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A new chaotic oscillator with free control

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A novel chaotic system is explored in which all terms are quadratic except for a linear function. The slope of the linear function rescales the amplitude and frequency of the variables linearly while its zero intercept allows offset boosting for one of the variables. Therefore, a free-controlled chaotic oscillation can be obtained with any desired amplitude, frequency, and offset by an easy modification of the linear function. When implemented as an electronic circuit, the corresponding chaotic signal can be controlled by two independent potentiometers, which is convenient for constructing a chaos-based application system. To the best of our knowledge, this class of chaotic oscillators has never been reported. *Published by AIP Publishing.*
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For many years, engineering applications for chaos have been considered and described. However, due to their finite word length, chaotic signals generated by digital technology are of limited use. Even with analog electronics, it is still difficult to provide an engineering-applicable chaotic oscillator with special requirements of frequency, amplitude, and polarity. Although hundreds of dissipative chaotic systems have been described and studied, little attention has been given to finding systems that permit simple and continuous adjustment of the signal without encountering undesirable bifurcations and variations of the power spectrum. Therefore, it is important to find even one example of a chaotic oscillator with free and unlimited control of its frequency, amplitude, and polarity by means of simple adjustable resistors.

I. INTRODUCTION

Chaos has attracted great interest in theoretical physics and corresponding engineering fields. Chaos exists widely in nature and society, and it has broad application in engineering for its noise-like property, broadband frequency spectrum, and sensitivity to initial conditions. Specifically, chaos has great advantage in image encryption,[1–3] secure communication,[4–6] and weak signal detection.[7,8] However, for the same reason, it is difficult to design a suitable amplifier with polarity conversion and broad bandpass characteristics. Specifically, amplifying or attenuating a chaotic signal may require extra hardware and pose risks to its bandpass characerisitics;[9–14] and even more challengingly, some integrated circuit chips require a transformation between a bipolar and a unipolar signal, while the offset control (OC) of the chaotic signal is also vital for chaos applications. As shown in Fig. 1, a suitable signal control unit can replace the signal modulator in chaos-based application systems because it provides efficient frequency control,[15,16] amplitude rescaling,[17,18] and offset boosting.[19,20]

Based on the above considerations, a chaotic oscillator with the free control of amplitude/frequency and offset is attractive for engineering applications where proper amplitude, frequency, or the offset level can be obtained by simple control knobs. Some work has been done towards this end. Chaotic systems with invariable Lyapunov exponents have the property of amplitude control (AC).[21,22] Chaotic flows with a single nonquadratic term[15,16] have an intrinsic amplitude-frequency controller (AFC) in the constant or coefficient of the nonquadratic linear term, and some chaotic flows provide offset control (OC) by the introduction of a new constant in any of the governing equations.[19,20] Inspired by the results in Refs. 19 and 20, we give here another case of a chaotic system with both amplitude-frequency control and offset control (known collectively as free control), which means that the chaotic signal can be freely controlled in its various aspects including amplitude, frequency, and polarity. That means the system proposed here can provide a full self-modulation of AFC and OC. The model description is given in Sec. II, the dynamical analysis including amplitude-frequency control and offset control is provided in Sec. III, and the circuit realization is described in Sec. IV. Some other known dynamical systems with the same properties are mentioned in the discussion and conclusions.

II. MODEL DESCRIPTION

In searching for a chaotic system with the free control of amplitude, frequency, and offset, a simple general structure (1) is designed based on the quadratic nonlinearity

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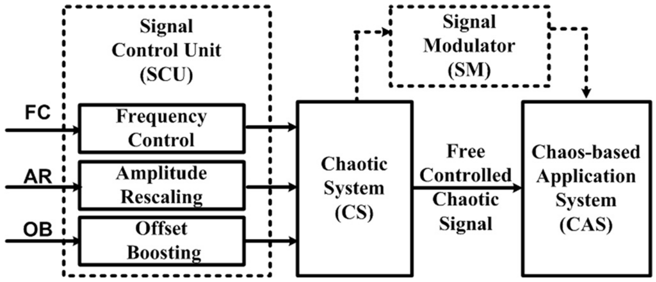

FIG. 1. Diagram of an application system based on chaos.

