Optik 127 (2016) 10389–10398

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Original research article

Variable-boostable chaotic flows

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ARTICLE INFO

Article history:
Received 15 June 2016
Accepted 22 August 2016

PACS:
05.45

Keywords:
Offset boosting
Amplitude control
Multistability

ABSTRACT

A new regime of chaotic flows is explored in which one of the variables has the freedom of offset boosting. By a single introduced constant, the DC offset of the variable can be boosted to any level, and therefore the variable can switch between a bipolar signal and a unipolar signal according to the constant. This regime of chaotic flows is convenient for chaos applications since it can reduce the number of components required for signal conditioning. Offset boosting can be combined with amplitude control to achieve the full range of linear transformations of the signal. The symmetry of the variable-boostable system may be destroyed by the new introduced boosting controller; however, a different symmetry is obtained that preserves any existing multistability.

1. Introduction

Amplitude control is an important issue in engineering applications for optimizing the amplitude [1–7] and achieving stability [6–9]. Partial amplitude control changes the amplitude of some of the variables using a partial controller [1], while a total amplitude controller adjusts the amplitude of all the variables simultaneously [2–6]. Moreover, it is often useful to transform a bipolar signal to a unipolar signal or vice versa [10–12]. For example, an ADC chip usually needs a non-negative analog signal as the input signal, which thus demands a unipolar signal. Unipolar signals are easier to transmit in directly-coupled integrated circuits. Many of the signals from physical sensors are unipolar. However, bipolar signals have lower levels of DC component, which reduces the power requirements and reduces the attenuation at high voltage and is thus good for signal transmission.

A natural question is how best to modify a bipolar signal in a differential chaotic system to make it unipolar or vice versa. A simple capacitor can transform a unipolar signal to a bipolar one since the DC component is blocked by the capacitor. But the capacitance must be large enough to have negligible reactance compared with the load. Alternately, a single-supply operational amplifier can offset the voltage and transform a unipolar signal to a bipolar one. Chaotic signals are broadband with low-frequency components, which complicates the process. A large capacitor or a broadband adder circuit is needed to achieve the transformation between the unipolar signal and a bipolar one.

In Section 2, we consider examples of chaotic flows that provide offset boosting by a single constant in the governing equations. In Section 3, we combine offset boosting with amplitude control to achieve a wide range of signals without

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http://dx.doi.org/10.1016/j.ijleo.2016.08.046

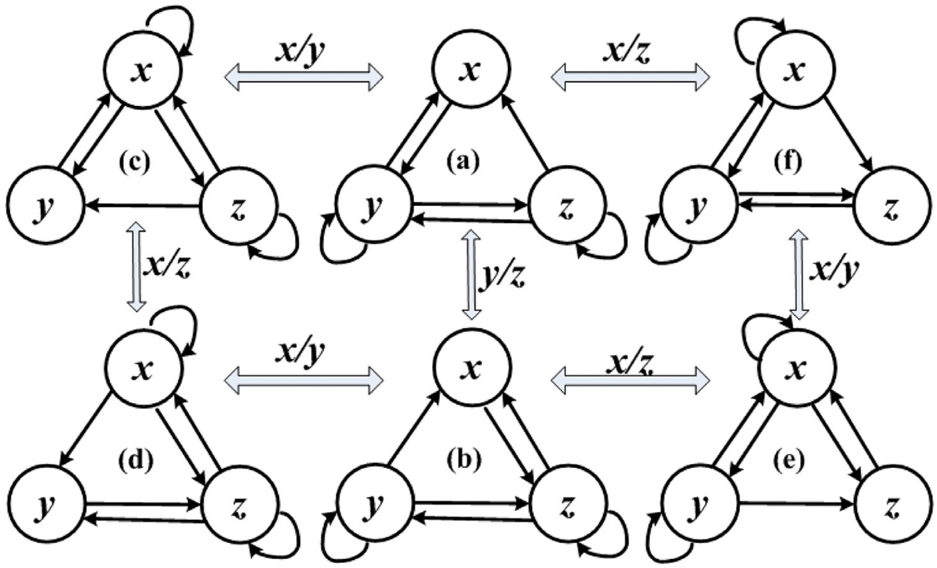


Fig. 1. The inner structure of the variable-boostable chaotic flows.

affecting their dynamical properties such as their power spectra and Lyapunov exponents. In Section 4, we show that offset boosting in symmetric systems can preserve the bistability, and the offset-boosted symmetric system can also give a symmetric pair of coexisting attractors in coordinate-shifted basins of attraction. A short conclusion and discussion are given in the last section.

2. Variable-boostable chaotic flows

Since the derivative of a constant is zero, a differential equation will not change its form if a constant is added to a variable, provided that variable does not appear explicitly. For example, replacing the variable x with $x+c$ (here c is a constant) in the equation $\dot{x} = f(y, z)$ has no effect on the dynamics. Consequently, if the other equations of the system have only a single linear occurrence of the variable x, the introduction of the constant into that equation will produce an offset of the variable x and thus give the freedom to alter the chaotic signal from unipolar to bipolar or vice versa.

Definition 1.

