INDUCTANCE AND RESISTANCE OF RFP DISCHARGES

J.C. Sprott

PLP 1010

September 1987

Plasma Studies

University of Wisconsin

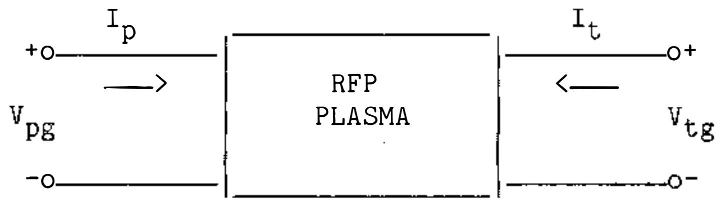
Inductance and Resistance of RFP Discharges

J. C. Sprott

I. Introduction

The goal of this paper is to use the modified Bessel function model (MBFM) results of PLP 1008 to derive simple expressions that permit the determination of time-dependent plasma quantities such as conductivity temperature and energy confinement time from external electrical circuit measurements in an RFP. The results can also be used in the inverse way to predict the electrical waveforms using assumptions about the variation of the conductivity temperature. Although it is possible to obtain exact results in closed, analytic form, the complexity of the expressions renders their use difficult and is not warranted given the somewhat arbitrary assumptions of the MBFM. Rather, the strategy will be to evaluate the MBFM results numerically and to fit them to simple polynomial expressions that are at least as accurate as the MBFM itself.

As a starting point, we consider the RFP plasma to be a non-linear, passive, two-port network,



in which the ports represent the poloidal-field and toroidal-field circuits, respectively. The existence of a closely fitting, highly conducting shell with poloidal and toroidal gaps establishes a convenient boundary between what we will consider to be the plasma (internal) region and the external region in which the electrical circuit elements are more conventional. In the present notation, V_{pg} and V_{tg} represent, respectively, the voltages at the poloidal and toroidal gaps in the shell, I_p represents the current in the poloidal-field circuit (plasma current), and I_t represents the (total) current (ampere-turns) in the toroidal-field winding. In the large-aspect-ratio ($R_o/a >> 1$) approximation, I_t and V_{tg} are related to the toroidal field at the wall B_{tw} and the average toroidal field $\langle B_t \rangle$ through the relations,

$$I_t = 2\pi R_o B_{tw} / \mu_o$$

$$V_{tg} = \pi a^2 \, d\langle B_t \rangle / dt$$

These relations provide the means by which B_{tw} and $\langle B_t \rangle$ are determined from the experimentally measured currents and voltages.

The coupling between the poloidal and toroidal field circuits can be calculated from the power balance equation,

$$V_{pg} I_p + V_{tg} I_t = dW_m/dt + P_{oh}$$

where W_m is the inductive energy stored in the magnetic field B,

$$W_m = \frac{1}{2\mu_o} \int B^2 \, dV$$

and P_{oh} is the ohmic heating power dissipated in the plasma resistance R_p,

$$P_{oh} = I_p{}^2 R_p$$

If we define the loop voltage V_ℓ according to

$$V_\ell = I_p R_p$$

it can be written in terms of the measured quantities as

$$V_\ell = V_{pg} + V_{tg}I_t/I_p - dW_m/dt/I_p$$

Note that in the steady state (V_{tg} = 0 and dW_m/dt = 0), V_ℓ is just equal to V_{pg}, but that when the fields are changing in time, a correction is needed to determine the loop voltage from the poloidal gap voltage. The first task will be to determine this correction. The result will then be used to derive a formula useful for determining the electron conductivity temperature.

