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Constructing Infinitely Many Attractors in a Programmable Chaotic Circuit

CHUNBIAO LI[1,2], (Member, IEEE), WESLEY JOO-CHEN THIO[3], JULIEN CLINTON SPROTT[4], HERBERT HO-CHING IU[5], (Senior Member, IEEE), AND YUJIE XU[1,2]

[1]Jiangsu Key Laboratory of Meteorological Observation and Information Processing, Nanjing University of Information Science and Technology, Nanjing 210044, China
[2]School of Electronic and Information Engineering, Nanjing University of Information Science and Technology, Nanjing 210044, China
[3]Department of Electrical and Computer Engineering, The Ohio State University, Columbus, OH 43210, USA
[4]Department of Physics, University of Wisconsin–Madison, Madison, WI 53706, USA
[5]School of Electrical, Electronic, and Computing Engineering, The University of Western Australian, Crawley, WA 6009, Australia

Corresponding author: Chunbiao Li (chunbiaolee@nuist.edu.cn).

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ABSTRACT In this paper, we modify the Sprott M chaotic system to provide infinitely many co-existing attractors by replacing the offset boosting parameter with a periodic function giving what we call a self-reproducing system. Consequently, a chaotic signal with either polarity can be obtained by selecting different initial conditions. Various periodic functions are introduced in the same offset-boostable system for producing coexisting attractors. We used a field programmable analog array to construct a programmable chaotic circuit, and the predicted attractors were observed on an oscilloscope.

INDEX TERMS Field programmable analog array (FPAA), infinitely many attractors, offset boosting, polarity control.

I. INTRODUCTION

Chaos exists widely in nature and engineering. Scientists and engineers have made great effort for chaos applications rather than chaos suppression. When the problem of synchronization between two chaotic systems was theoretically solved [1]–[6], it set off another surge in chaos-based engineering [7]–[12]. Specifically, chaotic signals and systems are widely used in image encryption [13]–[15], secure communications [16], [17], weak signal detection [18], [19], and Radar systems [20, p. 22]. A natural question is how to generate a chaotic signal with suitable polarity given that some hardware only allows unipolar or bipolar signals. This is not an easy question since the broadband of chaotic signals may pose risks to the traditional polarity convertor with its bandpass characteristics. To solve this problem, we turn back to system design. Two special regimes of chaotic systems may accomplish our goals. For an offset-boostable system [23]–[25], polarity control can be realized by modulating an extra DC-power source; while for a self-reproducing system [25], the selection of initial conditions may give the chaotic signal any desired polarity. The latter method is based on a self-reproducing system with many coexisting chaotic attractors with the same Lyapunov exponents in the solution space, which can be extracted using different initial conditions.

Here the multistability is a virtue for polarity control. In contrast, it is often problematic in engineering applications. Scientist and engineers have found that multistability exists in many physical systems, such as a gas laser [26], a delayed system [27], a biological system [28], atoms [29], a lactose utilization network [30], fiber lasers [31], phosphorylation systems [32], electroencephalograms [33], neural networks [34], ice sheets [35], and even in semiconductor superlattices [36]. The symmetric structure of these systems allows them to have a pair of coexisting attractors [37]–[47]. It is relatively rare to find coexisting attractors in asymmetric systems [48]–[50]. We find that systems with offset boosting [25], where the average of one of the signals can be controlled by a constant, can produce infinitely many attractors when replaced by a slowly changing periodic signal. Recognizing the connection between the chaotic signal with a desired polarity and multistability motivates the design of a chaotic system with infinitely many attractors. In this way, the needed unipolar or bipolar chaotic signal can be obtained by selecting a proper attractor with initial conditions. By this method, the complex hardware for polarity control is unnecessary. Note that unlike other references [51]–[54], here infinitely many attractors refer to the coexisting infinitely many attractors with the same shape of a unified Lyapunov exponent spectrum.

Field Programmable Analog Array (FPAA) technology has the advantages of both hardware and software, which is reliable and flexible and has the advantage of fast response, rapid prototyping, adaptation, and reduced cost or simplicity of design for the programmable architecture [55], [56]. FPAA provides such a workbench for circuit realization especially since it is more convenient for analog chaotic circuit design, and therefore recently FPAA is becoming more popular and is used in many applications such as system modeling, signal processing, fault-tolerance, and computing feature extraction. In this paper, we give an example of constructing chaotic systems with infinitely many strange attractors based on the newest integrated circuit technique of FPAA. In Section II, the mechanism for polarity control by constructing a self-reproducing system with infinitely many attractors is analyzed. In Section III, infinitely many attractors are coined in a simple offset-boostable system. In Section IV, the system is implemented on an FPAA along with different non-zero initial conditions. Conclusions and discussion are given in the last section.

II. APPROACHES FOR POLARITY CONTROL

As we mentioned, for the broadband frequency spectrum of a chaotic signal, the polarity transformation cannot be realized by traditional methods. A variable-boostable system [23]–[25] can output chaotic signals with any desired offset, and correspondingly can provide unipolar or bipolar chaotic signals. We call this class of system offset-boostable. In this case, the polarity control is realized by a variable DC voltage in the circuit which corresponds to a constant in the dynamical system equations. By introducing a periodic function of one of the variable into the offset-boostable system, as shown in Fig. 1, the system is converted into a self-reproducing system producing infinitely many attractors. The polarity control is realized by an attractor selection, which locates in the solution space with different offset and can be probed using different initial conditions. In a dynamical system initial condition refers to the initial value for an established solution while in an electronic oscillator the initial condition refers to the initial voltage in the capacitor or the initial current in the inductor. The scheme can be understood from the following definitions.

Definition 1: Define a differential dynamical system $\dot{X} = F(X)$ $(X = (x_1, x_2, \ldots, x_n), 1, 2, \ldots, n \in N)$ as an offset-boostable system if there exists a variable substitution $y_k = x_k - c_k$ subjected to $\dot{Y} = F(Y) + D(Y = (y_1, y_2, \ldots, y_n), C = (c_1, c_2, \ldots, c_n), D = (d_1, d_2, \ldots, d_n))$. Here when $k \in \{i_1, i_2, \ldots, i_m (1 \le i_1 < i_2 < \cdots < i_m \le n$, c_k is non-zero constant but when $k \in \{1, 2, \ldots, n i_1, i_2, \ldots, i_m$, c_k is zero.