$$\begin{cases} \dot{x} = a_1y^2 + a_2z^2 + a_3yz, \\ \dot{y} = a_4y^2 + a_5z^2 + a_6yz, \\ \dot{z} = a_7y^2 + a_8z^2 + a_9yz + a_{10}x. \end{cases} \tag{1}$$

In System (1), the coefficient a_{10} of x in the z-dimension is an amplitude-frequency controller, while an additional constant term can also be introduced in the z-dimension for offset boosting for the variable x. Many chaotic candidates were found from an exhaustive computer search, one simple example of which is

$$\begin{cases} \dot{x} = yz, \\ \dot{y} = y^2 - az^2 + yz, \\ \dot{z} = by^2 + F(x). \end{cases} \tag{2}$$

When $a = 8.5$, $b = 0.5$, $F(x) = x$, System (2) is chaotic with Lyapunov exponents (0.0417, 0, −0.9575) and Kaplan-Yorke dimension $D_{KY} = 2.0436$. The strange attractor is asymmetric, as shown in Fig. 2. The cross section of the attractor in the plane $z = -0.4$ is nearly one-dimensional, and the basin of attraction shown as red in Fig. 3 indicates that System (2) has a relatively large unbounded (white) region. According to the classification in Ref. 23, the basin is Class-3 in which an arbitrary point at a distance r from the attractor has a probability P of being in the basin given approximately by $P = 16/r$ in the limit of large r. This implies that the basin is essentially two-dimensional at large distances from the attractor. The attractor is self-excited since its basin is adjacent to the equilibrium point shown as a small green dot in Fig. 3.

System (2) has chaotic and periodic oscillations over a range of the bifurcation parameters a and b. Figure 4 shows the regions of chaos (C) and periodicity (P) in the ab-plane for initial conditions (−0.4, −0.4, −0.4). No evidence for multistability (coexisting attractors) was found anywhere in the ab-plane. When $F(x) = x$, System (2) has one equilibrium point at (0, 0, 0) with the characteristic equation $\lambda^3 = 0$, and thus eigenvalues (0, 0, 0), which implies that the origin is a

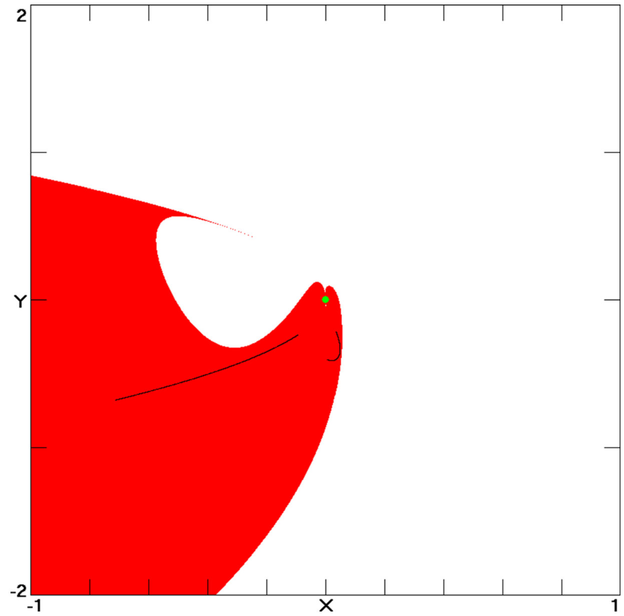

FIG. 3. Cross section of the basin of attraction for System (2) with $a = 8.5$, $b = 0.5$, and $F(x) = x$ in the plane $z = -0.4$.

