RFP INDUCTIVE VOLT-SECOND AND ENERGY REQUIREMENTS

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The purpose of this note is to calculate the inductive part of the volt-second and energy requirements for both the poloidal-field and toroidal-field systems of an RFP and to develop an optimal strategy for maximizing the plasma current subject to core saturation and capacitor-bank limits. The model will use the finite-pressure, modified Bessel function model of PLP 1008 and the plasma inductance and coupling model of PLP 1010. Resistive volt-second and energy consumption in both the plasma and external circuitry will be ignored, and thus the results represent an optimistic upper limit of the performance that can be expected from a given RFP device such as MST. The neglect of resistive losses is equivalent to assuming that the plasma current reaches its maximum value in a time short compared to τ/β where τ is the plasma energy confinement time and β is the ratio of plasma pressure to magnetic pressure.

From PLP 1010 with the plasma resistance set equal to zero, the poloidal gap voltage is given by

$$V_p = LdI_p/dt - AV_t$$

where L is the profile-dependent plasma inductance,

$$L \approx (1/4+\theta^2/2)\mu_o R_o$$

I_p is the plasma current, V_t is the toroidal gap voltage, A is the coupling between the poloidal and toroidal circuits,

$$A \approx R_o\theta(\theta^2-1) / 2a$$

θ is the pinch parameter,

$$\theta = \mu_o I_p / 2\pi a\langle B_t\rangle$$

R_o is the major radius, a is the minor radius of the (assumed) circular, high-aspect-ratio (R_o>>a) plasma, and $\langle B_t\rangle$ is the average toroidal magnetic field. The toroidal gap voltage is given by

$$V_t = \pi a^2 d\langle B_t\rangle/dt$$

and the current in the toroidal-field circuit is given by

$$I_t = 2\pi R_o\langle B_t\rangle F / \mu_o$$

where F is the ratio of toroidal field at the wall to the average toroidal field given in terms of θ by

$$F \approx 1 - \theta^2/2$$

The equations above do not form a closed set from which the volt-seconds and energy required to achieve a given plasma current at a given value of θ can be determined. This fortunate circumstance means that there is room to optimize waveforms so as to improve performance. We are thus allowed to introduce an additional equation which we take to be the variation of average toroidal field with plasma current as the current is increased from an initial value of zero to some final value,

$$\langle B_t \rangle = \langle B_t \rangle (I_p)$$

The final state is characterized by a plasma current I_{pf} and a pinch parameter θ_f. With the hardware this would be done be controlling the voltage and current at the toroidal gap during the time the plasma current is building up to its final value. When cast in this form, the problem becomes time-independent and reduces to a problem of comparing the volt-seconds and energy required to achieve a given plasma current and θ for various trajectories through $\langle B_t \rangle - I_p$ space. We will now consider several special cases:

Case I. Ramped Mode

Suppose that $\langle B_t \rangle$ is a linear function of I_p, corresponding to a constant value of θ,

$$\langle B_t \rangle = \mu_o I_p \ / \ 2\pi a \theta_f$$

The volt-seconds consumed by the poloidal-field circuit is

$$\Phi_p = \int V_p dt = (1/2+\theta_f^2/4)\mu_o R_o I_{pf}$$

and the energy supplied by the poloidal-field source is

$$U_p = \int V_p I_p dt = (1/4+\theta_f^2/8)\mu_o R_o I_{pf}^2$$

The volt-seconds consumed by the toroidal-field circuit is

$$\Phi_t = \int V_t dt = \mu_o a I_{pf} / 2\theta_f$$

and the energy supplied by the toroidal-field source is

$$U_t = \int V_t I_t dt = (1/4\theta_f^2-1/8)\mu_o R_o I_{pf}^2$$

Note that U_t is positive for $\theta_f^2 < 2$ and negative for $\theta_f^2 > 2$ and that $\theta_f^2 = 2$ is the value for which F = 0. This means that the external toroidal-field circuit must supply energy for a non-reversed ramp-up and must sink energy for a reversed ramp-up. A ramp-up at F = 0 involves no energy flow at the toroidal gap since the current is zero. In principle this could be done by simply leaving the toroidal gap open-circuited. A ramp-up at negative F does not require a toroidal-field power supply, but does require an impedance across the toroidal gap so that current can flow. This impedance must be inductive so as not to violate the initial assumption of the calculation, that there be no resistive losses.