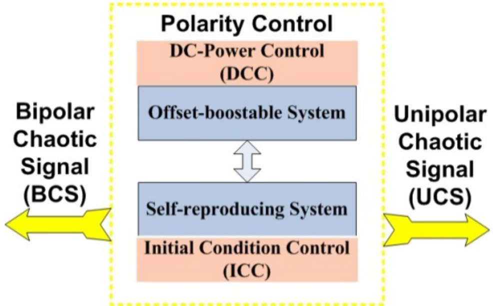

FIGURE 1. Polarity control of chaotic signal from system design.

Definition 2: Define a differential dynamical system $\dot{X} = F(X)$ $(X = (x_1, x_2, \ldots, x_n), 1, 2, \ldots, n \in N)$ as a self-reproducing system if it has many similar attractors (solution sets are marked as $S = S_1 \cup S_2 \ldots \cup S_m (1, 2, \ldots, m \in N)$) with the same Lyapunov exponent spectrum when the initial condition locates in the corresponding basins of attraction.

Theorem 3: A dynamical System (1) can be revised to System (2) for self-reproducing,

$$\begin{cases} \dot{x}_1 = F_1(x_1, x_2, \ldots, x_n), \\ \dot{x}_2 = F_2(x_1, x_2, \ldots, x_n), \\ \ldots\ldots \\ \dot{x}_n = F_n(x_1, x_2, \ldots, x_n). \end{cases} \tag{1}$$

$$\begin{cases} \dot{x}'_1 = G_1(x'_1, x'_2, \ldots, x'_n), \\ \dot{x}'_2 = G_2(x'_1, x'_2, \ldots, x'_n), \\ \ldots\ldots \\ \dot{x}'_n = G_n(x'_1, x'_2, \ldots, x'_n). \end{cases} \tag{2}$$

when the periodic functions are introduced into $F_i(x_1, x_2, \ldots, x_n)(i \in \{1, 2, \ldots, n)$ as: $x_k = g_k\left(x'_k\right)$, $(k \in \{j_1, j_2, \ldots, j_m\}, 1 \le j_1 < j_2 < \ldots < j_m \le n)$ and $x_k = x'_k, (k \in \{1, 2, \ldots, n\} \backslash \{j_1, j_2, \ldots, j_m\})$ supposing System (2) still has a bounded solution.

Proof 4: Since $g_{l1}(x_{l1}{'})$ is a periodic function, there exists a constant P_{l1} and an integer S_{l1} subject to $g_{l1}\left(x'_{l_1} + S_{l_1} P_{l_1}\right) = g_{l_1}\left(x'_{l_1}\right)$, $l_1 \in \{j_1, j_2, \ldots, j_m\}$. Make a variable substitution: $x''_{l_1} = x'_{l_1} + S_{l_1} P_{l_1}$, $x''_{l_2} = x'_{l_2}$, $l_2 \in \{1, 2, .., n\} \backslash \{j_1, j_2, \ldots, j_m\}$, and the following system is obtained,

$$\begin{cases} \dot{x}''_1 = G_1(x''_1, x''_2, \ldots, x''_n), \\ \dot{x}''_2 = G_2(x''_1, x''_2, \ldots, x''_n), \\ \ldots\ldots \\ \dot{x}''_n = G_n(x''_1, x''_2, \ldots, x''_n). \end{cases} \tag{3}$$

Equation (3) equates to Eq. (2), which indicates that System (2) is a self-reproducing system giving the same solution except for a shift in phase space according to the period of the new introduced function.

An offset-boostable system can be revised to be a self-reproducing system when some of the offset-boostable variables are revised to be a periodic function if the bounded

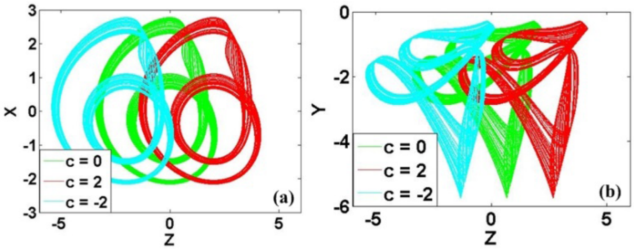

FIGURE 2. Strange attractor from System (4) with $a = b = 1.7$ and initial conditions $(-1, 0, -1+c)$.

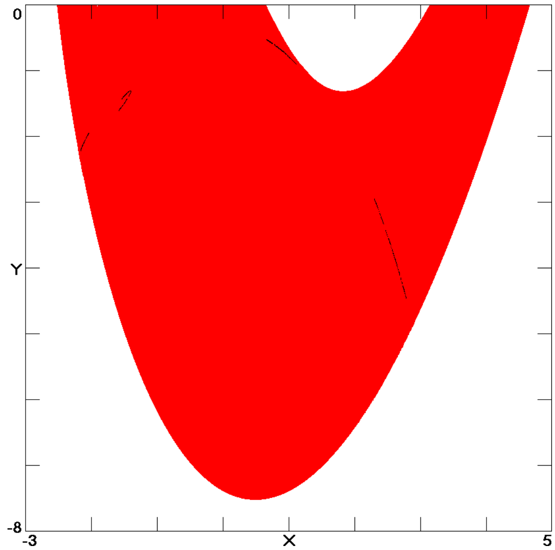

FIGURE 3. Cross section of the basin of attraction for System (4) with $a = b = 1.7$, $c = 0$ in the plane $z = 0$.

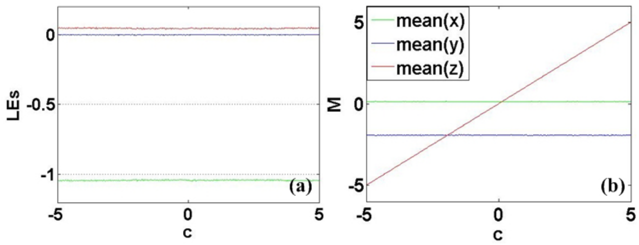

FIGURE 4. Lyapunov exponents and average value of the variables for initial conditions $(1, -1, c)$ when c varies from -5 to 5.