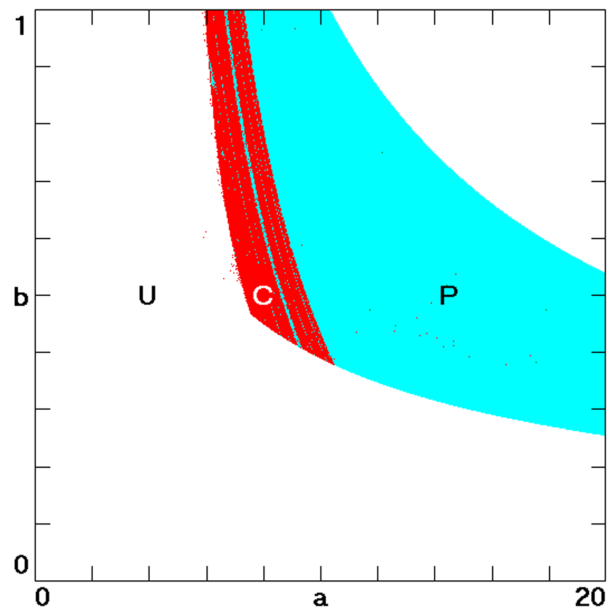

FIG. 4. Dynamical regions of System (2) with $F(x) = x$ and initial conditions (−0.4, −0.4, and −0.4).

center, but it is nonlinearly unstable. For $F(x) = mx + n$, the equilibrium moves to $(-n/m, 0, 0)$, but the stability does not change since the characteristic equation and eigenvalues remain the same,[17] implying invariance of the dynamics with respect to the parameters m and n.

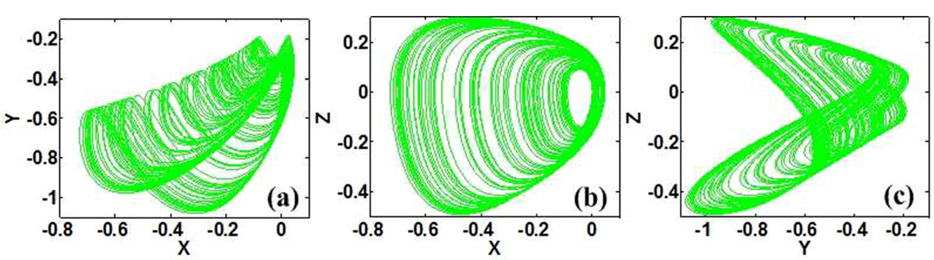

FIG. 2. Strange attractor of System (2) with $a = 8.5$, $b = 0.5$, and $F(x) = x$ for initial conditions (−1, −1, −1).

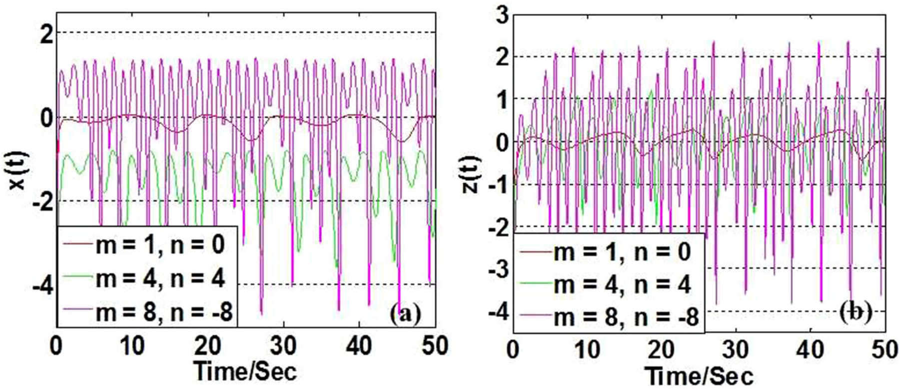

FIG. 5. Chaotic signals with different controllers: (a) $x(t)$ and (b) $z(t)$.

There is no chaotic solution of general equation (1), if the only term in the z-equation is linear because the coefficient of the linear term determines only the amplitude and frequency of the variables, and thus, the dynamics is solely governed by the other two dimensions. To prove this, in Eq. (1), when $a_7 = a_8 = a_9 = 0$, $\dot{z} = a_{10}x$, so $\dot{x} = \frac{1}{a_{10}}\ddot{z} = a_1y^2 + a_2z^2 + a_3yz$, which means that $\dot{z} = f(y,z)$ and $\dot{y} = a_4y^2 + a_5z^2 + a_6yz = g(y,z)$ indicating that System (1) degenerates into a two-dimensional system.