Suppose there is a differential dynamical system, $\dot{X} = F(X)(X = (x_1, x_2, x_3, \ldots x_{i,} \ldots) (i \in N)$. If $x_i = u_i + c$ is subject to the same governing equation except through introducing a single constant into one of the other equations, *i.e.*, $\dot{Y} = F(Y, c)(Y = (x_1, x_2, x_3, \ldots u_{i,} \ldots) (i \in N))$, then the system is a variable-boostable system since it has the freedom for offset boosting the variable x_i. Setting x_i with $x_i + c$ will introduce in the x_i variable a new constant c which will change the average value of the variable x_i. Thus the transformation is convenient to offset the bipolar signal x_i by a unipolar DC voltage in the circuit, or vice versa.

For a three-dimensional system, any of the variables (x, y, z) can be boosted by a constant. Consequently, there are three cases for variable boosting. To limit the complexity of the examples, we consider only quadratic nonlinearities. For such a dynamic system, if the variable x needs to be boosted by a constant, the equation can be in the form of Eq. (1), where the extra constant is introduced in the dimension of z.

$$\begin{cases} \dot{x} = a_1 y + a_2 z + a_3 y^2 + a_4 z^2 + a_5 yz + a_{16}, \\ \dot{y} = a_6 y + a_7 z + a_8 y^2 + a_9 z^2 + a_{10} yz + a_{17}, \\ \dot{z} = a_{11} y + a_{12} z + a_{13} y^2 + a_{14} z^2 + a_{15} yz + a_{18} x. \end{cases} \tag{1}$$

Similarly, the variables y and z can be boosted by a new introduced constant in other dimensions. However, all such cases can be written in the form of Eq. (1) without loss of generality through a simple transformation of variables. This can be confirmed from the topological structure as shown in Fig. 1. The variable without self-feedback can receive offset boosting control from the variable in the arm with dual-direction connectivity. For example, in the structure (b), the variable x has no self-feedback; the variable x influences the dynamics only by the dimension of z leading to a dual-direction connection. Therefore, the variable x can obtain offset boosting from the dimension of z. All these cases can be transformed from any of the other cases by the variable substitutions marked in Fig. 1.

Some cases conforming to the above topological structures have been given by Sprott [13,15,16]. For electrical circuit implementation, a standard jerk equation usually leads to a compact circuit topology. Therefore, for the convenience of reference, we transformed all those cases to the form of Eq. (1) corresponding to the structure of (b) in Fig. 1. To find further examples, we employed a systematic numerical search procedure developed in [13,14]. In this procedure, the space of the control parameters embedded in the differential equations and the initial conditions were scanned to find a positive Lyapunov exponent, which is a signature of chaos. Table 1 shows all the elementary examples (those that cannot be further simplified by elimination of terms) of chaotic 3-D quadratic variable-boostable (VB) systems that were found along with the numerically calculated Lyapunov exponents and Kaplan-Yorke dimensions. From Table 1, we see that most of the chaotic flows have an equilibrium point of saddle-focus. VB5 and VB13 have no equilibria and therefore corresponding attractors are hidden [17–23]. The corresponding strange attractors are shown in Fig. 2.

