. II, Express Briefs 61 (2014) 977.
[13] B. Munmuangsaen, B. Srisuchinwong, J.C. Sprott, Phys. Lett. A 375 (2011) 1445.
[14] J.C. Sprott, Phys. Lett. A 266 (2000) 19.
[15] J.R. Piper, J.C. Sprott, IEEE Trans. Circuits Syst. II, Express Briefs 57 (2010) 730.
[16] A.S. Elwakil, S. Özoğuz, M.P. Kennedy, IEEE Trans. Circuits Syst. I, Fundam. Theory Appl. CS-49 (2002) 527.
[17] C. Li, J.C. Sprott, Phys. Lett. A 378 (2014) 178.
[18] J.C. Sprott, Elegant Chaos: Algebraically Simple Chaotic Flows, World Scientific, Singapore, 2010.
[19] C. Li, J.C. Sprott, W. Thio, J. Exp. Theor. Phys. 118 (2014) 494.
[20] R. Lozi, J. Phys. (Paris) 39 (1978) 9.
[21] S.J. Linz, J.C. Sprott, Phys. Lett. A 259 (1999) 240.
[22] C. Li, J.C. Sprott, Nonlinear Dyn. 73 (2013) 1335.
[23] C. Li, D. Wang, Acta Phys. Sin. 58 (2009) 764 (Chinese edition).
[24] W.J. Grantham, B. Lee, Dyn. Control 3 (1993) 159.
[25] R.F. Gans, J. Sound Vib. 188 (1995) 75.
[26] W. Zhou, Y. Xu, H. Lu, L. Pan, Phys. Lett. A 372 (2008) 5773.
[27] G. Van der Schrier, L.R.M. Maas, Physica D 141 (2000) 19.
[28] L.P. Shilnikov, Sov. Math. Dokl. 6 (1965) 163.
[29] J.C. Sprott, Int. J. Bifurc. Chaos 24 (2014) 1450009.