III. CHAOTIC SIGNAL CONTROL BY A LINEAR FUNCTION

When $F(x) = mx + n$, the new introduced coefficient m controls the amplitude and frequency of the chaotic signal[15,16] while the constant n provides an offset allowing a transformation between a bipolar signal and a unipolar signal. As shown in Fig. 5, when $m = 1$, chaotic signals in red are of low frequency; when $m = 4$ and 8, chaotic signals in green and pink increase their frequency along with an increase in the amplitude. When the constant n changes from 0 to 4 or to -8, the average level of x changes correspondingly, while the average level of z remains the same, indicating that n is an offset booster for the variable x. A more clear identification of this is seen in the phase trajectory, where it is controlled and shifted by these two parameters with different frequency spectra as shown in Fig. 6.

We can prove the free control of amplitude/frequency scaling and offset boosting theoretically and give a further demonstration with the linearly scaled Lyapunov exponent spectrum. As an illustration, making a simultaneous amplitude, frequency, and the offset control of Eq. (2) where $F(x) = x$ by the transformation $x \to (x + n/m)/m$, $y \to y/m$, $z \to z/m$, $t \to mt$, then Eq. (2) becomes

$$\begin{cases} \dot{x} = yz, \\ \dot{y} = y^2 - az^2 + yz, \\ \dot{z} = by^2 + mx + n. \end{cases} \tag{3}$$

Thus, if the coefficient m of the linear term x in the z-equation changes, it will rescale the amplitude and frequency according to m while the constant n will boost the offset of the variable x according to n/m in the negative direction. Figure 7 shows that when the coefficient m varies, the Lyapunov exponents are rescaled proportionally along with the modified frequency and amplitude of the chaotic signals. Figure 8 shows that the offset controller n will only change the average value of the signal x but will not influence the dynamics of the chaotic system, leading to the same Lyapunov exponents.

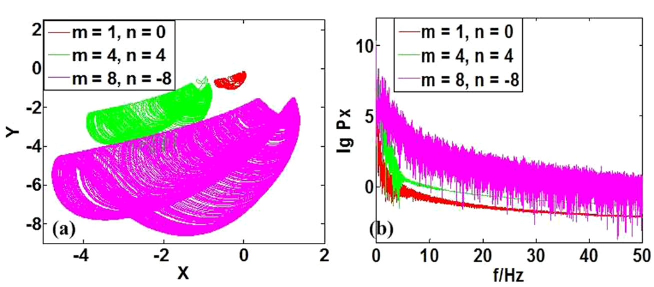

FIG. 6. Controlled phase trajectory (a) strange attractor in the x-y plane (b) the frequency spectrum of the signal $|x|$.

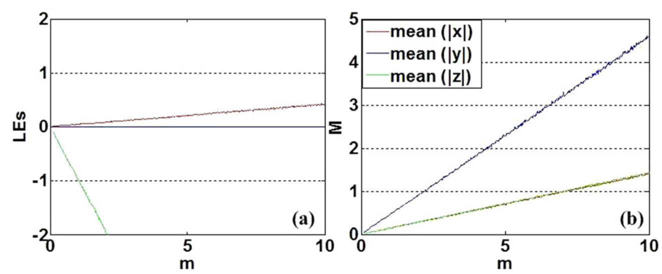

FIG. 7. Amplitude-frequency control with linearly rescaled Lyapunov exponents and amplitude when $a = 8.5$, $b = 0.5$, $n = 0$ for $F(x) = mx + n$ when m varies in [0, 10]: (a) Lyapunov exponents (b) average of absolute value signals $|x|$, $|y|$, and $|z|$.

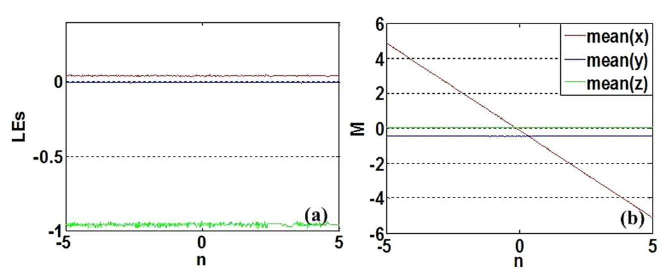

FIG. 8. Offset control with the same Lyapunov exponents when $a = 8.5$, $b = 0.5$, $m = 1$ for $F(x) = mx + n$ when n varies in $[-5, 5]$: (a) Lyapunov exponents and (b) average of signals x, y, and z.