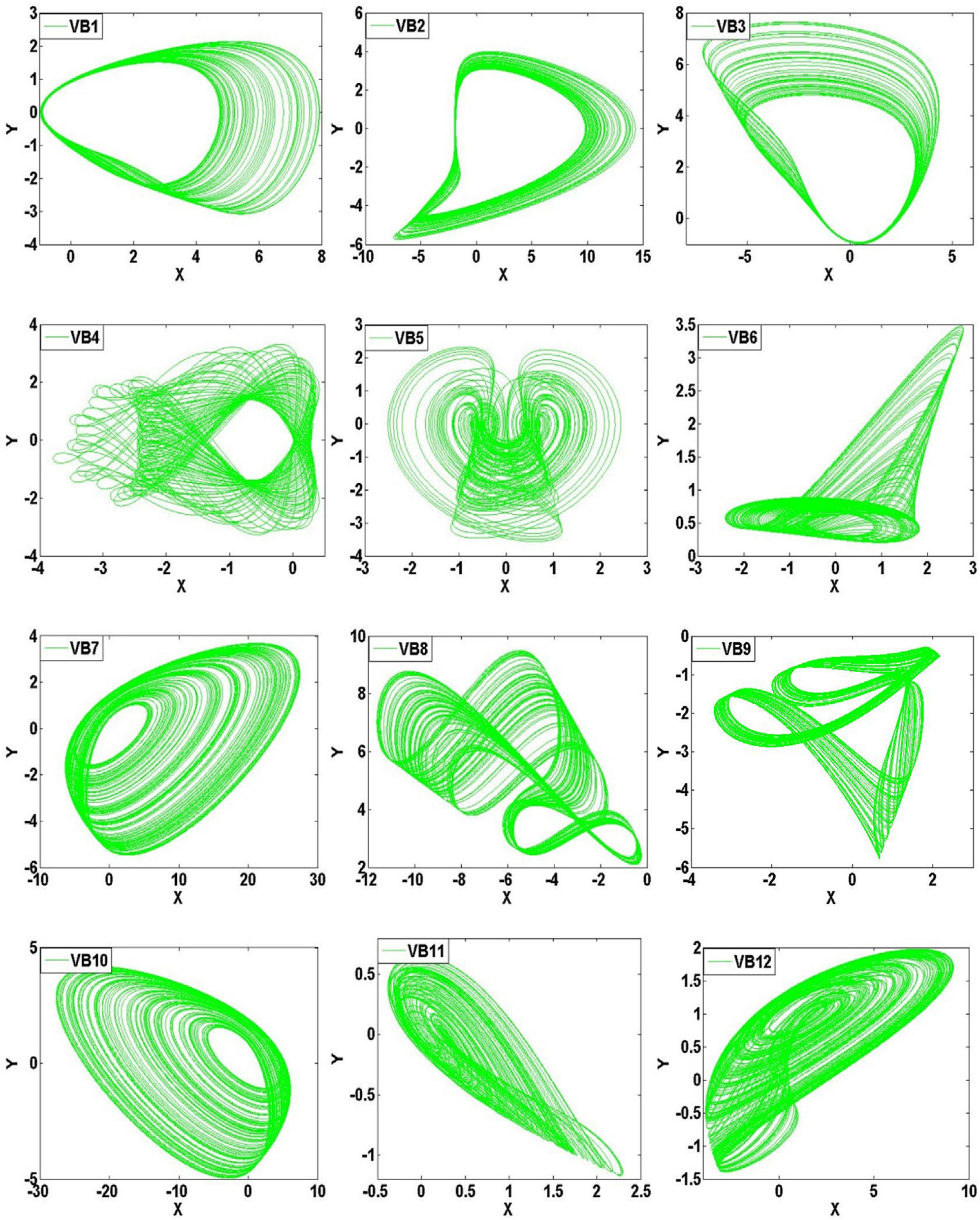


Fig. 2. Strange attractors in variable-boostable chaotic flows.