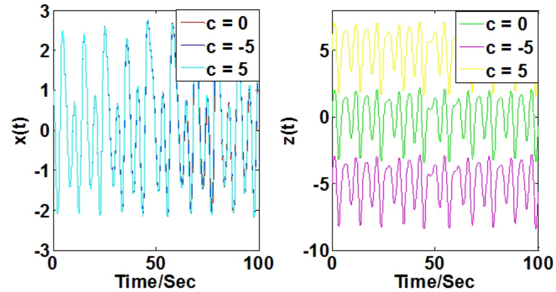

FIGURE 5. Offset-boosting-based polarity converting with an offset control c and initial conditions $(1, -1, c)$.

solution is preserved in one period. In this case, polarity control of the chaotic signal can be realized only with an initial condition rather than a DC voltage control. We will give a demonstration in the following section.

III. SELF-REPRODUCING CHAOTIC SYSTEM WITH INFINTELY MANY ATTRACTORS

As reported in [23]–[25] and [57], many systems have an offset boosting property. The Sprott M system is an example [57] where the variable z can be offset boosted by a new introduced constant c in the x dimension,

$$\begin{cases}\dot{x} = -z + c,\\ \dot{y} = -x^2 - y,\\ \dot{z} = a + bx + y.\end{cases} \tag{4}$$

When $a = b = 1.7$ and $c = 0$, the system is chaotic with corresponding Lyapunov exponents (0.044, 0, -1.044) and Kaplan-Yorke dimension 2.042, as shown in Fig. 2. The basin of attraction as shown in Fig. 3 shows a large region with unbounded solutions. The system is asymmetric and has two equilibrium points of saddle-foci: (2.406, -5.791, 0) is of index-1 with eigenvalues (0.9074, $-0.9537\pm1.5878i$), and (-0.706, -0.499, 0) is of index-2 with eigenvalues (-1.3892, $0.1946\pm1.4842i$). Here the constant c is available for offset boosting of the variable z.

The constant c in the x equation only affects the average level of the variable z. To show this, let $x = u$, $y = v$, $z = w + c$ to obtain new equations in the variables u, v, w that are identical to System (4) with $c = 0$, as shown in Fig. 4. It is shown that when the variable c increases in $[-5, 5]$, the average of the variable z increases accordingly while the averages of x and y remain unchanged. This property gives an opportunity to get a unipolar signal z as shown in Fig. 5 where the constant c represents a DC voltage in the circuit. To guarantee the desired oscillation, the initial condition should shift in the z direction simultaneously, which indicates that this offset-boosting-based polarity convertor needs a DC voltage for offset control and also needs special equipment for presetting the initial conditions. The constant c can be hidden in a periodic function as a period for giving such an offset boosting.

Self-reproducing systems [25] can be coined by introducing a periodic function into System (4). Consequently, infinitely many attractors can be obtained if constant c turns to be discrete numbers and is placed in a periodic function of z. This will generate different attractors according to the initial condition of the variable z without influencing its basic dynamics. For example, a sinusoidal function can be introduced into the first dimension as follows:

$$\begin{cases}\dot{x} = f(z),\\ \dot{y} = -x^2 - y,\\ \dot{z} = a + bx + y,\end{cases} \tag{5}$$

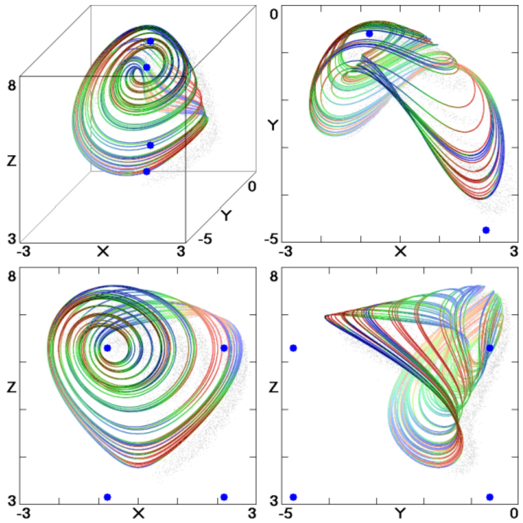

FIGURE 6. Strange attractor from System (5) with $a = 1.7$, $b = 1.4$, $m = 3$, $n = 1$, and initial conditions (2, 2, 0), the colors indicate the value of the local largest Lyapunov exponent with positive values in red and negative values in blue. Equilibrium points are shown as blue dots.

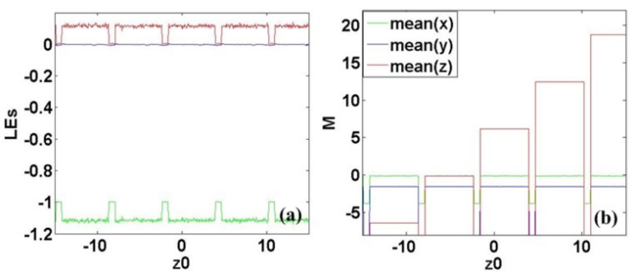

FIGURE 7. Lyapunov exponents for System (5) when $a = 1.7$, $b = 1.4$, $m = 3$, $n = 1$ and the average value of the variables for different initial conditions $(2, 2, z_0)$ when z_0 varies from -15 to 15.

Here $f = -m\sin(nz)$, but with slightly revised parameters such as $a = 1.7$, $b = 1.4$, $m = 3$, $n = 1$ which will preserve the chaos but with larger Lyapunov exponents (0.1151, 0, -1.1151) and a Kaplan-Yorke dimension 2.1032 and an attractor as shown in Fig. 6. System (5) remains the asymmetric structure, and has four groups of infinite equilibrium points of saddle-foci: $(2.1799, -4.7518, 2k\pi(k \in Z))$, which are of index-1 with eigenvalues $(1.2600, -1.1300\pm2.4022i)$, $(2.1799, -4.7518, (2k + 1)\pi(k \in Z))$, which are of index-2 with eigenvalues $(-3.1896, 1.0948\pm1.2591i)$, $(-0.7799, -0.6082, 2k\pi(k \in Z))$, which are of index-2 with eigenvalues $(-1.6696, 0.3348\pm2.2817i)$ and $(-0.7799, -0.6082, (2k + 1)\pi(k \in Z))$, which are of index-1 with eigenvalues $(2.3645, -1.6822\pm0.9620i)$.

