MODIFIED POLYNOMIAL FUNCTION MODEL FOR REVERSED FIELD PINCHES

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Modified Polynomial Function Model for Reversed Field Pinches. J. C. SPROTT, W. SHEN, University of Wisconsin - Madison. --A new model for the magnetic field and current density profiles in a reversed field pinch is proposed. The model has finite beta, a relatively constant ratio of j/B and vanishing current density at the wall. The fields and currents are simple polynomial functions of radius whose coefficients depend in a simple way on the field reversal parameter (F) and pinch parameter (Θ). The profiles are used to derive expressions for many useful quantities such as magnetic energy, beta, inductance, resistance and ohmic input power. These quantities, in turn, are used for electrical circuit modeling of RFP discharges. Under some conditions the equations lead to an instability that is characterized by a peaking of the current density profile and an increase in beta reminiscent of the sawtooth oscillations often observed in RFP discharges. The conditions under which such instabilities are expected to occur will be discussed.

RFP Profile Models

The Modified Polynomial Function Model generalizes the Polynomial Function Model in the same way that the Modified Bessel Function Model generalizes the Bessel Function Model:

Model:	BFM	MBFM	PFM	MPFM
$j(a)=0$	no	yes	yes	yes
$\beta>0$	no	no	yes	yes
Parameters	1	2	1	2

MPFM Expressions for B(r) and j(r)

$$B_\phi = B_\phi(0)\ [1 - \Theta_o^2(r/a)^2 + \Theta_o^2(r/a)^4/2]$$

$$B_\theta = B_\phi(0)\ (r/a)\ [\Theta_o - (2\Theta_o-3C)\ (r/a)^2 + (\Theta_o-2C)\ (r/a)^4]$$

$$j_\phi = 2\ B_\phi(0)\ [\Theta_o - 2\ (2\Theta_o-3C)\ (r/a)^2 + 3\ (\Theta_o-2C)\ (r/a)^4]\ /\ \mu_o a$$

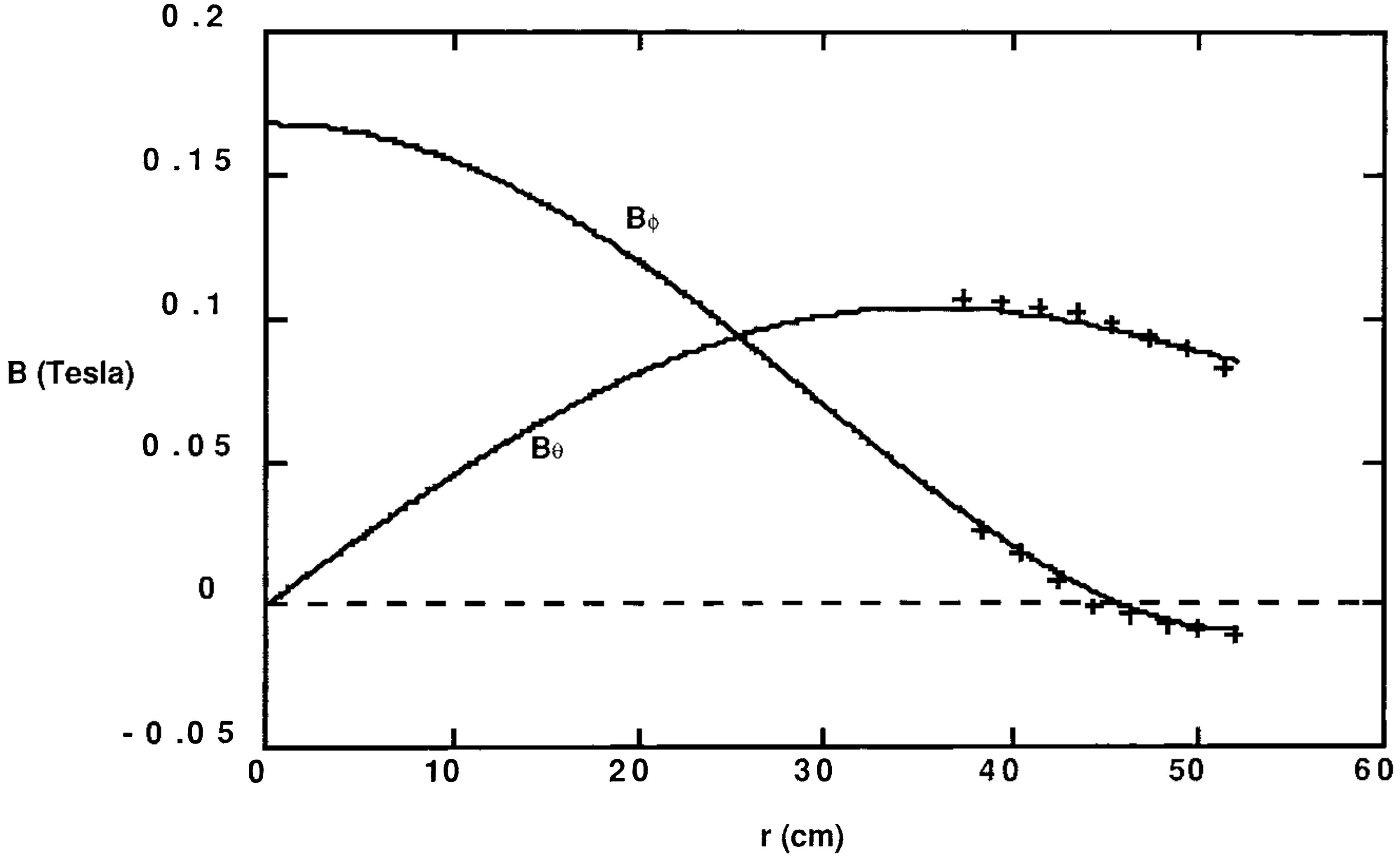
$$j_\theta = 2\ B_\phi(0)\ \Theta_o^2(r/a)\ [1 - (r/a)^2]\ /\ \mu_o a$$