II. Loop Voltage Calculation

In the large-aspect-ratio ($R_o/a >> 1$), cylindrical approximation, the magnetic energy can be written

$$W_m = \frac{2\pi^2 R_o}{\mu_o} \int_o^a (B_t^2 + B_p^2)\, r\, dr$$

where the fields are given by the MBFM result of PLP 1008,

$$B_t(r)/\langle B_t\rangle = 3 - 2F - 6(1-F)(r/a)^2 + 3(1-F)(r/a)^4$$

and

$$B_p(r)/\langle B_t\rangle = [18-30F+12F^2]^{1/2}[r/a - 3(1-F)(r/a)^3/(3-2F) + (3-4F)(r/a)^5/3(3-2F)]$$

In the above equations, F is defined by

$$F = B_{tw} / \langle B_t\rangle$$

A graph of the magnetic energy density normalized to $\langle B_t\rangle^2/2\mu_o$ as a function of F in the range of interest ($-1 < F < 1$) is shown in figure 1. It is fit quite well (to within a few percent) by the function

$$\mu_o W_m/\pi^2 R_o a^2 \langle B_t\rangle^2 \approx 4 - 5F + 2F^2$$

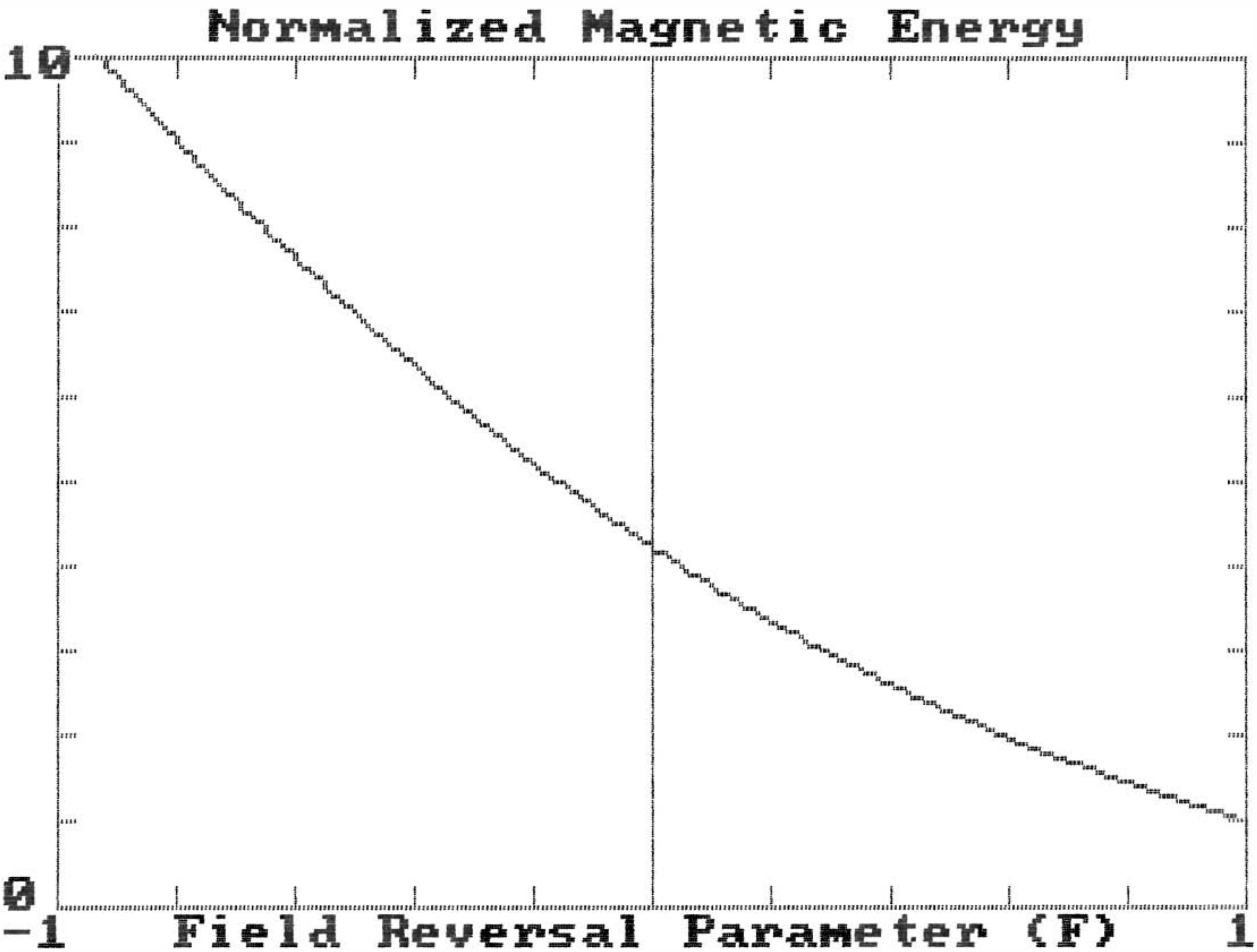

Normalized Magnetic Energy
10
0
-1
Field Reversal Parameter (F)
1

Figure 1

A time-variation of magnetic energy can result from either a variation of $\langle B_t \rangle$ or F. Consider first the matched-mode case of constant $\langle B_t \rangle$ (or $V_{tg}=0$), corresponding to a perfectly crowbarred toroidal gap. If the discharge remains on the F-θ curve for the MBFM, then the variation in F is due entirely to a variation in I_p, and we can write,

$$dW_m/dt/I_p = L_{sc}(F)dI_p/dt$$

where $L_{sc}(F)$ is an effective inductance that depends upon the value of F. The subscript sc is used to indicate that this inductance is appropriate only to the case with the toroidal gap short-circuited ($V_{tg}=0$). Applying the chain rule of differentiation,