Here the constant c in Eq.(4) disappeared, but the period in the first dimension in Eq. (5) gives a periodic offset boosting. As predicted, System (5) displays chaos with countless coexisting strange attractors. As shown in Fig. 7, when the initial condition in the z dimension varies from -15 to 15, the average value of z jumps to different levels corresponding to different attractors, each of which share the same

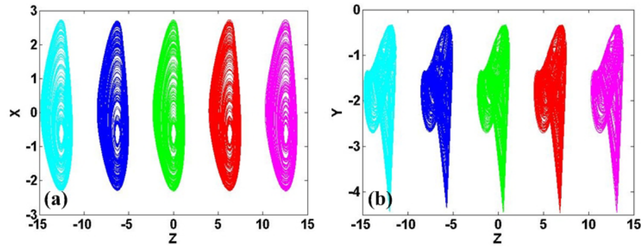

FIGURE 8. Some of the coexisting strange attractors for initial conditions: $(1, -1, k\pi)$, $k = 0$ for green, $k = -2$ for blue, $k = -4$ for cyan, $k = 2$ for red, and $k = 4$ for pink.

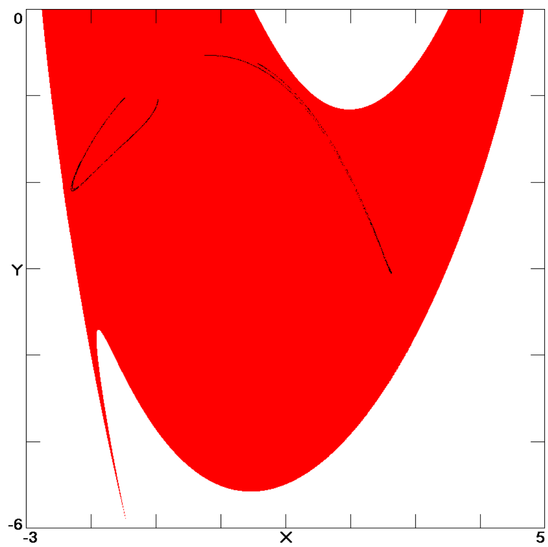

FIGURE 9. Cross section of the basin of attraction for System (5) with $a = 1.7$, $b = 1.4$, $m = 3$, $n = 1$ in the plane $z \pmod{2\pi} = 0$.

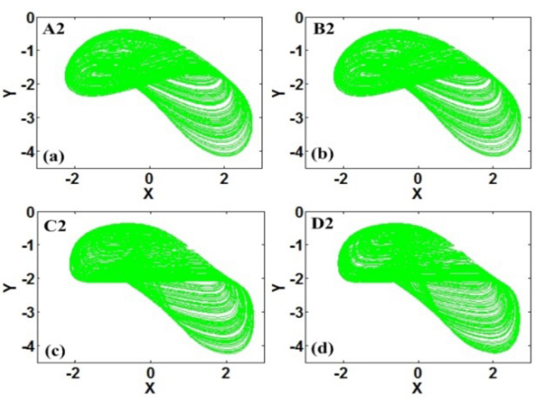

FIGURE 10. Typical attractors of System (5) with different periodic functions.

Lyapunov exponents. Note that there is a third state between the predicted strange attractors, where the system diverges in the direction of z and evolves to an oscillation in the direction of x and y. In Fig. 8 five coexisting strange attractors are shown for different initial conditions. The selection of suitable initial conditions is not critical, but values close to $(1, -1, k\pi)$ are in the basins of the corresponding attractors. Here we can clearly see that the polarity of the signal z can

TABLE 1. Typical periodic functions giving multistability.

Cases	Periodic functions	Parameters	Initial conditions	Period	LEs
A1	$-m\sin(nz)$	$m = 5$ $n = 1$	2, -2, 0	2π	0.2171,0, -1.2171
A2	$-m\sin(nz)$	$m = 10$ $n = 0.5$	2, -2, 0	4π	0.2322,0, -1.2322
A3	$-m\sin(nz)$	$m = 20$ $n = 0.25$	2, -2, 0	8π	0.2329,0,-1.2329
A4	$-m\sin(nz)$	$m = 40$ $n = 0.125$	2, -2, 0	16π	0.2307,0,-1.2307
B1	$m\cos(nz)$	$m = 6$ $n = 1$	2, -2, 1	2π	0.1544,0, -1.1544
B2	$m\cos(nz)$	$m = 12$ $n = 0.5$	2, -2, 3	4π	0.1620, 0, -1.1620
B3	$m\cos(nz)$	$m = 24$ $n = 0.25$	2, -2, 6	8π	0.1580,0,-1.1580
B4	$m\cos(nz)$	$m = 48$ $n = 0.125$	2, -2, 12	16π	0.1591,0, -1.1591
C1	$-m\tan(nz)$	$m = 1.1$ $n = 2$	2, -2, 0	$\pi/2$	0.2794,0, -1.2794
C2	$-m\tan(nz)$	$m = 1.6$ $n = 2$	2, -2, 0	$\pi/2$	0.2509,0 ,-1.2509
C3	$-m\tan(nz)$	$m = 2$ $n = 2$	2, -2, 0	$\pi/2$	0.1510,0,-1.1510
C4	$-m\tan(nz)$	$m = 2.4$ $n = 2$	2, -2, 0	$\pi/2$	0.1275,0 ,-1.1275
D1	$m\cot(nz)$	$m = 1.1$ $n = 2$	2, -2, 1	$\pi/2$	0.2795, 0, -1.2795
D2	$m\cot(nz)$	$m = 1.6$ $n = 2$	2, -2, 1	$\pi/2$	0.2506,0,-1.2506
D3	$m\cot(nz)$	$m = 2$ $n = 2$	2, -2, 1	$\pi/2$	0.1518,0,-1.1518
D4	$m\cot(nz)$	$m = 2.4$ $n = 2$	2, -2, 1	$\pi/2$	0.1272,0,-1.1272

be selected by initial condition from the green, blue and red attractor corresponding to bipolarity, negative and positive polarity.