One can calculate the value of external inductance L_{ext} required at the toroidal gap to allow a ramp-up at a given value of θ (or F),

$$L_{ext} = -V_t \;/\; dI_t/dt = -\mu_o a^2 \;/\; 2R_o F$$

or in terms of the internal toroidal inductance of the machine in the absence of plasma, $L_{ext} = -L_{int}/F$. Note that for a ramp-up at F=0, an infinite external inductance is required, and that only negative values of F are allowed. In practice, it may not be possible to start up the plasma with zero initial toroidal field, but the result above represents an ideal limiting case.

Case II. Matched Mode

The second case to consider is one in which an initial average toroidal field is established and held constant while the plasma current is increased from zero to its final value. Constant $\langle B_t \rangle$ implies $V_t = 0$ (toroidal gap perfectly crowbarred) and no energy flow into or out of the toroidal gap while the plasma current is increasing. For such a case, θ is a linear function of I_p,

$$\theta = \mu_o I_p \,/\, 2\pi a \langle B_t \rangle$$

The volt-seconds consumed by the poloidal-field circuit is

$$\Phi_p = (1/4+\theta_f^2/6)\mu_o R_o I_{pf}$$

and the energy supplied by the poloidal-field source is

$$U_p = (1/8+\theta_f^2/8)\mu_o R_o I_{pf}^2$$

All the toroidal flux and toroidal-field energy are supplied before the plasma current begins to increase and are given by

$$\Phi_t = \pi a^2 \langle B_t \rangle = \mu_o a I_{pf} \,/\, 2\theta_f$$

and

$$U_t = \pi^2 a^2 R_o \langle B_t \rangle^2 \,/\, \mu_o = \mu_o R_o I_{pf}^2 \,/\, 4\theta_f^2$$

Note that matched mode requires less poloidal flux swing and poloidal-field source energy but more toroidal-field source energy than ramped mode, but that the toroidal flux swing is the same in the two cases. The total energy is the same for the two modes since it depends only on the final state,

$$U = U_p + U_t = (1/8+\theta_f^2/8+1/4\theta_f^2)\mu_o R_o I_{pf}^2$$

An interesting feature is that for a fixed plasma current, there is a minimum energy state of $U_{min} = 0.479\mu_o R_o I_p^2$ at $\theta^4 = 2$, corresponding to a not-quite-reversed condition of F = 0.293. The exact location of this minimum depends on assumptions about how the radial field profiles change as a function of θ (see PLP 1008).

Case III. Aided Reversal

We now examine a more general class of solution in which the average toroidal field is held at a fixed but arbitrary value until the plasma current reaches its final value and then the average toroidal field is readjusted to the level required for the desired θ while the plasma current is held constant. We thus have an additional parameter α which we define as the ratio of the initial average toroidal field (when I_p=0) to the final toroidal field (when I_p has reached its maximum value). Matched mode thus becomes a special case of aided reversal in which α = 1.

For this case, the volt-seconds consumed by the poloidal-field circuit consists of two parts:

$$\Phi_{p1} = (1/4+\theta_f^2/6\alpha^2)\mu_o R_o I_{pf}$$

$$\Phi_{p2} = (\theta_f^2/8-\theta_f^2/8\alpha^2-\ln\alpha/4)\mu_o R_o I_{pf}$$

Subscript $_1$ refers to the flux swing during the ramp-up of the current, and subscript $_2$ refers to the flux swing during the subsequent readjustment of the toroidal field. Note that Φ_{p1} is always positive, but that Φ_{p2} can be either positive or negative depending upon the values of α and θ_f. Similarly, the energy supplied by the poloidal-field source has two components:

$$U_{p1} = (1/8+\theta_f^2/8\alpha^2)\mu_o R_o I_{pf}^2$$

$$U_{p2} = (\theta_f^2/8-\theta_f^2/8\alpha^2-\ln\alpha/4)\mu_o R_o I_{pf}^2$$

As with the flux, the poloidal-field circuit supplies energy during the ramp-up of the current and is either a source or sink for energy during the subsequent readjustment of the toroidal field, depending upon α and θ_f. The toroidal flux is given by