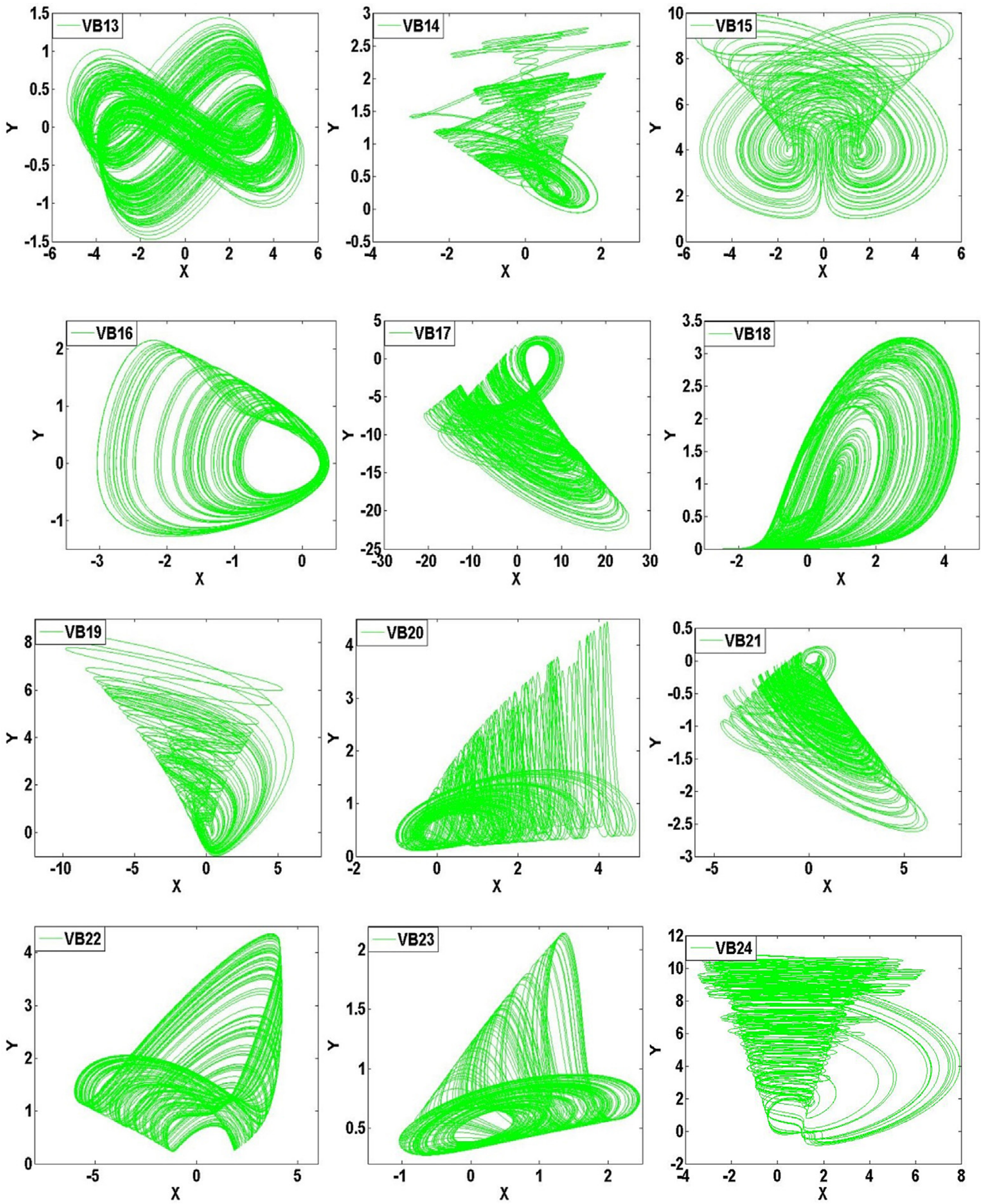


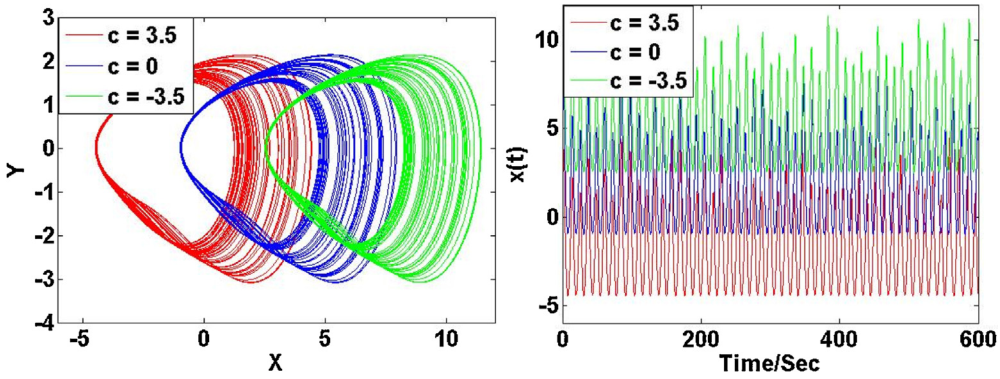
Fig. 2. (*Continued*)

3. Offset boosting and amplitude control

3.1. Offset boosting with total amplitude control

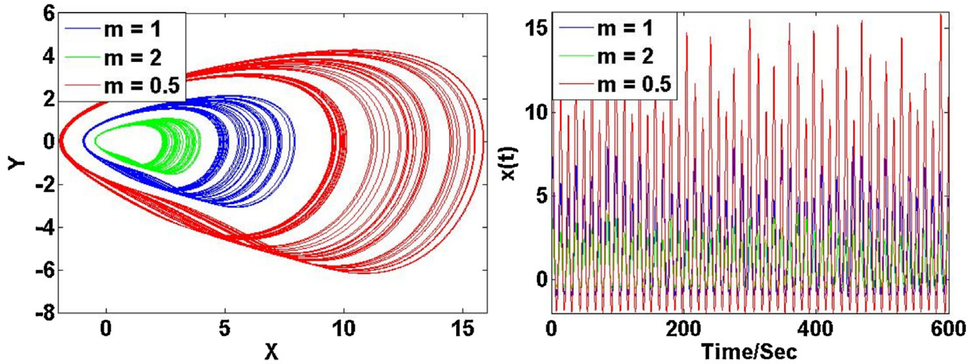
The simplest case of a variable-boostable flow is the case JD0 (VB1), which has five terms and a single quadratic nonlinearity,

$$\begin{aligned}\dot{x} &= y\\ \dot{y} &= z\\ \dot{z} &= -x + y^2 - az\end{aligned} \quad (2)$$



(a) (b)

Fig. 3. Chaotic attractors (a) and signal $x(t)$ with different boosting constants (b): red for $c = 3.5$ at (1.5, 2, 0), blue for $c = 0$ at (5, 2, 0), and green for $c = -3.5$ at (8.5, 2, 0). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)



(a) (b)

Fig. 4. Chaotic attractors (a) and signal $x(t)$ with different amplitude controllers (b): red for $m = 0.5$ at (10, 4, 0), blue for $m = 1$ at (5, 2, 0) and green for $m = 2$ at (2.5, 1, 0). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Replacing x with $x + c$ will introduce in the z dimension a new constant c, which boosts the amplitude of the variable x. The new equation is,

$$\begin{aligned} \dot{x} &= \mathrm{y} \\ \dot{y} &= \mathrm{z} \\ \dot{z} &= -\mathrm{x} + y^2 - \mathrm{az} - \mathrm{c} \end{aligned} \tag{3}$$

As shown in Fig. 3, when $a = 2.02$, $c = -3.5$,the chaotic signal x is boosted from a bipolar signal to a unipolar one indicated by the green attractor and green signal. Since the system has a small basin of attraction, the initial condition x_0 must be correspondingly adjusted to avoid unbounded solutions.

Since VB1 (Sprott JD0) has one quadratic term while the others are of degree 1, the coefficient of the single quadratic term m is a total amplitude controller,

$$\begin{aligned} \dot{x} &= \mathrm{y} \\ \dot{y} &= \mathrm{z} \\ \dot{z} &= -\mathrm{x} + \mathrm{m}y^2 - \mathrm{az} \end{aligned} \tag{4}$$

To show this, let $x = u/m$, $y = v/m$, $z = w/m$ to obtain new equations in the variables u, v, w that are identical to system VB1. Therefore, the coefficient m controls the amplitude of variable x according to $1/m$, as shown in Fig. 4.

Offset boosting has some connection with amplitude control. If the goal is to transform a bipolar signal to a unipolar one, the depth of offset boosting should be adjusted according to the signal amplitude. Amplitude control for larger amplitude requires a correspondingly larger boosting controller. Once the boosting controller is introduced in the system, the amplitude controller will consequently modify the depth of boosting. Specifically, in Eq. (4), variable substitutions $x = (u + c)/m$, $y = v/m$, $z = w/m$ result in new equations in the variables u, v, w that are identical to Eq. (2). Therefore, the amplitude controller m modifies the depth of variable boosting according to c/m.