where

$$C = B_\theta(a)\ /\ B_\phi(0), \qquad \Theta_o = \mu_o a\ j_\phi(0)\ /\ 2\ B_\phi(0)\ .$$

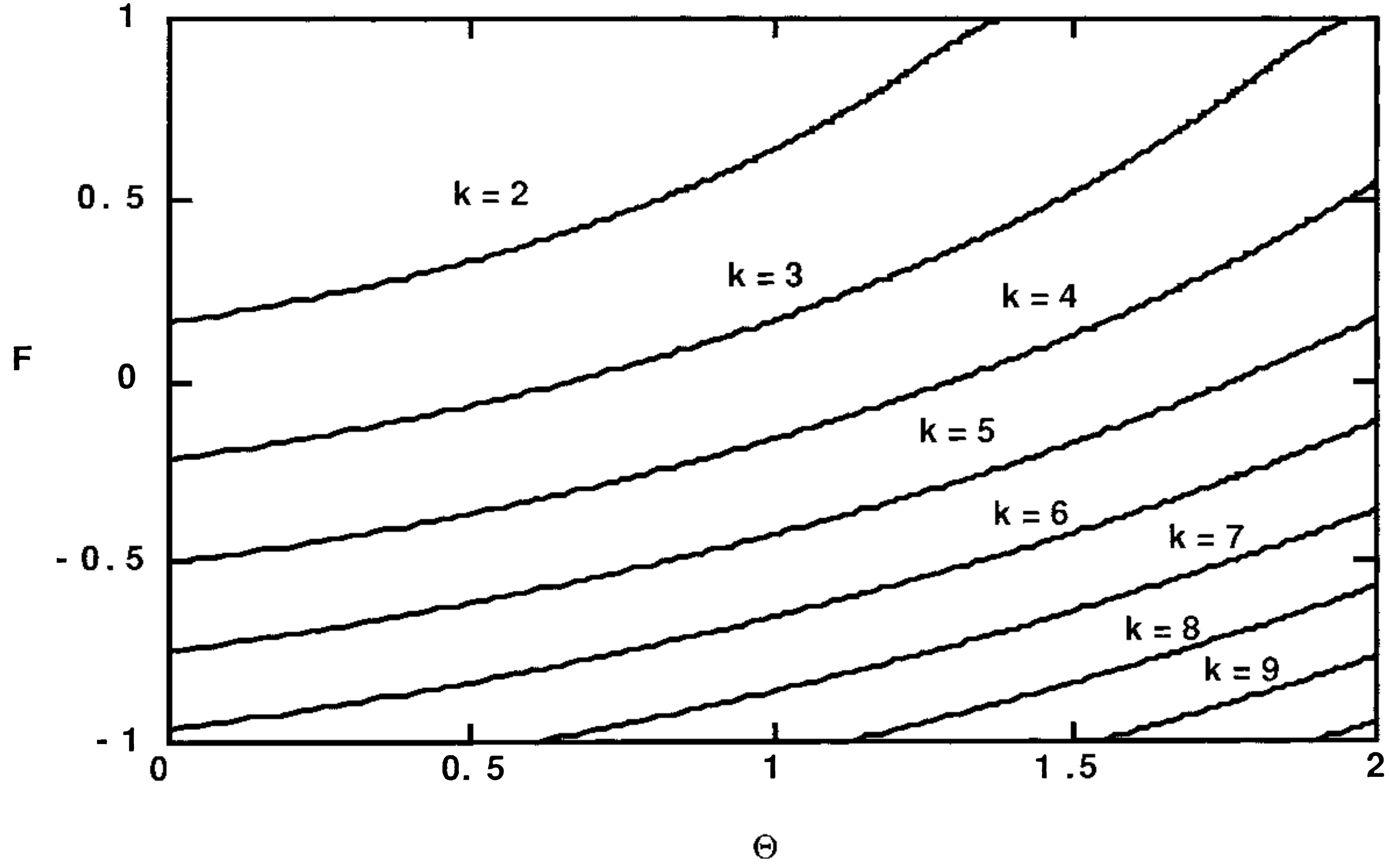
$$\Theta_o = \sqrt{\frac{6-6F}{3-2F}} \qquad C = \frac{\Theta}{3-2F}$$