$$L_{sc}(F) = \frac{1}{I_p}\frac{dW_m}{dt}\frac{dt}{dI_p} = \frac{1}{I_p}\frac{dW_m}{dF}\frac{dF}{d\theta}\frac{d\theta}{dI_p}$$

where θ is defined by

$$\theta = \mu_o I_p \,/\, 2\pi a\langle B_t \rangle$$

which from PLP 1008 is related to F for the MBFM by

$$F \approx 1 - \theta^2/2$$

The inductance is thus calculated to be

$$L_{sc}(F) \approx \mu_o R_o (5/4 - F)$$

This inductance, normalized to $\mu_o R_o$, is plotted in figure 2.

As a numerical example, in the tokamak limit (F=1) of MST (R_o = 1.5 m), the inductance is $L_{sc}(1) = 0.47$ μH. Neglecting resistive losses, a Φ = 2-volt-second iron core would thus permit a maximum plasma current of $\Phi/L_{sc}(1)$ = 4.2 MA if a $L_{sc}(1) I_p^2/2$ = 4.2-MJ power source were available. A 93-kG toroidal field would be required in MST to reach this current with a cylindrical q (= $2\pi a^2 \langle B_t \rangle / R_o \mu_o I_p$) of 2.0!

As another example, the flux required to just reach the RFP state (F=0) at a given current with matched-mode self-reversal ($\langle B_t \rangle$ = constant) can be calculated from

$$\Phi = \int_o^{I_p} L_{sc} dI_p \approx 7 \mu_o R_o I_p / 12$$

which evaluates to 2 volt-seconds at I_p = 1.8 MA for MST. The poloidal bank energy required to reach the RFP state at this current is

$$U_p = \int_o^{I_p} L_{sc} I_p \, dI_p \approx 3 \mu_o R_o I_p^2 / 8$$

or 2.3 MJ for MST, neglecting resistive losses. At this current, the average toroidal field is $\langle B_t \rangle$ = 4.9 kG at F = 0.