Table 1
Variable boostable chaotic systems.

Model	Equations	Parameters	Equilibria	Eigenvalues	x_0 y_0 z_0	LEs	D_{KY}
VB1 (Sprott JD0)	$\dot{x}=y$ $\dot{y}=z$ $\dot{z}=-x+y^2-az$	$a=2.02$	(0, 0, 0)	(−2.2225, $0.1012\pm0.6631i$)	5 2 0	0.0448 0 −2.0648	2.0217
VB2	$\dot{x}=y+yz$ $\dot{y}=-z$ $\dot{z}=x-az$	$a=2.04$	(0, 0, 0)	(−2.2394, $0.0997\pm0.6608i$)	1 1 −5	0.0327 0 −2.0727	2.0158
VB3	$\dot{x}=z^2-y$ $\dot{y}=z$ $\dot{z}=x-az$	$a=2.02$	(0, 0, 0)	(−2.2225, $0.1012\pm0.6631i$)	1 5 1	0.0478 0 −2.0678	2.0231
VB4 (revised Sprott A)	$\dot{x}=-z$ $\dot{y}=z^2-a$ $\dot{z}=x-yz$	$a=1$	none	none	0 0 5	0.0138 0 −0.0138	3
VB5	$\dot{x}=ayz$ $\dot{y}=1-z^2$ $\dot{z}=x+yz$	$a=1$	(0, 0, 1) (0, 0, −1)	(−0.7709, $0.3855\pm1.5639i$)	−1 1 −1	0.1271 0 −0.5526	2.2299
VB6	$\dot{x}=1-yz$ $\dot{y}=az^2-yz$ $\dot{z}=x$	$a=0.22$	$(0, \frac{\pm\sqrt{550}}{50}, \frac{\pm\sqrt{550}}{11})$	(−2.3050, $0.0865\pm0.9275i$) (1.7425, 1.2837, −0.8941)	−1 1 −1	0.0717 0 −1.6646	2.0431
VB7 (Sprott J)	$\dot{x}=az$ $\dot{y}=-by+z$ $\dot{z}=-x+y+y^2$	$a=2$ $b=2$	(0, 0, 0)	(−2.3146, $0.1573\pm1.3052i$)	−1 0.3 0	0.0787 0 −2.0787	2.0379
VB8 (revised Sprott L)	$\dot{x}=1-az$ $\dot{y}=bz^2-y$ $\dot{z}=y+x$	$a=3.9$ $b=3.5$	(−350/1521, 350/1521, 10/39)	(−1.4319, $0.2160\pm1.6361i$)	−1 0.01 0	0.0521 0 −1.0521	2.0495
VB9 (revised Sprott M)	$\dot{x}=a+y+bz$ $\dot{y}=-y-z^2$ $\dot{z}=-x$	$a=1.7$ $b=1.7$	$(0, \frac{-629\mp17\sqrt{969}}{200}, \frac{17\pm\sqrt{969}}{20})$	(0.9074, $-0.9537\pm1.5878i$) (−1.3892, $0.1946\pm1.4842i$)	−1 0.3 0	0.0435 0 −1.0435	2.0417
VB10 (revised Sprott N)	$\dot{x}=-az$ $\dot{y}=1-by+z$ $\dot{z}=x+y^2$	$a=2$ $b=2$	(−0.25, 0.5, 0)	(−2.3146, $0.1573\pm1.3052i$)	2 0 0.3	0.0746 0 −2.0746	2.0359
VB11 (revised Sprott P)	$\dot{x}=y+z$ $\dot{y}=y^2-az$ $\dot{z}=x+by$	$a=1$ $b=2.7$	(0, 0, 0) (2.7, −1, 1)	(−0.5101, $0.2551\pm1.3767i$) (0.3828, $-1.1914\pm1.0921i$)	0 0.3 0	0.0861 0 −0.4807	2.1790
VB12 (revised Sprott S)	$\dot{x}=ay^2+bz$ $\dot{y}=1+z$ $\dot{z}=-x-z$	$a=4$ $b=4$	(1, 1, −1) (1, −1, −1)	(−1.6075, $0.3038\pm2.2101i$) (1.2030, $-1.1015\pm2.3317i$)	4 0.01 0	0.1930 0 −1.1930	2.1617
VB13 (revised NE5)	$\dot{x}=az$ $\dot{y}=y^2-z^2+b$ $\dot{z}=-x-y$	$a=4$ $b=2$	none	none	8 −1 −0.01	0.0130 0 −0.3615	2.0359
VB14	$\dot{x}=1-ayz$ $\dot{y}=z^2-z$ $\dot{z}=x-bz$	$a=3.55$ $b=0.5$	(0.5, 20/71, 1)	(−1.4673, $0.4837\pm1.4783i$)	1 0 1	0.1510 0 −0.6510	2.2319
VB15	$\dot{x}=az-yz$ $\dot{y}=z^2-by$ $\dot{z}=x-z$	$a=4$ $b=0.4$	(0, 0, 0) $(\frac{\pm2\sqrt{10}}{5}, 4, \frac{\pm2\sqrt{10}}{5})$	(1.5616, −0.4, −2.5616) $(-2, 0.3\pm1.2288i)$	1 1 0	0.2229 0 −1.6229	2.1373

Table 1 (*Continued*)