MAGNETIC FIELD PROFILE FROM MST AND PREDICTED BY THE MPFM



CONTOURS OF CONSTANT MAGNETIC ENERGY U_m

$$K = \frac{U_m}{\left(\frac{R\,\Phi^2}{\mu_0\,\mathbf{a}^2}\right)}$$



MPFM Predictions

F-Θ Relation:

$$\beta_\theta = 1 - (9-8F-F^2) / 5\Theta^2$$

Resistive Voltage:

$$V_R = V_\phi - L\, dI_\phi/dt + M\, dI_\theta/dt - A\, V_\theta$$

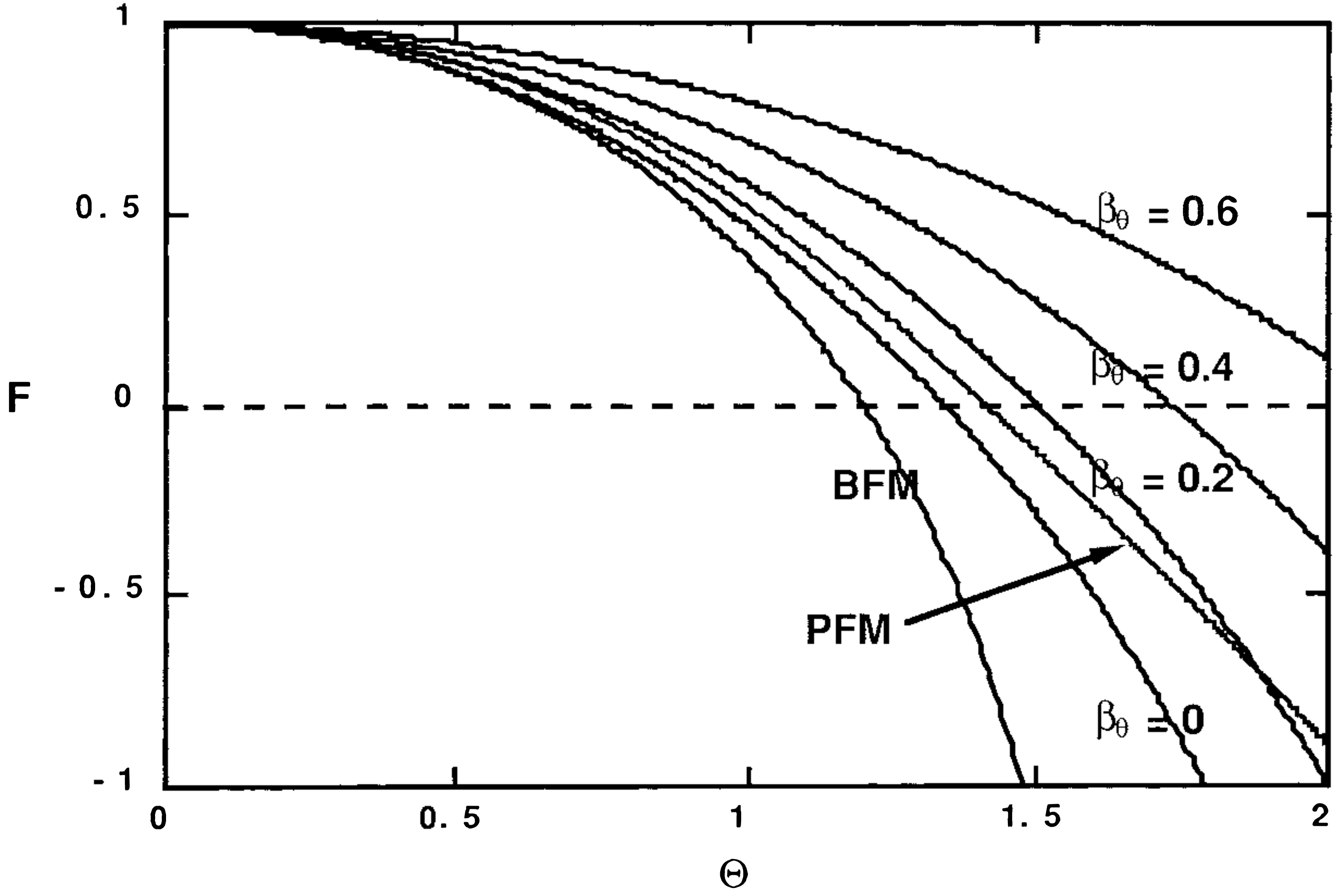
where

$$L = \frac{\mu_o R_o}{120} \left[31 + \frac{4}{\Theta}\sqrt{(6-6F)(3-2F)}\right]$$

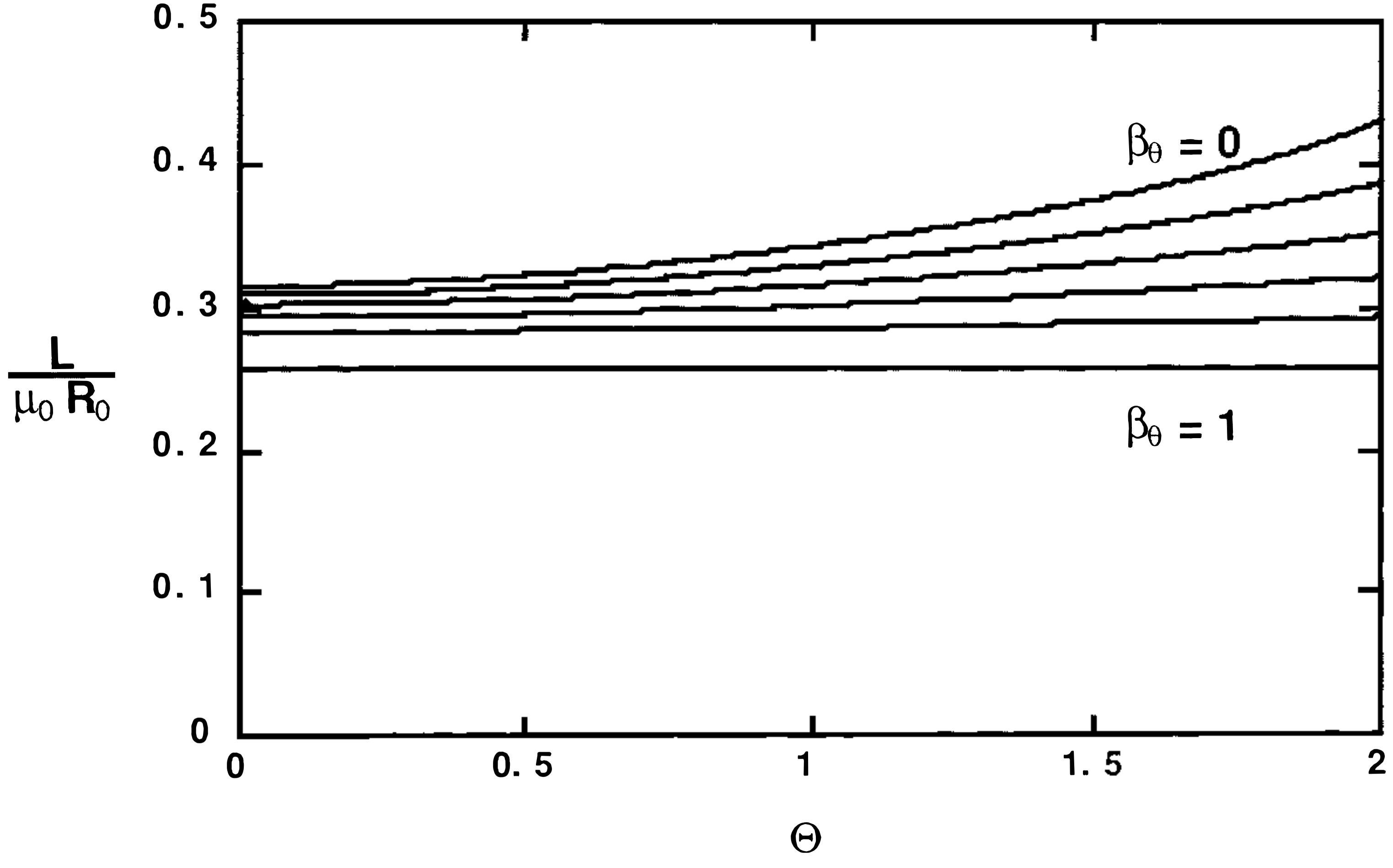
$$M = \frac{\mu_o a}{20\Theta}\left[13 - 12F + \frac{2\Theta\,(5-4F)}{\sqrt{(6-6F)(3-2F)}}\right]$$