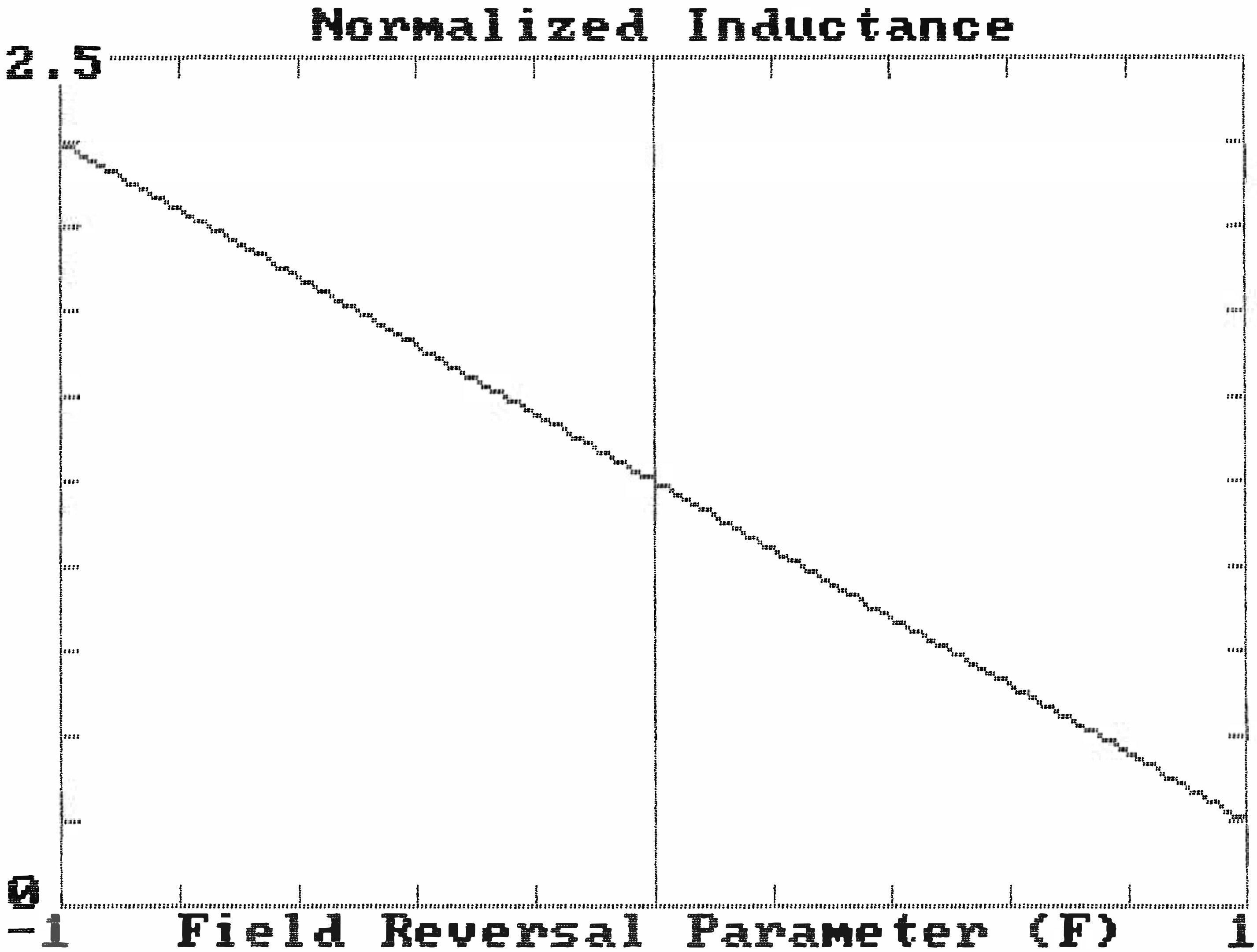


Figure 2

The other case that is easy to calculate is the one in which the plasma current if ramped up from a negligibly low value to full current at a constant and negligibly small value of F. If F remains at zero throughout the ramp-up, I_t is zero, and the toroidal-field circuit need not supply any energy. This is accomplished in principle by leaving the toroidal gap open-circuited throughout the ramp-up. In such a case, the inductance can be calculated from

$$L_{oc}(0) = \frac{1}{I_p} \frac{dW_m(0)}{dt} \frac{dt}{dI_p} = \frac{1}{I_p} \frac{dW_m(0)}{d\langle B_t\rangle} \frac{d\langle B_t\rangle}{dI_p} \approx \mu_o R_o$$