IV. CIRCUIT IMPLEMENTATION

The above system is desirable for providing signals in chaos-based applications where appropriate amplitude and polarity are necessary for signal processing. Furthermore, when considering the hardware limitations in the circuit design, since the above system provides easy control for amplitude, frequency, and polarity, the circuit implementation becomes more convenient without any requirement of linear transformation for pre-scaling to avoid saturating the analog multipliers and operational amplifiers. In Fig. 2, we see that all of the signals, x, y, and z oscillate in the interval $(-1.5, 1.5)$. Therefore, we can begin the circuit design from the original equation without considering the control knobs. From Eq. (2), we design the analog circuit shown in Fig. 9 where the circuit equations in terms of the circuit parameters are

$$\begin{cases} \dot{x} = \dfrac{1}{R_1C_1}yz, \\ \dot{y} = \dfrac{1}{R_2C_2}y^2 - \dfrac{1}{R_4C_2}z^2 + \dfrac{1}{R_3C_2}yz, \\ \dot{z} = \dfrac{1}{R_6C_3}y^2 + \dfrac{1}{R_5C_3}x + \dfrac{1}{R_9C_3}V_{dd}. \end{cases} \quad (4)$$

The circuit includes three channels to realize the integration, addition, and subtraction of the state variables x, y, and z, respectively. The operational amplifier OPA404/BB and its peripheral circuit perform the addition, inversion, and integration, and the analog multiplier AD633/AD performs the nonlinear product operation. The state variables x, y, and z in Eq. (4) correspond to the state voltages of the three channels, respectively. For the system parameters $a = 8.5$,