$$A = \frac{R_o}{10a}\left[\frac{2(6-5F)}{\sqrt{(6-6F)(3-2F)}} + \frac{24-13F}{\Theta}\right] - \frac{R_o F}{a\Theta}$$

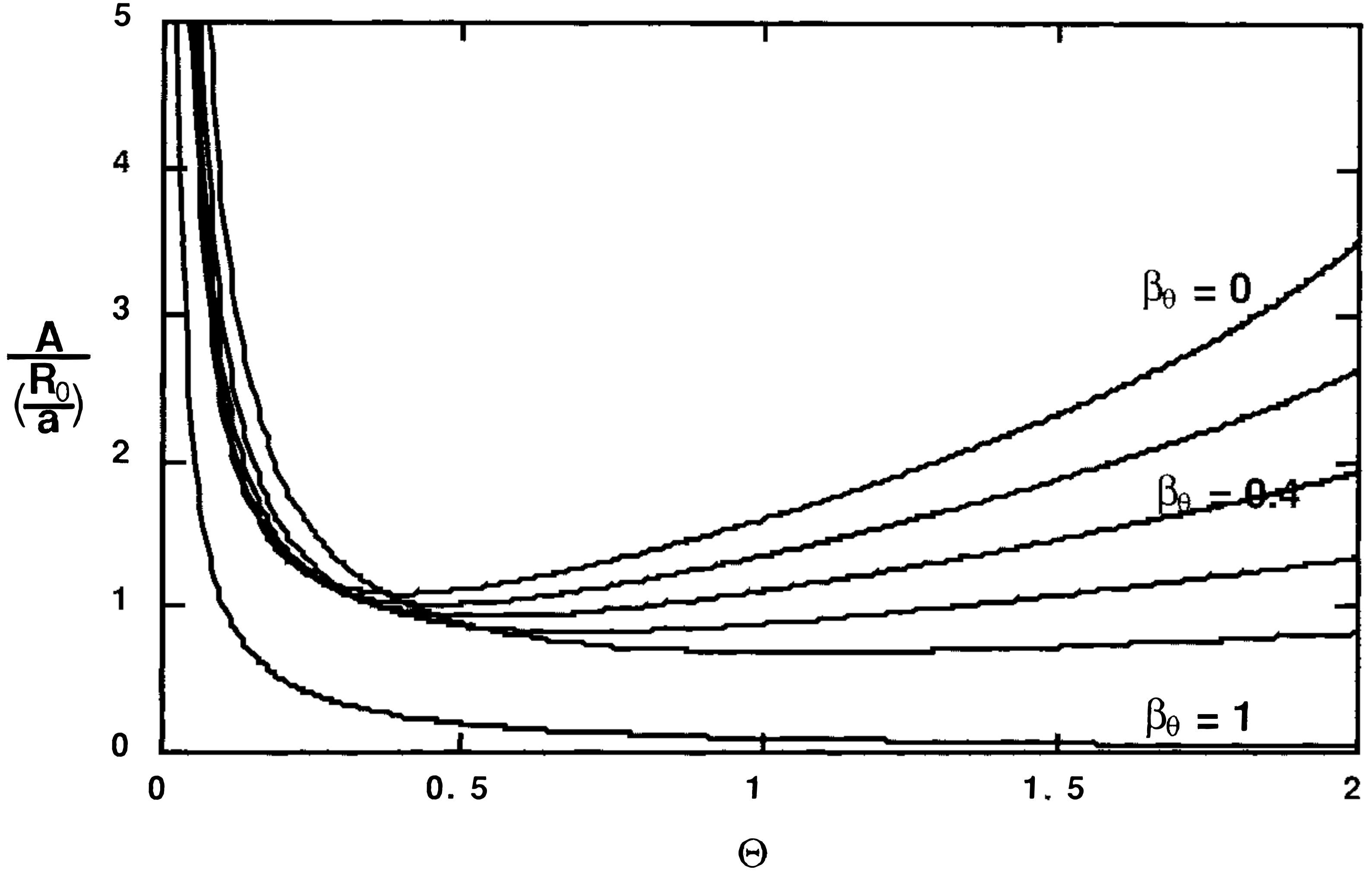
F - Θ CONTOURS FOR VARIOUS βθ



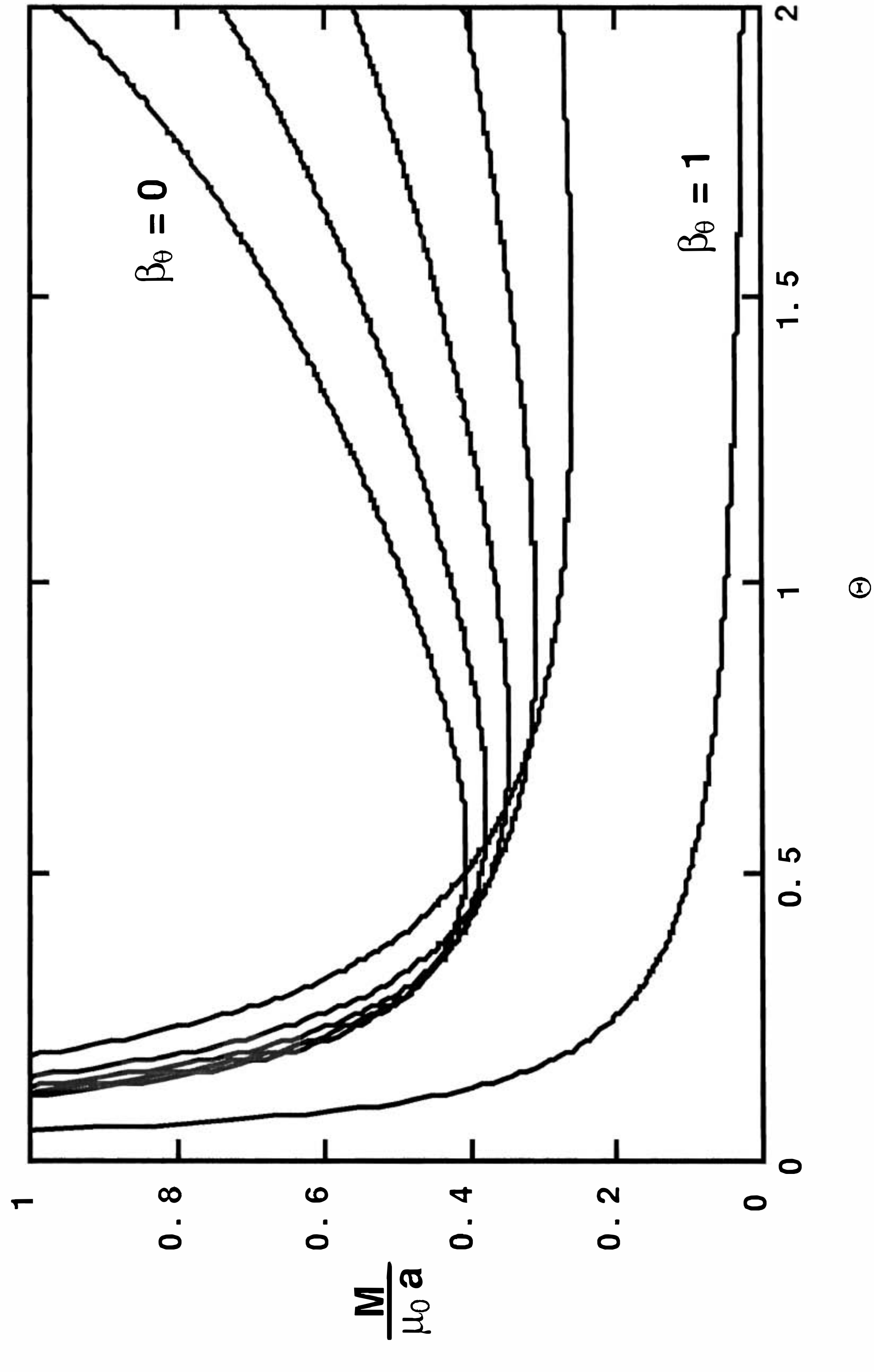
INDUCTANCE OF RFP PLASMA AS A FUNCTION OF Θ