Model	Equations	Parameters	Equilibria	Eigenvalues	x_0 y_0 z_0	LEs	D_{KY}
VB16	$\dot{x}=y$	$a=4.2$	(0, 0, 0)	(−3.3699, 0.1849±1.1010i)	1	0.0454	2.0149
	$\dot{y}=-az-yz$	$b=3$			−1	0	
	$\dot{z}=x+y^2-bz$				1	−3.0448	
VB17	$\dot{x}=yz$	$a=4$	(0, 0, 0) (2, 0, 4)	(0, −0.5, −1) (−3.0529, −0.7765 ± 2.1536i)	1	0.0947	2.0594
	$\dot{y}=az-y-z^2$	$b=0.5$			0	0	
	$\dot{z}=x-bz$				−1	−1.5948	
VB18	$\dot{x}=az+y^2-1$	$a=2.8$	(0, 1, 0) (0, −1, 0) (−5/14,0,5/14)	(−1.8404, 0.4202 ± 2.0421i) (1.4286, −0.5 ± 1.5969i)	1	0.1155	2.1035
	$\dot{y}=byz$	$b=4$			1	0	
	$\dot{z}=-x-z$				0	−1.1155	
VB19	$\dot{x}=z^2-ay-yz$	$a=1$	(0, 0, 0)	(−1.4059, 0.2530 ± 0.8045i)	1	0.1183	2.1162
	$\dot{y}=z$	$b=0.9$			−1	0	
	$\dot{z}=x-bz$				1	−1.0183	
VB20	$\dot{x}=z-1$	$a=4$	(16/81, 4/9, 1)	(−9.3750, 0.1875 ± 0.9617i)	1	0.2033	2.0221
	$\dot{y}=az^2-byz$	$b=9$			1	0	
	$\dot{z}=y^2-x$				1	−9.2170	
VB21	$\dot{x}=ayz-y^2$	$a=6$	(0, 0, 0) (0, 0, 1) (0, −30, −5)	(0,0,−1) (−2.2188, 0.6094 ± 1.5274i) (0.8264, −0.9132 ± 13.4416i)	1	0.1107	2.0996
	$\dot{y}=z-by-z^2$	$b=1$			1	0	
	$\dot{z}=x$				0	−1.1110	
VB22	$\dot{x}=y-ay^2-yz$	$a=1$	(0, 0, 0) $(0, 3\mp 2\sqrt{2}, -2\pm 2\sqrt{2})$	(0,0,−1) (−1.0553, 0.0276 ± 0.4787i) (1.0377, −1.0189 ± 2.6277i)	1	0.0689	2.0645
	$\dot{y}=bz^2-y$	$b=0.25$			1	0	
	$\dot{z}=x$				1	−1.0689	
VB23	$\dot{x}=z^2-1$	$a=1$	(0.25, 0.5, −1) (0.25, −0.5, 1)	(−2.3146, 0.1573 ± 1.3052i) (2.5616, 1, −1.5616)	1	0.1223	2.0721
	$\dot{y}=az^2+byz$	$b=2$			1	0	
	$\dot{z}=x-y^2$				1	−1.6965	
VB24	$\dot{x}=az^2-yz$	$a=1.5$	$(\frac{3}{4}, \frac{\pm 3\sqrt{3}}{4}, \frac{\pm\sqrt{3}}{2})$	(−2.4941, 0.3810 ± 0.6755i) (−0.5727, 1.1524 ± 1.1362i)	1	0.1144	2.1635
	$\dot{y}=z^2-b$	$b=0.75$			1	0	
	$\dot{z}=x-z^2$				1	−0.6994	

3.2. *Offset boosting with partial amplitude control*

For some chaotic flows with global attraction, it is simpler to realize offset boosting since the initial conditions need not be revised according to the boosted variable. System VB5 is such a case. Replacing x with $(x+c)$ gives

$$\begin{aligned}\dot{x}&=\mathrm{ayz}\\ \dot{y}&=1-z^2\\ \dot{z}&=(x+c)+\mathrm{yz}\end{aligned}\tag{5}$$

Here the variable x is boosted along the x-axis according to the constant c. Negative c boosts the variable in the positive direction, while positive c draws the variable backwards in the negative direction. As shown in Fig. 5, when the boosting controller c varies from −5 to 5, system VB5 gives the same Lyapunov exponents. We can check further the equilibrium points, which are two saddle-foci $(-c, 0, \pm 1)$ and their corresponding fixed eigenvalues $(-0.7709, 0.3855 \pm 1.5639i)$ giving another profile for invariable Lyapunov exponents [1,3]. To prevent the system for locating at the exact equilibria when the boosting controller c varies, the initial condition is set to (1, 0, 2). The average of the variable x varies with the boosting controller c decreasing from 5 to −5 while the average of the variables y and z remain unchanged.