The current required to consume 2 volt-seconds in MST is $\Phi/L_{oc}(0) = 1.1$ MA, and the energy required is $L_{oc}(0)I_p^2/2 = 1.1$ MJ. At this current, the average toroidal field is $\langle B_t\rangle = 3.0$ kG at F = 0.

A general conclusion is that a discharge ramped up to a given current at a low value of F consumes more inductive volt-seconds from the poloidal-field circuit than would be required to achieve the same current with self-reversal of a large, pre-existing toroidal field. In the former case, the poloidal-field circuit provides both the poloidal and toroidal fields, whereas in the latter case some of the pre-existing toroidal field gets converted into poloidal field with a smaller consumption of flux and energy from the poloidal-field circuit. Of course, when one includes the resistive volt-second consumption, the situation may change completely. These results are summarized in Table I.

Finally we consider the general case in which the current ramp-up is accompanied by power flow either from or to the toroidal-field circuit. However, we assume that the discharge remains on the F-θ curve for the MBFM model, and thus for a given $I_p(t)$, the loop voltage can depend on only one other parameter which we take to be $V_{tg}(t)$,

$$V_\ell = V_{pg} + A(F)V_{tg} - L_{sc}(F)dI_p/dt$$

$L_{sc}(F)$ is the inductance previously calculated for $V_{tg}=0$, and $A(F)$ is a coefficient of coupling between the toroidal-field and poloidal-field circuits, evaluated for I_p=constant,

$$A(F) = I_t/I_p - dW_m/dt/I_pV_{tg}$$

The first term on the right can be written as a function of F as

$$I_t/I_p = R_oF \ / \ a[2-2F]^{1/2}$$

Evaluation of the second term is a bit more complicated,

$$\frac{dW_m}{dt} = \left[\frac{\partial W_m}{\partial\langle B_t\rangle} + \frac{\partial W_m}{\partial F}\frac{\partial F}{\partial\theta}\frac{\partial\theta}{\partial\langle B_t\rangle}\right]\frac{d\langle B_t\rangle}{dt}$$

Considerable tedious algebra leads to the result,

$$A(F) \approx R_o(1-3F+2F^2) \;/\; a[2-2F]^{1/2}$$

which is plotted, normalized to the aspect ratio (R_o/a), in figure 3. The above expression is slightly simpler when expressed in terms of θ,

$$A(\theta) \approx R_o\theta(\theta^2-1) \;/\; 2a$$

Putting it all together leads to the following final result suitable for on-line determination of the loop voltage and electrical circuit modeling:

$$V_\ell = V_{pg} + A(\theta)V_{tg} - L_{sc}(F)dI_p/dt$$

$$\text{where } L_{sc}(F) \approx \mu_o R_o(5/4-F)$$

$$\text{and } A(\theta) \approx R_o\theta(\theta^2-1) \;/\; 2a$$

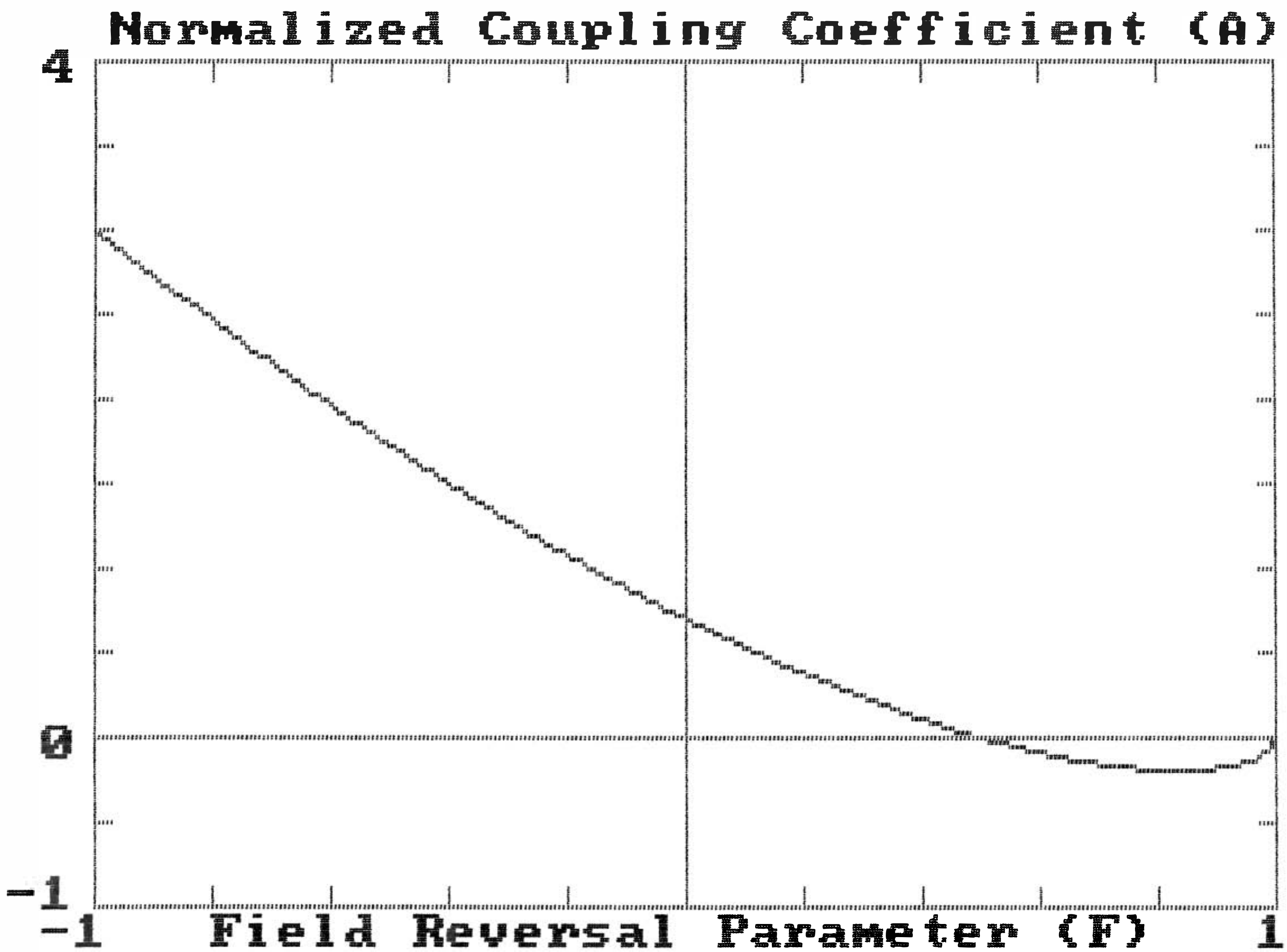

Normalized Coupling Coefficient (A)
4
0
-1
-1
Field Reversal Parameter (F)
1

Figure 3

Table I

Plasma current limits for MST, assuming MBFM profiles, neglecting ohmic losses, for a 2-volt-second flux swing in the iron core.