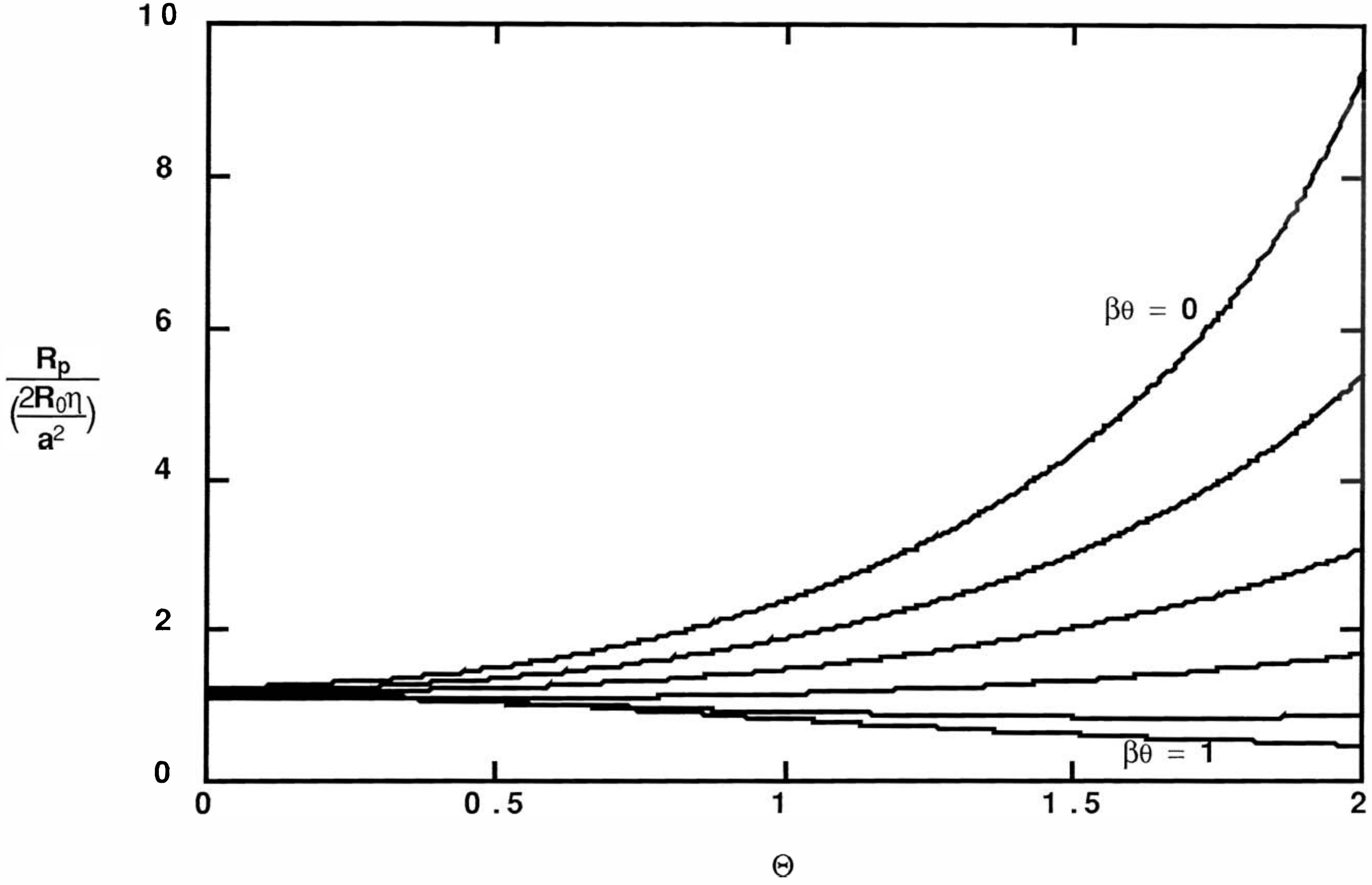
COUPLING COEFFICIENT AS A FUNCTION OF Θ



MUTUAL INDUCTANCE AS A FUNCTION OF Θ



NORMALIZED PLASMA RESISTANCE



Electrical Circuit Equations

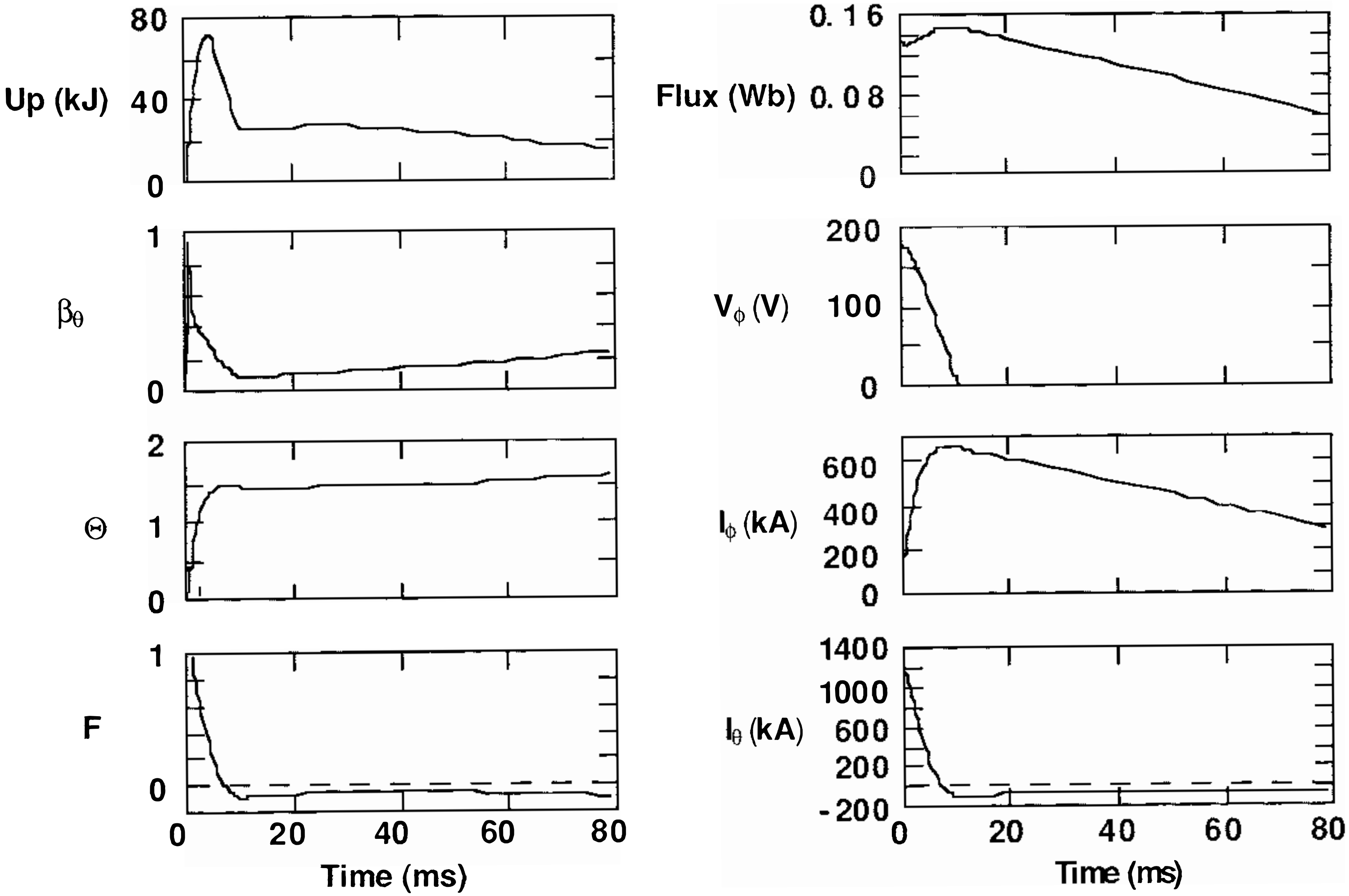
$$\frac{dV_\phi}{dt} = -\frac{I_\phi}{C_P}$$

$$\frac{dI_\phi}{dt} = \frac{1}{L}\left[V_\phi - V_R + M\frac{dI_\theta}{dt} - A\frac{d\Phi}{dt}\right]$$

$$\frac{d\Phi}{dt} = -L_T\frac{dI_\theta}{dt} - R_T I_\theta + V_T$$

$$\frac{dI_\theta}{dt} = \frac{(V_T - I_\theta R_T)(27 - 12 I_\theta L/\Phi) + 10L(V_R I_\phi - U_p/\tau)/\Phi - 15\ (V_\phi - V_R)R_0\Theta/a}{15\ (AL_T + M)R_0\Theta/a - 12L(I_\phi L_T/\Phi\ - 1) + 3FL + 18L_T}$$

SOLUTIONS OF CIRCUIT EQUATIONS FOR MST



Linear Stability

The circuit equations are linearly stable over the usual experimental range:

$$i\omega\tau \cong \beta_\theta/3 - 1$$

Perturbations damp on the time scale of the plasma energy confinement time.