$$\Phi_{t1} = \alpha\mu_o a I_{pf} / 2\theta_f$$

$$\Phi_{t2} = (1-\alpha)\mu_o a I_{pf} / 2\theta_f$$

and the energy supplied by the toroidal-field source is

$$U_{t1} = \alpha^2\mu_o R_o I_{pf}^2 / 4\theta_f^2$$

$$U_{t2} = (1-\alpha^2+\theta_f^2\ln\alpha)\mu_o R_o I_{pf}^2 / 4\theta_f^2$$

In the expressions above, Φ_{t1} and U_{t1} are, respectively, the flux and energy supplied by the external toroidal-field circuit prior to the start of the current ramp-up. During the ramp-up, the toroidal gap voltage is zero, and hence there is no flux or energy consumption. Subsequent to the current ramp-up, The toroidal field circuit is a source of flux and energy for $\alpha<1$ and a sink for $\alpha>1$. As a check, one can verify that the net energy consumption $U_{p1} + U_{p2} + U_{t1} + U_{t2}$ (which must depend only on the final state and not on how it was obtained) is independent of α and has the same value as calculated for the matched mode (α=1) case. The other expressions also reduce to the matched mode case for $\alpha = 1$.

Figure 1 shows the trajectories through $\langle B_t\rangle - I_p$ space for aided reversal with $\alpha = 1.5$ and $\alpha = 2$ as well as for matched mode and ramped mode, all for the same final value of $\theta_f = 1.5$ (barely reversed). The axes are normalized to the poloidal flux swing $\Delta\Phi_p$ and to the machine dimensions so as to produce a graph that is machine-independent. Plotted in this way, one can also see directly the increase in I_{pf} that results with aided reversal for a fixed poloidal volt-second consumption. Figure 2 shows the same graph but with the axes relabeled to correspond to MST with a 2.0-volt-second poloidal flux swing. Table I summarizes the results.