The basin for one of the attractors has the same shape as the original system as shown in Fig. 9, which also includes a large area of unbounded solutions. Note that there are infinitely many attractors located in Euclidean space. The corresponding circuit is more convenient for producing chaotic oscillations since infinitely many solutions are available. Furthermore, this time a chaotic signal with the desired polarity can complement by selecting an attractor with specific initial conditions.

If the periodic function is replaced with $\dot{x} = -m\cos(nz)$, System (5) will still give the same countless coexisting strange attractors. Moreover, changing the polarity of

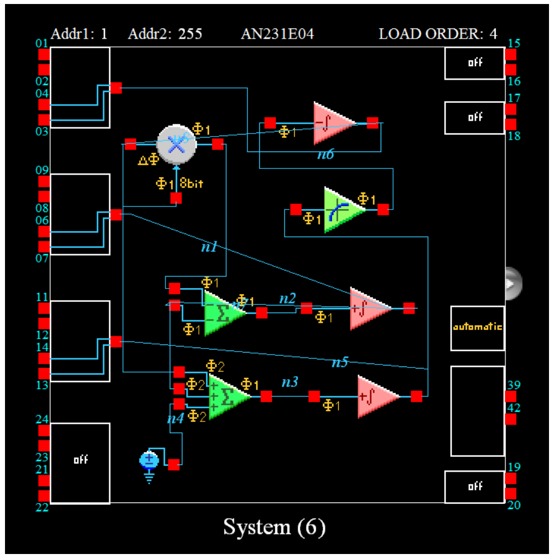

FIGURE 11. Circuit schematic of System (6) in AnadigmDesigner2.

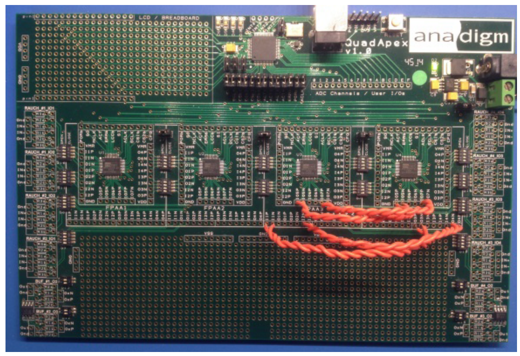

FIGURE 12. Anadigm QuadApex Development Board.

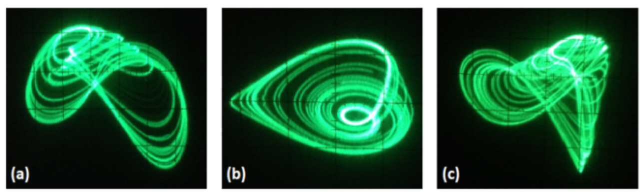

FIGURE 13. Phase trajectory of System (6) from oscilloscope (a) *x-y* (b) *z-x* (c) *z-y*.

the function will not affect the system significantly, then $\dot{x} = m\sin(nz)$ and $\dot{x} = m\cos(nz)$ also give the same attractors. Some other typical functions and parameters can be applied to replace the sinusoidal function to obtain infinitely many attractors as shown in Table 1. Four attractors from Table 1 are shown in Fig. 10. Here the parameters are selected for their relatively larger Lyapunov exponent, and initial conditions are given arbitrarily for obtaining those attractors.

IV. CIRCUIT REALIZATION BASED ON FPAA

Here an Anadigm QuadApex Development board with four AN231E04 chips [58]–[61] was used to construct a circuit

TABLE 2. CAM parameters of the circuit in Fig. 7.

Name	Options	Parameters
Integrator1 (Integrator v1.1.1) *Anadigm (Approved)*	Polarity: *Non-inverting* Input Sampling Phase: *Phase 1* Compare Control To: *No Reset*	Integration Const. [1/us]: *0.00250*
Integrator2 (Integrator v1.1.1) *Anadigm (Approved)*	Polarity: *Non-inverting* Input Sampling Phase: *Phase 1* Compare Control To: *No Reset*	Integration Const. [1/us]: *0.00250*
Integrator3 (Integrator v1.1.1) *Anadigm (Approved)*	Polarity: *Non-inverting* Input Sampling Phase: *Phase 1* Compare Control To: *No Reset*	Integration Const. [1/us]: *0.00250*
Multiplier1 (Multiplier v1.0.2) *Anadigm (Approved)*	Sample and Hold: *Off*	Multiplication Factor: *1.00*
Voltage1 (Voltage v1.0.1) *Anadigm (Approved)*	Polarity: *Positive (+2V)*	
SumDiff1 (SumDiff v1.0.1) *Anadigm (Approved)*	Output Phase: *Phase 1* Input 1: *Inverting* Input 2: *Inverting* Input 3: *Off* Input 4: *Off*	Gain 1 (UpperInput): *8.00* Gain 2 (LowerInput): *1.00*
SumDiff2 (SumDiff v1.0.1) *Anadigm (Approved)*	Output Phase: *Phase 1* Input 1: *Non-inverting* Input 2: *Non-inverting* Input 3: *Non-inverting* Input 4: *Off*	Gain 1 (UpperInput): *1.12* Gain 2 (MiddleInput): *0.400* Gain 3 (LowerInput): *0.160*
TransferFunction1 (TransferFunction v1.0.1) *Anadigm (Approved)*	Output Hold: *Off*	

implementation of System (5) with non-zero initial conditions and to experimentally verify the infinitely many coexisting attractors. Each chip contains four Configurable Analog Blocks (CABs) which perform the analog processing through fully differentiable switched capacitor technology. Since the dpASP has a differential output voltage level of ±3V, amplitude rescaling is necessary. To put it within the +/−3V signal range of the FPAA, System (5) was first rescaled by $x \to 4x$, $y \to 2y$, $z \to 5z$ as follows:

$$\begin{cases} \dot{x} = -\dfrac{m}{4}\sin(5nz), \\ \dot{y} = -8x^2 - y, \\ \dot{z} = \dfrac{a}{5} + \dfrac{4bx}{5} + \dfrac{2y}{5}, \end{cases} \tag{6}$$

The circuit implementation of System (6) was designed using AnadigmDesigner2 as shown in Fig. 11 and then downloaded into an Anadigm QuadApex Development Board consisting of four AN231E04 chips as shown in Fig. 12. Configurable Analog Module (CAM) values are given in Table 2, and the corresponding phase trajectories on the oscilloscope are shown in Fig. 13. The clock frequency of the CAMS is 250kHz.