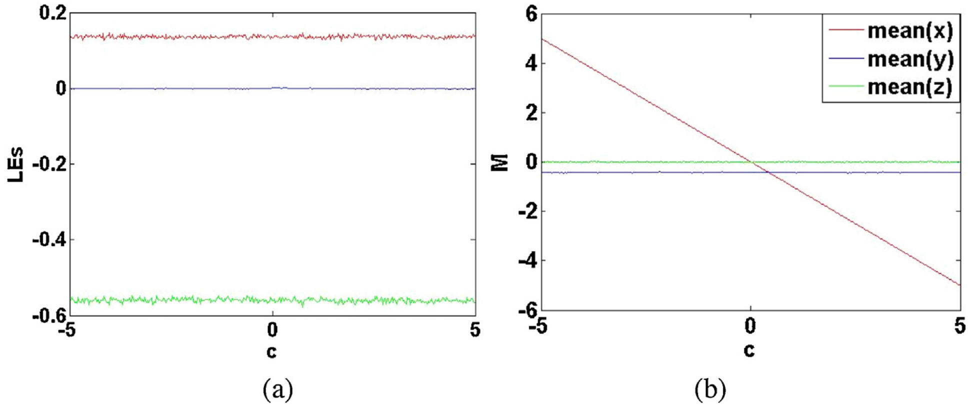


Fig. 5. Lyapunov exponents (a) and the average values (b) when $a = 1$ for initial conditions (1, 0, 2) when the boosting controller c varies from −5 to 5.

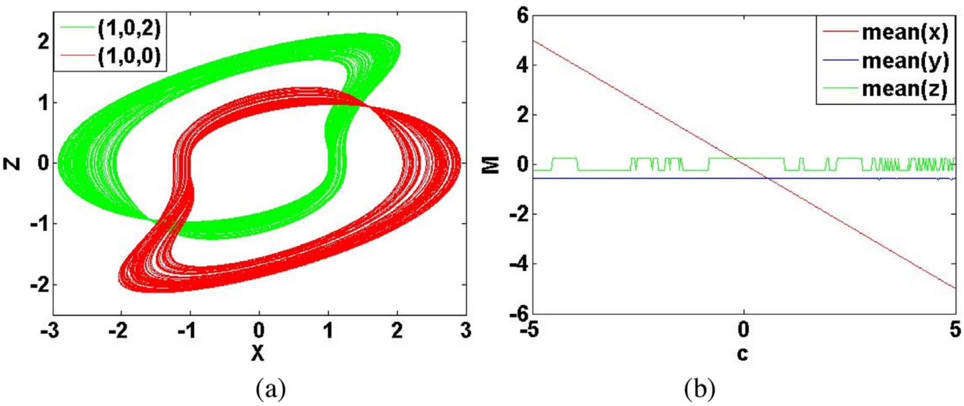


Fig. 6. Coexisting strange attractors when $a = 3.6$ (a) and the average values (b): the boosting controller c varies from −5 to 5 under the fixed initial condition (1, 0, 2).

System VB5 has one coefficient m for partial amplitude control,

$$\begin{aligned} \dot{x} &= ayz \\ \dot{y} &= 1 - mz^2 \\ \dot{z} &= x + yz \end{aligned} \tag{6}$$

To show this, let $x = u/\sqrt{m}$, $y = v$, $z = w/\sqrt{m}$ to obtain new equations in the variables u, v, w that are identical to system VB5. Therefore, the coefficient m controls the amplitude of variable x and z according to $1/\sqrt{m}$ while the amplitude of y remains unchanged. Variable boosting is direct when it is arranged after the partial amplitude control. A pre-fixed boosting controller will be modified by the amplitude controller according to $1/\sqrt{m}$, and thus the transformation between a bipolar signal and a unipolar one should consider this. The new introduced boosting controller can offset the bipolar signal x by a unipolar DC voltage in the circuit, or vice versa.

4. Bistability with offset boosting

Partial amplitude control usually indicates some kind of symmetry. System VB5 is symmetric with respect to a 180° rotation about the y-axis, which can be shown by the coordinate transformation $(x, y, z) \to (-x, y, -z)$, and thus its basin of attraction may be divided into at least two parts leading to a symmetric pair of coexisting attractors [2,5,6,24].

In this case, the boosting controller c may select different attractors since it moves the basin of attraction in the direction of the boosted variable x, and correspondingly a given initial condition may drop into different basins. As shown in Fig. 6(a), when $a = 3.6$, $c = 0$, system (5) has a symmetric pair of strange attractors with basins of attraction as shown in Fig. 8. The regions in light blue and red representing two different attractor basins are asymmetric with respect to $x = 0$, and the strange attractors, represented in cross section by black lines, nearly touch their basin boundaries. The boosting controller c varies from −5 to 5 giving two symmetric attractors, which have different average value of the variable z. For the variable x the switch in polarity in different attractors does not influence their size, and the average value of x declines with increasing c. As shown in Fig. 6(b), the switch between two symmetric attractors for fixed initial conditions leads to two different levels