For values of α that satisfy the equation

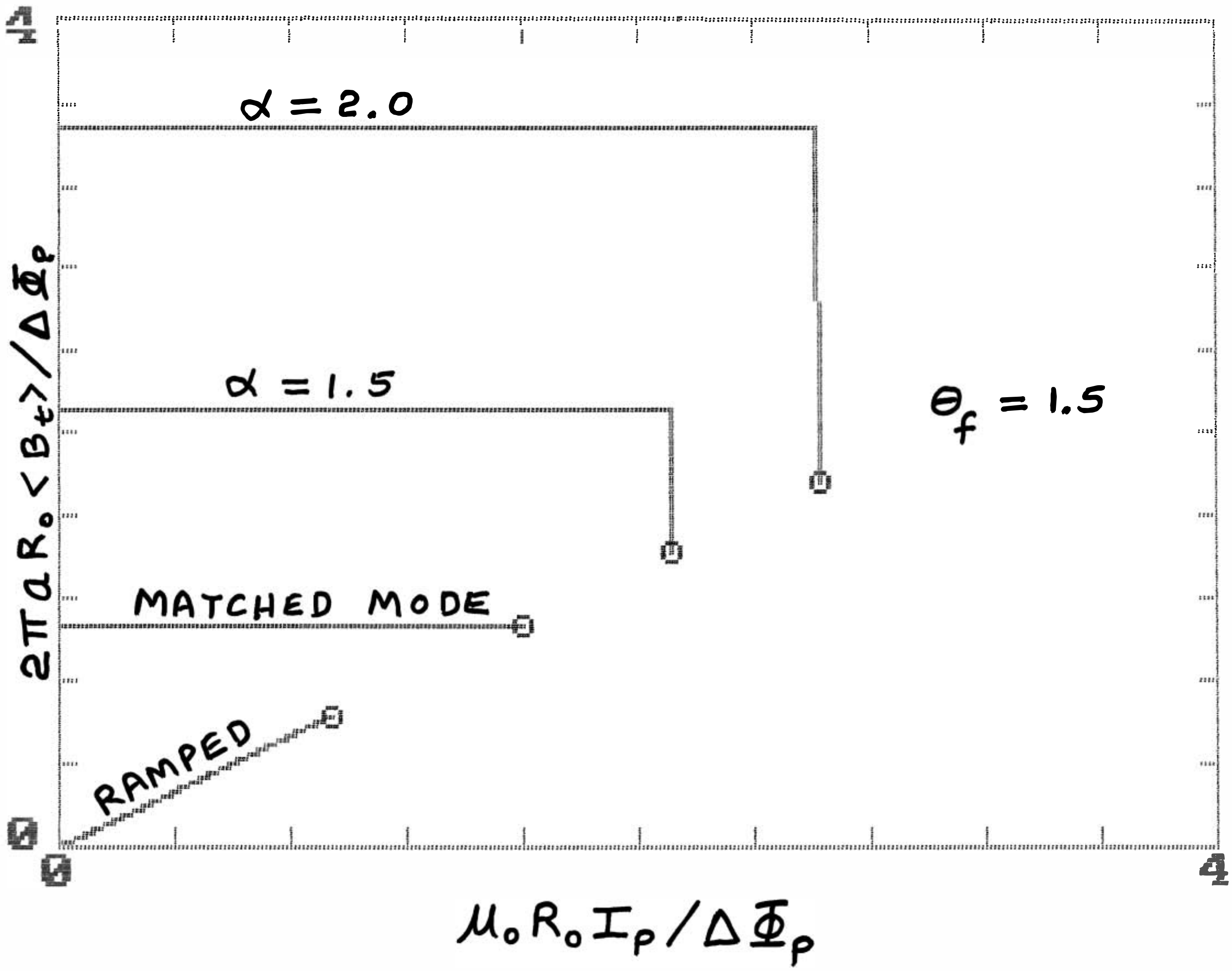

Figure 1. Trajectories through normalized $\langle B_t \rangle$-I_p space for various startup modes terminating in a final theta value of θ_f = 1.5 at a maximum plasma current I_p that consumes a given poloidal flux swing $\Delta\Phi_p$.