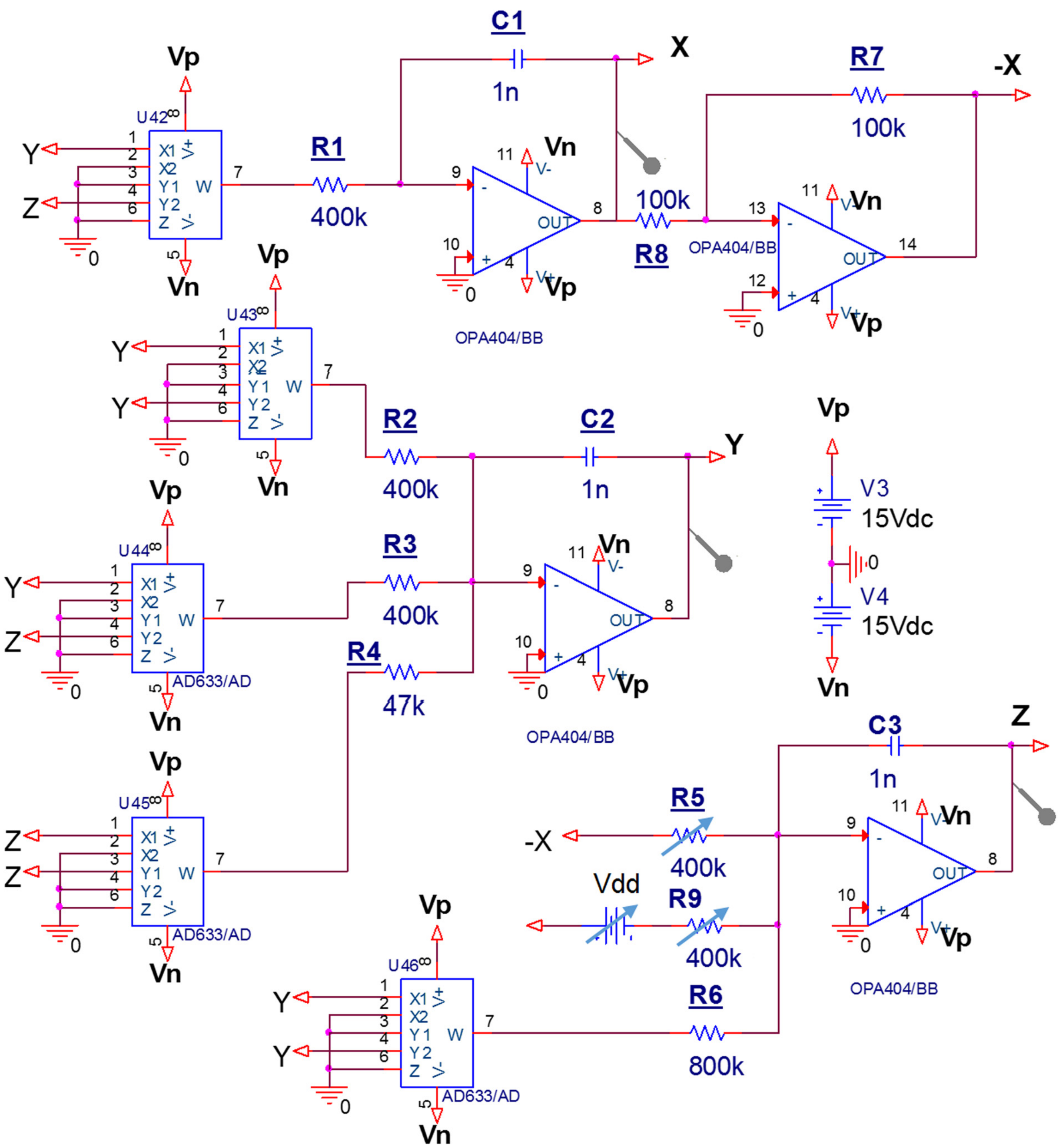

FIG. 9. Electronic circuit schematic of System (2) where $F(x) = mx + n$ $(m = 1, n = 0)$.

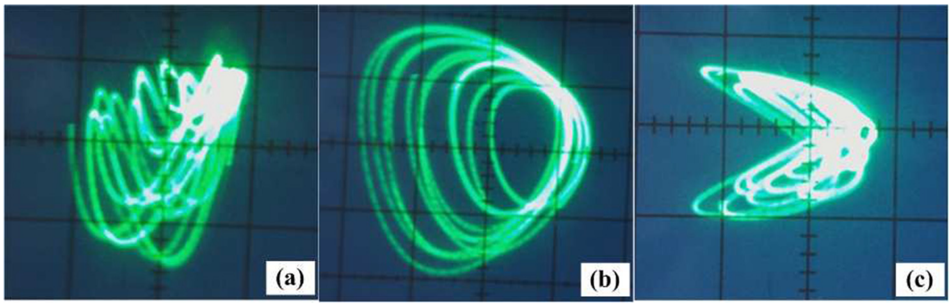

FIG. 10. Phase portrait of chaotic System (2) when $F(x) = mx + n$ ($m = 1$, $n = 0$) (a) *x-y* plane, (b) *x-z* plane, and (c) *y-z* plane. Volt/div = 0.1 in channel 1 and Volt/div = 0.2 in channel 2.

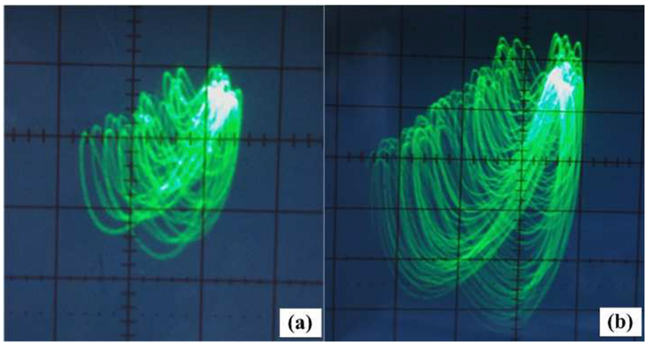

FIG. 11. Phase portrait of chaotic System (2) ($F(x) = mx + n$) in the *x-y* plane (a) $m = 4$, $n = 4$ (b) $m = 8$, $n = -8$. Volt/div = 0.5 in channel 1 and Volt/div = 1 in channel 2.