TABLE 3. CAM parameters of the circuit in Fig. 14 for System(6A) and System(6B).

Name	Options	Parameters
Integrator8 (Integrator v1.1.1) *Anadigm (Approved)*	Polarity: *Non-inverting* Input Sampling Phase: *Phase 1* Compare Control To: *No Reset*	Integration Const. [1/us]: *0.00250*
Integrator9 (Integrator v1.1.1) *Anadigm (Approved)*	Polarity: *Non-inverting* Input Sampling Phase: *Phase 1* Compare Control To: *No Reset*	Integration Const. [1/us]: *0.00250*
Integrator7 (Integrator v1.1.1) *Anadigm (Approved)*	Polarity: *Non-inverting* Input Sampling Phase: *Phase 1* Compare Control To: *No Reset*	Integration Const. [1/us]: *0.00250*
SumDiff4 (SumDiff v1.0.1) *Anadigm (Approved)*	Output Phase: *Phase 1* Input 1: *Non-inverting* Input 2: *Inverting* Input 3: *Off* Input 4: *Off*	Gain 1 (UpperInput): *0.125* Gain 2 (LowerInput): *1.00*
Voltage4 (Voltage v1.0.1) *Anadigm (Approved)*	Polarity: *Positive (+2V)*	
(SumDiff v1.0.1) *Anadigm (Approved)*	Input 1: *Inverting* Input 2: *Inverting* Input 3: *Off* Input 4: *Off*	Gain 2 (LowerInput): *1.00*
Voltage6 (Voltage v1.0.1) *Anadigm (Approved)*	Polarity: *Positive (+2V)*	

Name	Options	Parameters
Multiplier2 (Multiplier v1.0.2) *Anadigm (Approved)*	Sample and Hold: *Off*	Multiplication Factor: *1.00*
Voltage2 (Voltage v1.0.1) *Anadigm (Approved)*	Polarity: *Positive (+2V)*	
SumDiff1 (SumDiff v1.0.1) *Anadigm (Approved)*	Output Phase: *Phase 1* Input 1: *Inverting* Input 2: *Inverting* Input 3: *Off* Input 4: *Off*	Gain 1 (UpperInput): *8.00* Gain 2 (LowerInput): *1.00*
SumDiff4 (SumDiff v1.0.1) *Anadigm (Approved)*	Output Phase: *Phase 1* Input 1: *Non-inverting* Input 2: *Non-inverting* Input 3: *Non-inverting* Input 4: *Off*	Gain 1 (UpperInput): *1.12* Gain 2 (MiddleInput): *0.400* Gain 3 (LowerInput): *0.160*
TransferFunction2 (TransferFunction v1.0.1) *Anadigm (Approved)*	Output Hold: *Off*	

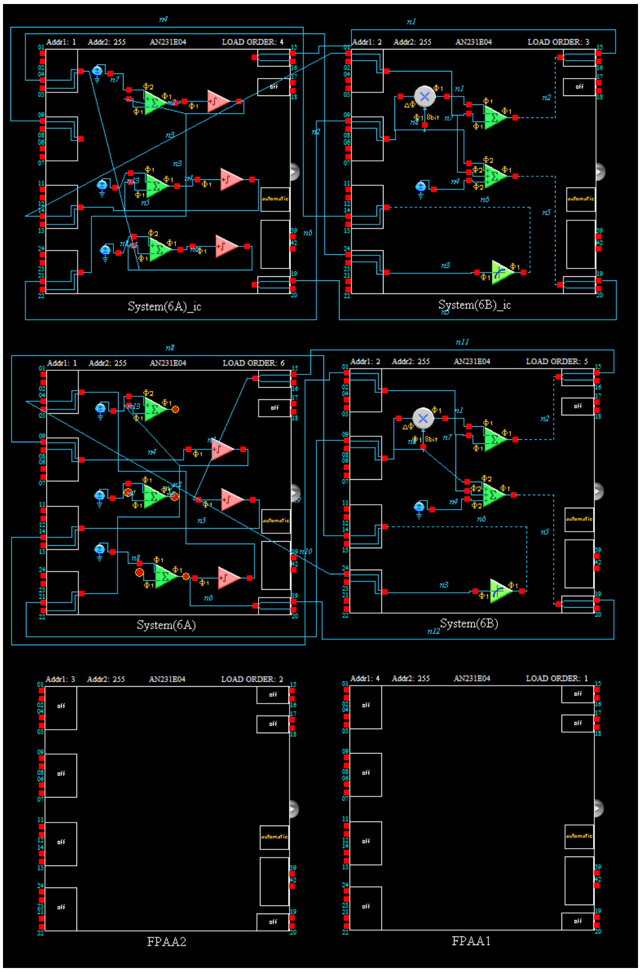

FIGURE 14. Circuit schematic of System (6) in AnadigmDesigner2 for nonzero initial conditions.

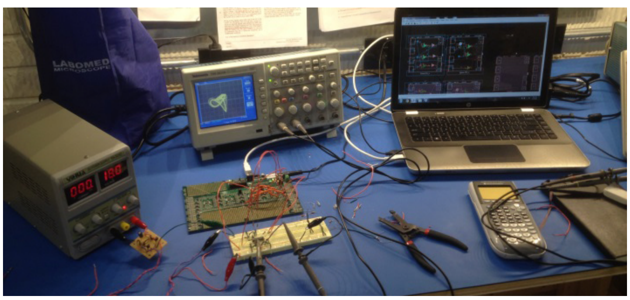

FIGURE 15. Hardware for the implementation of System (6).