Case	L	I_p	U_p	$\langle B_t \rangle$
Tokamak (F=1)	0.47 μH	4.2 MA	4.2 MJ	93 kG*
Matched Mode RFP (F=0)	1.10 μH	1.8 MA	2.3 MJ	4.9 kG
Ramped RFP (F=0)	1.88 μH	1.1 MA	1.1 MJ	3.0 kG

*For a cylindrical q of 2.0

III. Conductivity Temperature Calculation

The final task will be to determine the plasma conductivity-temperature from the loop voltage V_ℓ and the plasma current I_p. The ohmic power input to the plasma at low beta can be written as

$$V_\ell I_p = I_p{}^2 R_p = \int \eta_{||} J_{||}{}^2 \, dV$$

where $\eta_{||}$ is the electrical resistivity parallel to B. Classically, the (Spitzer) resistivity depends on electron temperature as

$$\eta_{||} \approx 7 \times 10^{-4} Z_{eff} \,/\, T_e{}^{3/2}$$

where T_e is in electron volts and Z_{eff} is the effective ionic charge. Since T_e in general has a spatial dependence, the calculation of T_e from the plasma resistance involves an assumption about the temperature (or resistivity) profile. Rather than be concerned about such details, we will assume a spatially constant resistivity and understand that the temperature derived therefrom will be an underestimate (perhaps by about a factor of two) of the temperature on the axis and an overestimate of the temperature near the edge. It represents a reasonable value to use in combination with the line-averaged density to determine the plasma energy content and energy confinement time,

$$\tau = 6\pi^2 R_o a^2 \langle n \rangle k T_e \; / \; P_{oh}$$

assuming $T_i = T_e$. Furthermore, we will assume $Z_{eff}=1$, which is tantamount to neglecting impurities and other causes of resistivity anomaly such as fluctuations. Under these conditions, the conductivity electron temperature is given by

$$T_e \approx 0.0125[R_o S(F)/a^2 R_p]^{2/3}$$

where S(F) is the so called "screw-up factor" that accounts for the increased resistivity due to the fact that the current follows field lines and the field lines are not purely in the toroidal direction, especially for low and negative values of F. Also included in S(F) is the resistance enhancement (about 20%) due to the non-uniform current density over the poloidal cross section. For a spatially uniform resistivity, S(F) can be calculated from

$$S(F) = \frac{2\pi^2 a^2}{I_p^2} \int_o^a \frac{(J_p B_p + J_t B_t)^2}{B_p^2 + B_t^2} \, r \, dr$$

where J(r) and B(r) are given as functions of F by the MBFM of PLP 1008. The factor S(F) has been calculated numerically and is shown in figure 4. The result can be fit quite accurately (to within a few percent) by the function,