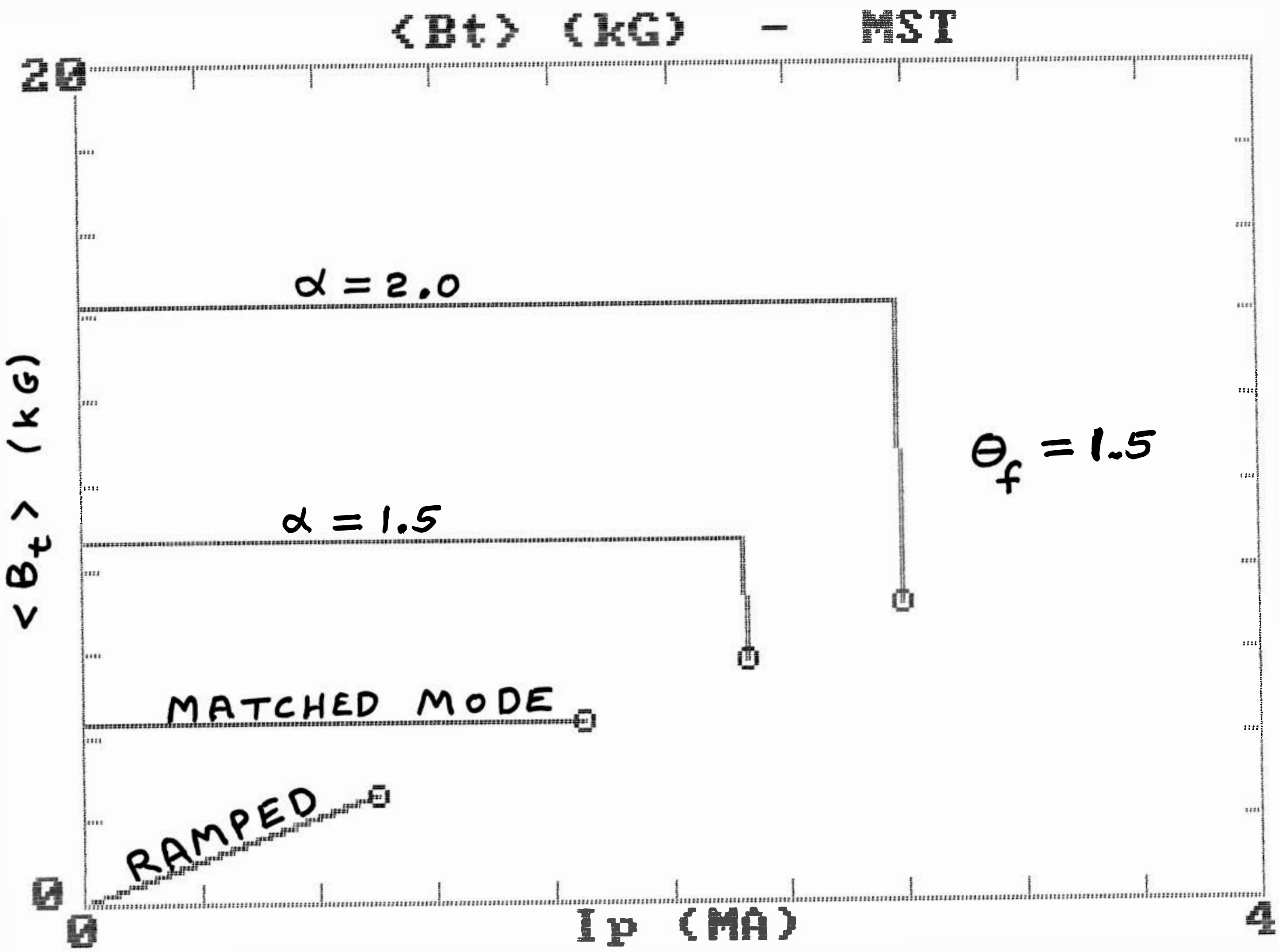

Figure 2. Same data as in figure 1 but applied to MST (R_o/a = 1.5/0.52 m) for a poloidal flux swing of $\Delta\Phi_p$ = 2.0 Webers.

Table I

Summary of parameters for various startup modes of MST for $\Delta\Phi_p$ = 2.0 Webers and θ_f = 1.5.

			AIDED REVERSAL		
	RAMP	MM	α = 1.5	α = 2.0	
I_{pf}	1.0	1.7	2.3	2.8	MA
$\langle B_t \rangle(0)$	0.0	4.4	8.7	14.3	kG
$\langle B_{tf} \rangle$	2.6	4.4	5.8	7.1	kG
U_p	1.0	2.2	2.9	3.4	MJ
U_t	-0.03	0.6	2.4	6.5	MJ
$\Delta\Phi_t$	0.2	0.4	1.1	2.4	Wb

$$1 - \ln\alpha + \theta_f^2/2 + \theta_f^2/6\alpha^2 = 0$$

the net flux swing at the poloidal gap is zero. For a typical θ_f of 1.5, the corresponding value of α is 8.4, requiring an unreasonably high initial $\langle B_t \rangle(0)$. Although the net flux swing is zero, it consists of a positive swing during the ramp-up of the plasma current and an equal and opposite negative flux swing while the toroidal field is readjusted. Similarly, for values of α given by

$$\alpha = \exp[1/2+\theta_f^2/2]$$

the net energy supplied by the poloidal-field circuit is zero. For a typical θ_f of 1.5, the corresponding value of α is 5.1. As before, this result implies a positive energy flow into the plasma followed by an equal outward energy flow.

Case IV. Theta Pinch Mode

Many other trajectories through $\langle B_t \rangle$-I_p space were examined numerically without revealing any surprises. However, it is interesting to ask whether an RFP state can be achieved with the poloidal-field voltage V_p identically zero at all times. If such a solution were possible, one could eliminate the poloidal-field power supply, iron core and shell gap. Field errors would be extremely small, and the plasma current would be limited only by the capabilities of the toroidal-field power supply. Since the space through the center of the toroid is not energized with magnetic flux, the coupling efficiency of flux and energy to the plasma region is essentially perfect. Such operation resembles that of a theta pinch.

Setting V_p equal to zero gives the equation

$$L dI_p/dt - \pi a^2 A d\langle B_t\rangle/dt = 0$$

which can be rewritten in a time-independent form as

$$dI_p/d\langle B_t\rangle = (\theta^2-1)I_p \;/\; (2\theta^2+1)\langle B_t\rangle$$

Since θ depends on $I_p/\langle B_t\rangle$, this equation cannot be simply integrated, but one can make several observations about the character of the solution. For $\theta = 1$, we have $dI_p/d\langle B_t\rangle = 0$, corresponding to a maximum of I_p. For $\theta \ll 1$, $\langle B_t\rangle$ is proportional to $1/I_p$, and for $\theta \gg 1$, $\langle B_t\rangle$ is proportional to I_p^2. However, recall that the expressions for L and A are valid only over the range $0 < \theta < 2$, and thus the high θ values are not quantitatively accurate and may not even be physical.

A more detailed examination of the equation requires a numerical solution, the results of which are shown in figure 3. A given final state can be approached in one of two ways, by lowering $\langle B_t\rangle$ from infinity or by raising $\langle B_t\rangle$ while maintaining