To set non-zero initial conditions, the circuit implementation of System (6) was constructed on two FPAA chips, System (6A) and System (6B) as shown in Fig. 14. The physical connections are shown in Fig. 14. Non-zero initial conditions were then applied to the integrators and a primary configuration AHF file consisting of chips System (6A)_ic, System (6B)_ic, FPAA2 and FPAA1 was downloaded to the board. Then a dynamic configuration AHF file consisting of chips System (6A), System (6B), FPAA2 and FPAA1 was

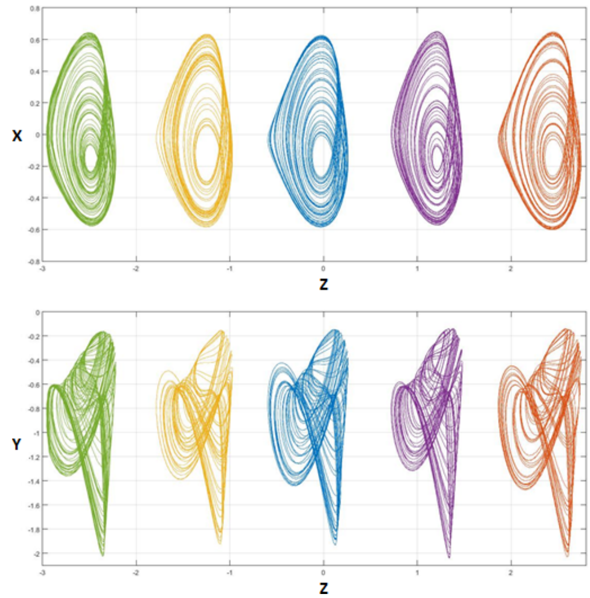

FIGURE 16. Coexisting strange attractors from the circuit.

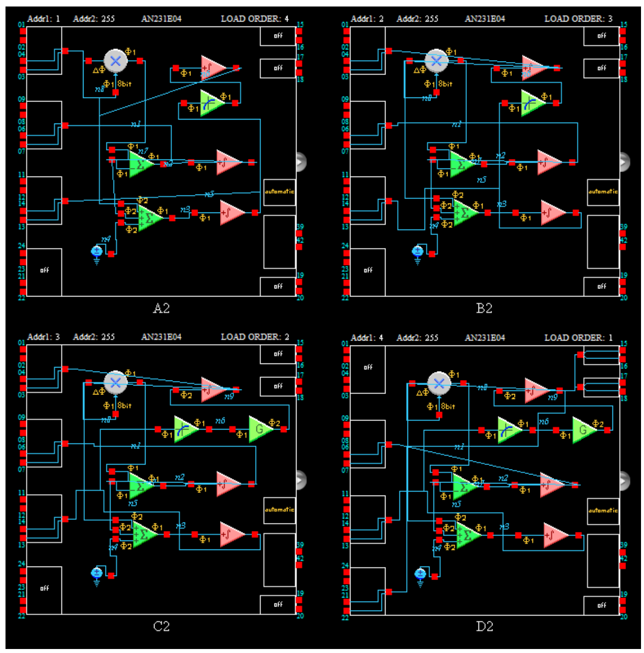

FIGURE 17. Circuit schematic of A2–D2 in AnadigmDesigner2.

downloaded to start the circuit. The CAM values are given in Table 3, using a clock frequency of 250 kHz.

The whole hardware implementation scheme is shown in Fig. 15. The signals were collected through a NI USB 6002 at a sample rate of 16 kHz after time rescaling and are shown in Fig. 16 to demonstrate multistability of the system.

Circuits for various periodic functions corresponding to A2, B2, C2, and D2 in Fig. 10 were also implemented. To fit the functions of C2 and D2 into the +/−3V range, system C2 was scaled by $x \to 5x$, $y \to 2y$, $z \to z/4$, and system D2 was scaled by $x \to 8x$, $y \to 2y$, $z \to z/2$. The

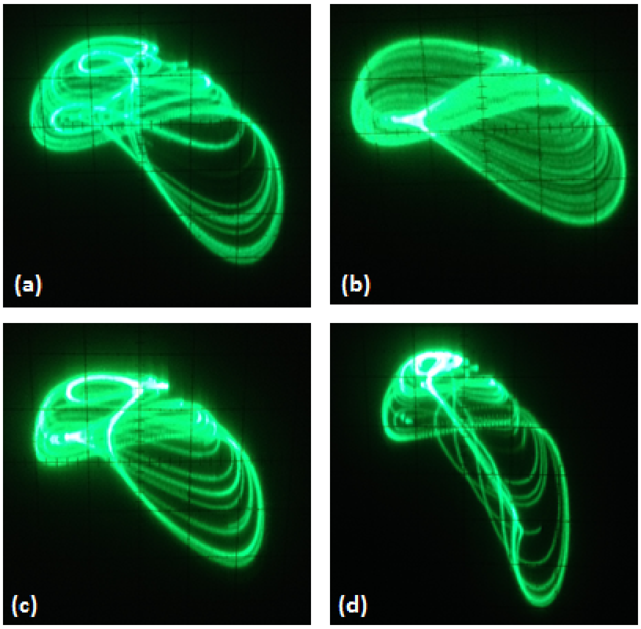

FIGURE 18. Phase trajectory of System (5) from oscilloscope (a) A2 (b) B2 (c) C2 (d) D2.

schematic is shown in Fig. 17, and the corresponding phase trajectories are shown in Fig. 18. The configuration files for the AHF files and transfer function block can be downloaded at http://wesleythio.com/.

V. CONCLUSION AND DISCUSSION

The polarity control of chaotic signals is an important issue in engineering applications like amplitude control [58]–[60]. A feasible approach for realizing chaotic polarity control may entail system selection. We describe two regimes of chaotic systems which can output unipolar or bipolar chaotic signals. One regime is defined as an offset-boostable system, where a DC voltage is applied for polarity control. Considering the existence of unbounded solutions, additional hardware may be necessary for initial condition presetting. The other regime is defined as a self-reproducing system with infinitely many attractors coexisting in the solution space, any of which can be extracted by choosing an initial condition. However, when the offset constant is replaced by a periodic function in the offset-boostable system, a series of initial-condition-periodic attractors are born giving infinite multistability.

The mechanism for constructing dynamical systems with infinitely many attractors is based on the fact that some of the variables in the system can be offset boosted by a newly introduced constant. In this paper, we show that when applying this method in the Sprott M system, we get the predicted coexisting strange attractors. Note that the periodic function can be rectangular or some other nonlinear function so long as it preserves the desired oscillation and the method proposed in this paper is also effective for those symmetric cases [72], [73]. An effective circuit for demonstrating the infinitely many attractors was built based on the FPAA, and the output from the circuit demonstrates the predicted multistability.