$$S(F) \approx 3.2 - 2F$$

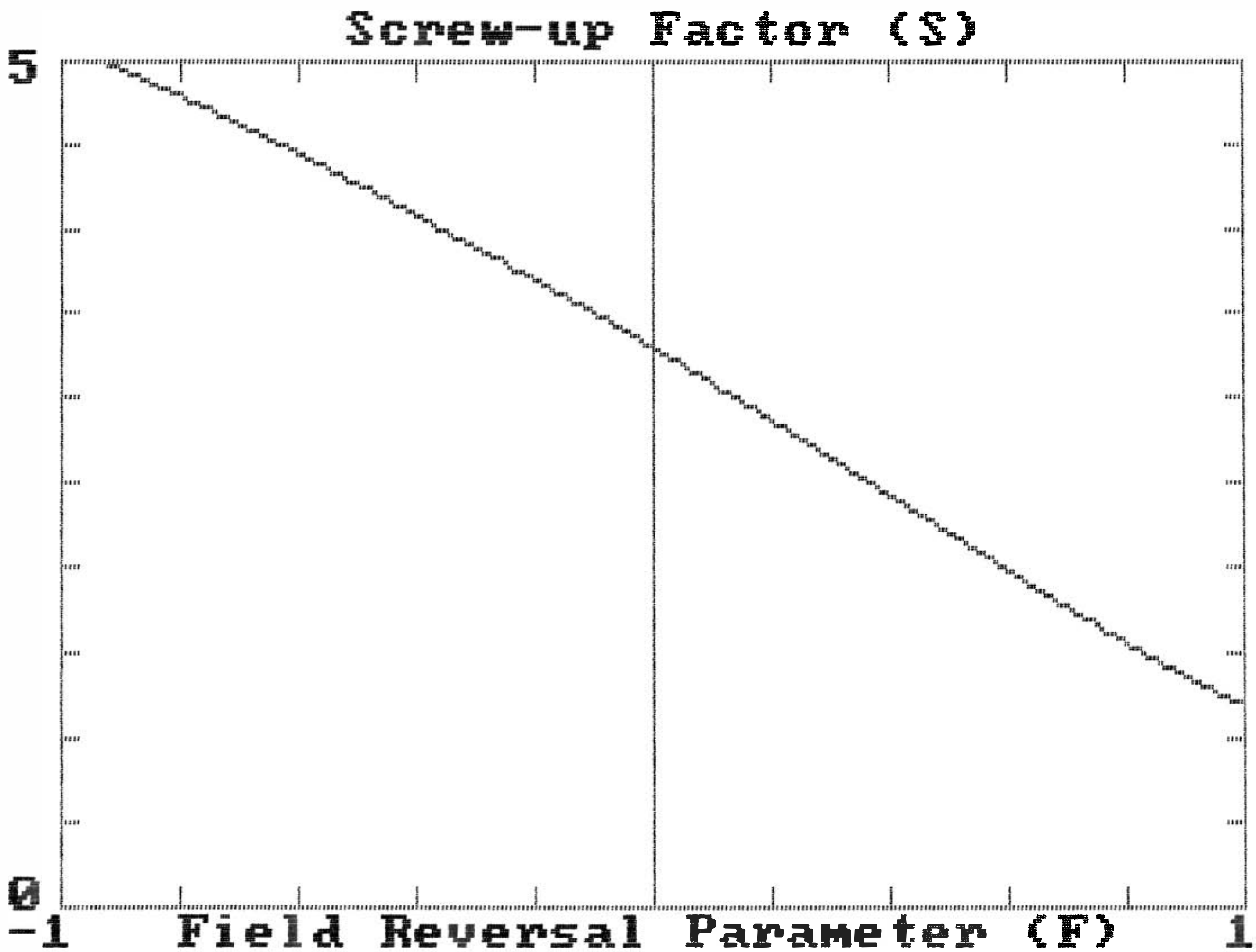


Figure 4

Note that for F=1, all the current is toroidal, and the factor S(1) = 1.2 is due entirely to the effect of the peaking of the current density profile in the MBFM. By contrast, a tokamak parabolic current density profile would have a peaking factor of S(1) = 4/3.

At this point, it is useful to compare the results derived in this and the previous section with the results used, for example, in PLP 965 for the design of MST and for the IBM monitor program used for the construction-phase RFP experiments. The old values were obtained from Los Alamos numerical modeling of ZT-40M and from old Culham results from ZETA. The old and new values of the various quantities are compared in Table II at a value of F=0 for MST. The differences are not large compared to other uncertainties in the use of the MBFM to represent MST. The new values will be used for future electrical circuit modeling and on-line data analysis of MST plasmas.

In summary, the electron conductivity temperature is calculated from the loop voltage V_ℓ (as derived in the previous section) and the plasma current I_p using the relation,

$$T_e \approx 0.0125[R_o S(F) I_p / a^2 V_\ell]^{2/3}$$

$$\text{where } S(F) \approx 3.2 - 2F$$

Table II

Comparison of old and new values of various quantities at F=0 for MST.

quantity	old	new	change
θ	1.25	1.41	+13%
L_{sc}	2.76 μH	2.36 μH	-14%
A	3.11	2.04	-34%
S	2.79	3.20	+15%