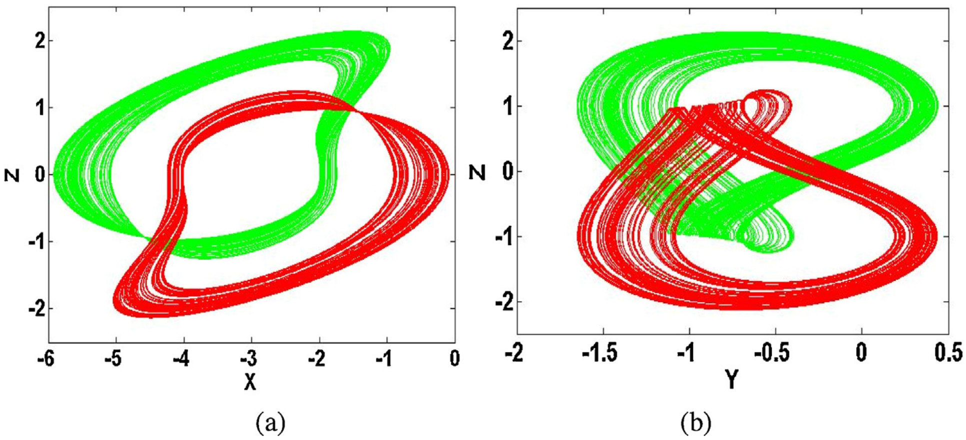


Fig. 7. Coexisting strange attractors when $a = 3.6$, $c = 3$: (a) projection on x-z plane and (b) projection on y-z plane; green with initial condition (1, 0, −2) and red with initial condition (1, 0, 2). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

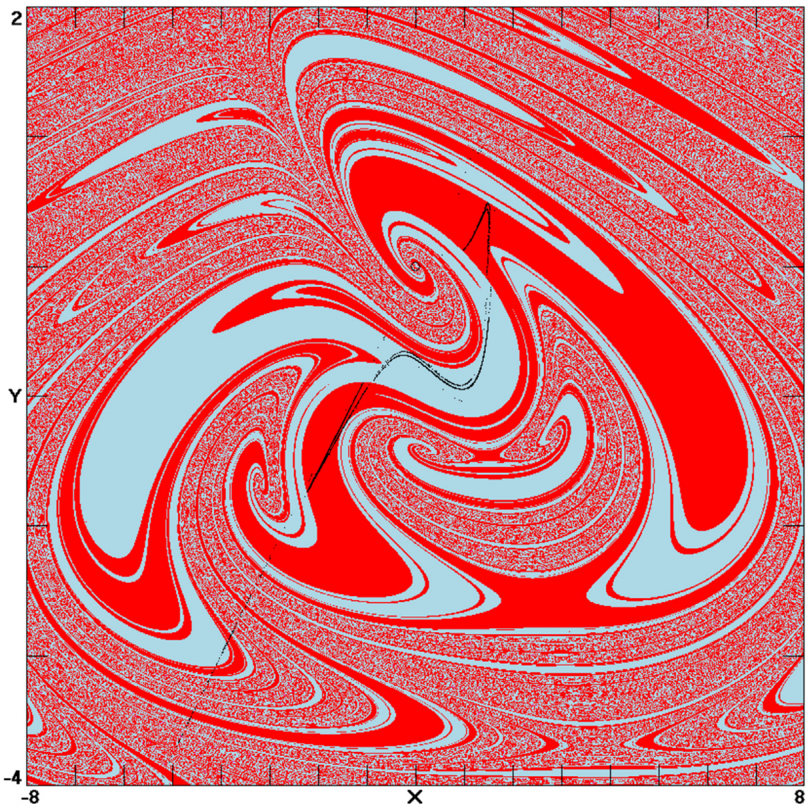


Fig. 8. Cross section for $z = 1$ of the basins of attraction for the symmetric attractors of system (5) at $a = 3.6$, $c = 0$; light blue and red for two coexisting attractors whose cross section are shown in black. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

of the average value of z giving an irregular waveform as expected from the complex fractal basins of attraction shown in Fig. 8. Moreover, since system VB5 is symmetric about the y-axis, the average value of the variable y is independent of c.

It is interesting that the two coexisting symmetric attractors survive in the system while its symmetry is destroyed by the new introduced boosting controller. This is because there is a degenerate conditional symmetry [25] given by the pre-boosted variable x. For system (5), suppose $x = -u - c$, $y = v$, $z = -w$ (here c is the boosting controller), *i.e.*, if $c = -x - u$, the new deduced system has a conditional rotational symmetry, which can be proved by the transformation $\dot{u} = avw$, $\dot{v} = 1 - w^2$, $\dot{w} = u + vw$. In the variables u, v, w, the equations are identical to the original system VB5. When $a = 3.6$, $c = 3$, the asymmetric system (5) has also a symmetric pair of coexisting attractors, as shown in Fig. 7, whose basins of attraction are like the ones shown in Fig. 8 except for a coordinate shift in the negative x direction.

5. Conclusions and discussion

As with amplitude control, variable boosting is important in chaos applications since they can both reduced the required hardware and give a desired voltage level and amplitude for signal transformation and transmission. In this paper, a number of elementary 3-D variable-boostable chaotic flows with quadratic nonlinearities are described. When the system has unbounded solutions, the variable boosting should be accompanied with a modification of the initial conditions, while the initial conditions can be ignored in the systems with global attraction. If the amplitude parameter changes the amplitude of the boosting variable, the boosting controller should be readjusted if the goal is to maintain a unipolar or bipolar signal. Variable boosting breaks the symmetry of the system with respect to $x = 0$, but the symmetry can be recovered by an additional counter-boosting, thus giving coexisting symmetric solutions.

Acknowledgements

This work was supported financially by the Startup Foundation for Introducing Talent of NUIST (Grant No.: 2016205), the Natural Science Foundation of the Jiangsu Higher Education Institutions of China (Grant No.: 16KJB120004), the 4th 333 High-level Personnel Training Project (Grant No.: BRA2013209) and a Project Funded by the Priority Academic Program Development of Jiangsu Higher Education Institutions.

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