The experiment shows that it is more convenient to transform unipolar and bipolar signals based on the selection of any of the coexisting attractors. There are three distinct advantages for polarity control based on infinite multistability. Firstly a desired signal (corresponding to an attractor) can be freely selected by the initial conditions rather than different hardware equipment. Secondly the unipolar signal and the bipolar signal can exchange smoothly. Moreover, more coexisting attractors in a limited solution space reduce the risk of oscillation death since some systems may have unbounded solutions or coexisting stable point attractors (correspondingly the chaotic attractor is hidden [65]–[71]) and can provide more available chaotic sources for signal processing.

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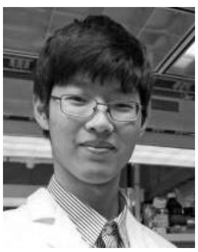
CHUNBIAO LI received the master's and Ph.D. degrees from the Nanjing University of Science and Technology, in 2004 and 2009, respectively. From 2010 to 2014, he was a Post-Doctoral Fellow with the School of Information Science and Engineering, Southeast University. From 2012 to 2013, he was a Visiting Scholar with the Department of Physics, University of Wisconsin-Madison.

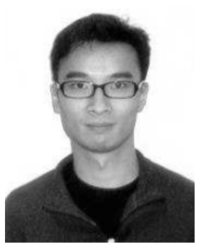
He is currently an Associate Professor with the School of Electronic and Information Engineering, Nanjing University of Information Science and Technology. His research interests include the areas of nonlinear dynamics and chaos including nonlinear circuits and systems, memristive circuits and corresponding applications. He was a recipient of several awards for his teaching and research in Jiangsu Province.



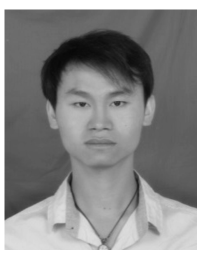
WESLEY JOO-CHEN THIO received the B.S. degree in electrical and computer engineering from The Ohio State University in 2018. His current research interests include the design of chaotic memristive circuits, and the engineering applications of batteries and fuel cells. He has authored several publications in these fields, and authored the book *Elegant Circuits: Simple Chaotic Oscillators* in 2020.

JULIEN CLINTON SPROTT received the bachelor's degree from MIT in 1964 and the Ph.D. degree in physics from the University of Wisconsin–Madison in 1969. His professional interests include heating and confinement of plasmas, especially electron and ion cyclotron resonance heating in magnetic mirrors and toroidal confinement devices, extraterrestrial plasmas and cosmic rays, nonlinear dynamics, chaos, fractals, complex systems, numerical simulation, time-series analysis, and physics education. Most of his professional career has been devoted to experimental plasma physics with an application to the development of controlled nuclear fusion. He played major roles in the design and construction of several magnetic confinement devices, including Tokapole II, a toroidal divertor tokamak, and MST, a reversed field pinch, with the University of Wisconsin. Since 1989 his work has been mostly in nonlinear dynamics and chaos. He developed several computer programs to demonstrate chaos and to perform time-series analysis of experimental data with the aim of clarifying the underlying dynamics. He has discovered a variety of especially simple chaotic systems and electrical circuits and done statistical analyses of large collections of numerically simulated chaotic systems. He has studied the chaotic and self-organizing properties of large artificial neural networks and other high-dimensional dynamical systems. In 1984 he began a program called The Wonders of Physics, aimed at generating interest in science and encouraging students to consider scientific careers. This effort has included public presentations, workshops, development of educational software, videos, and a lecture kit, and the training and supervision of graduate students and teachers in employing these techniques.

He has published over 400 journal articles and abstracts, mostly in experimental plasma physics and computational nonlinear dynamics, plus 10 books, four educational software packages, 30 educational videos, and numerous popular lectures. He is an American Physical Society Fellow (Division of Plasma Physics), and in University Fusion Association, American Association of Physics Teachers, Sigma Xi, Society for Chaos Theory in Psychology and Life Sciences, and New England Complex Systems Institute. He was a recipient of the fellowship in the American Physical Society in 1980, and was the winner of the first annual Computers in Physics software contest for innovative software in physics education in 1990, got John Glover Award-Dickinson College in 1994, and Van Hise Outreach Award for Excellence in Teaching-University of Wisconsin-Madison in 1997, the Lifetime Achievement Award-Wisconsin Association of Physics Teachers in 1999 and the Distinguished Service Award-UW Department of Physics in 2013.



HERBERT HO-CHING IU received the B.Eng. degree (Hons.) in electrical and electronic engineering from The University of Hong Kong, Hong Kong, in 1997, and the Ph.D. degree from The Hong Kong Polytechnic University, Hong Kong, in 2000. In 2002, he joined the School of Electrical, Electronic and Computer Engineering, The University of Western Australia, as a Lecturer, where he is currently a Professor. His research interests include power electronics, renewable energy, nonlinear dynamics, current sensing techniques, and memristive systems. He has authored over 100 papers in these areas. He was a recipient of the two IET Premium Awards in 2012 and 2014. He was also a recipient of the Vice-Chancellor's Mid-Career Research Award in 2014 and the IEEE PES Chapter Outstanding Engineer in 2015. He currently serves as an Associate Editor of the IEEE Transactions on Circuits and Systems II, the IEEE Transactions on Power Electronics, the IEEE Access, the IEEE *Circuits and Systems Magazine*, the *IET Power Electronics*, and the *International Journal of Bifurcation and Chaos*, an Editorial Board Member of the IEEE Journal of Emerging and Selected Topics in Circuits and Systems and the *International Journal of Circuit Theory and Applications*. He is a Co-Editor of the *Control of Chaos in Nonlinear Circuits and Systems* (Singapore: World Scientific, 2009) and a Co-Author of *Development of Memristor Based Circuits* (World Scientific, 2013).



YUJIE XU received the B.S. degree from Shihezi University in 2013. He is currently pursuing the master's degree in information and communication engineering with the Nanjing University of Information Science and Technology, China. His current research interest is in nonlinear and chaotic circuits.