$b = 0.\ 5$, $m = 1$, and $n = 0$, the circuit element values are $R_1 = R_2 = R_3 = 40\,\mathrm{k\Omega}$, $R_4 = 4.7\,\mathrm{k\Omega}$, $R_5 = R_9 = 400\,\mathrm{k\Omega}$, $R_6 = 80\,\mathrm{k\Omega}$ and $R_7 = R_8 = 100\,\mathrm{k\Omega}$, $V_{\mathrm{dd}} = 0$ (or $R_9 = \infty$). We select the capacitors $C_1 = C_2 = C_3 = 1$ nF to obtain a stable phase portrait which only affects the time scale of the oscillation. Figure 10 shows the phase portraits observed on the oscilloscope. Unlike other chaotic oscillators, a potentiometer R_5 is used to control the amplitude/frequency. By this knob, we can adjust the signals x, y, and z for any desired voltages and frequencies. With a decrease in R_5, the amplitude and frequency of all variables increase accordingly. The polarity control of the signal x can be implemented by the adjustable DC source in the last dimension. The corresponding phase trajectories and signals are shown in Figs. 11 and 12 when the controllers are adjusted.

V. DISCUSSION AND CONCLUSIONS

A new class of chaotic oscillators with free control of amplitude, frequency, and offset is proposed in this paper, where two parameters in the linear function rescale the amplitude/frequency and boost the offset of the signal x independently. These controls only require two independent potentiometers or a variable resistor and an adjustable DC source, which is convenient for chaotic signal adjusting in engineering applications. Other simple cases of chaotic systems with free control are $\dot{x} = -3y^2 + 0.85z^2$, $\dot{y} = y^2 + yz$, $\dot{z} = -y^2 + F(x)$ and $\dot{x} = yz$, $\dot{y} = 0.13y^2 - z^2 + yz$, $\dot{z} = 2z^2 + F(x)$, where $F(x) = mx + n$, and the coefficients m and n have the same function as demonstrated earlier. Note that these newly found systems are based on the exhaustive computer searching, but degree modification[18] of the structure in the dynamical system may provide another approach to find systems with free control since the absolute value function and signum function can return the necessary degree balance. Since the new type of chaotic system with free control can provide chaotic signals with any desired amplitude,

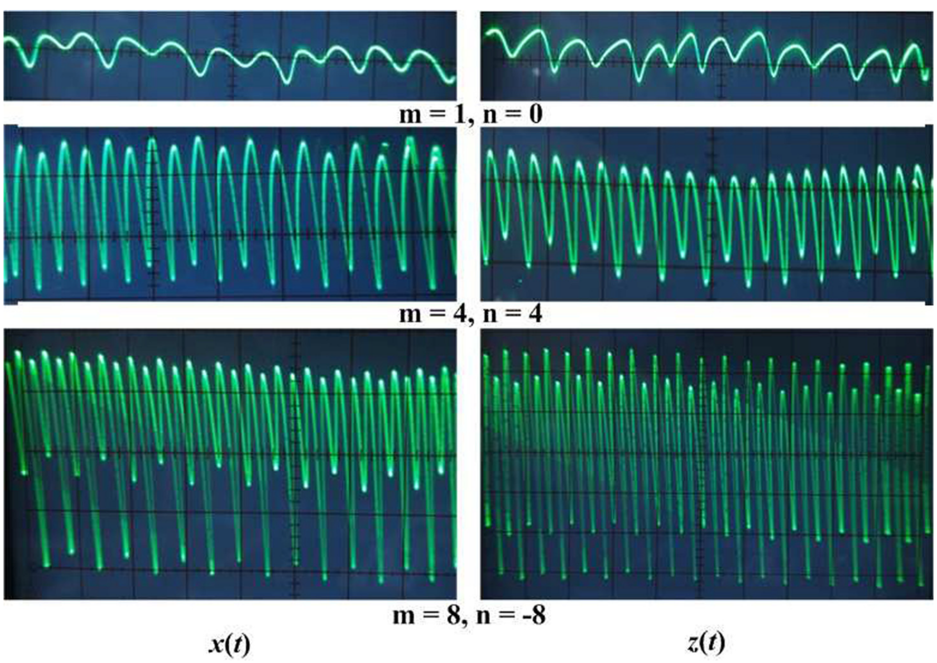

FIG. 12. Chaotic signals $x(t)$ and $z(t)$ from System (2). Volt/div = 0.5 in channel 1 and Volt/div = 1 in channel 2.

frequency, and polarity, it should have a broad application in chaos-based information systems.

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