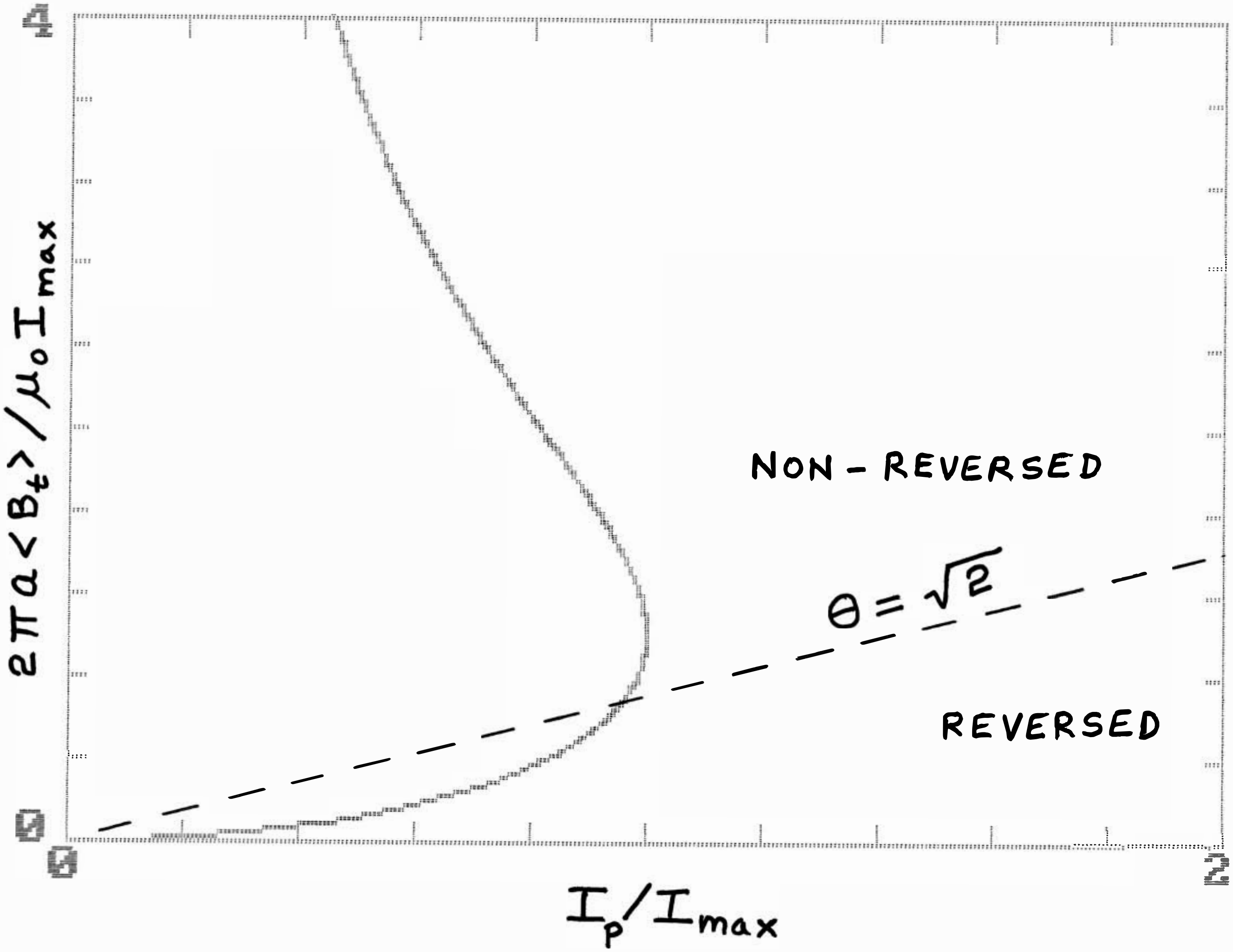

Figure 3. Trajectory through normalized $\langle B_t\rangle$-I_p space for a theta-pinch-like startup in which the poloidal-field drive voltage is maintained at zero.

deep reversal. Neither case is particularly physical, requiring in the first case an infinite $\langle B_t \rangle(0)$ and in the other an infinite $\theta(0)$. Furthermore, note that in both cases energy is flowing out of the plasma and into the toroidal-field circuit during most of the plasma current ramp-up. Such is reasonable for the case with infinite $\langle B_t \rangle(0)$, but the case with $\langle B_t \rangle(0) = 0$ also must have a large initial energy. This is possible with deep values of field reversal since $\langle B_t \rangle = 0$ does not require $\langle B_t^2 \rangle = 0$.

It is not clear how, in practice, one might achieve the required initial conditions, but theta pinches are known to operate, and under the right circumstances might be made to evolve into something resembling the RFP state without the need for poloidal-field circuitry. It would make an interesting experiment on MST or even Tokapole II to attempt to form an RFP state with the poloidal-field primary short-circuited.