CONTROLLING CHAOS IN HIGH-DIMENSIONAL SYSTEMS WITH PERIODIC PARAMETRIC PERTURBATIONS

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Controlling Chaos in High-Dimensional Systems with Periodic Parametric Perturbations

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The application of a small amplitude periodic perturbation to an accessible parameter of chaotic systems has been found to significantly decrease the dimension of the systems or produce limit cycle behavior. This effect occurs at minimal perturbation amplitudes when the perturbation frequency is at or near the frequency of unstable periodic orbits embedded in the attractor of the unperturbed system. Such periodic perturbations will be shown to suppress the chaos in a system of coupled Lorenz equations (a 96-dimensional polynomial flow), a tearing mode model of plasma fluctuations (a nine-dimensional polynomial flow), and a neural net model for poloidal magnetic field fluctuations in the Madison Symmetric Torus (MST) (a 64-dimensional nonlinear map).

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Outline

1. Introduction

- Fluctuating fields in fusion plasmas result in confinement degradation.

- One largely unexplored means of reducing these fluctuations is to apply recent developments in chaos control techniques.

- Here, a number of numerical chaotic systems of varying dimension were controlled with small amplitude (<10%) periodic perturbations at numerous frequencies.

- These frequencies do not always correspond to power spectral peaks.

- The most effective frequencies are found by identifying underlying structures in the dynamics: unstable periodic orbits (UPOs).

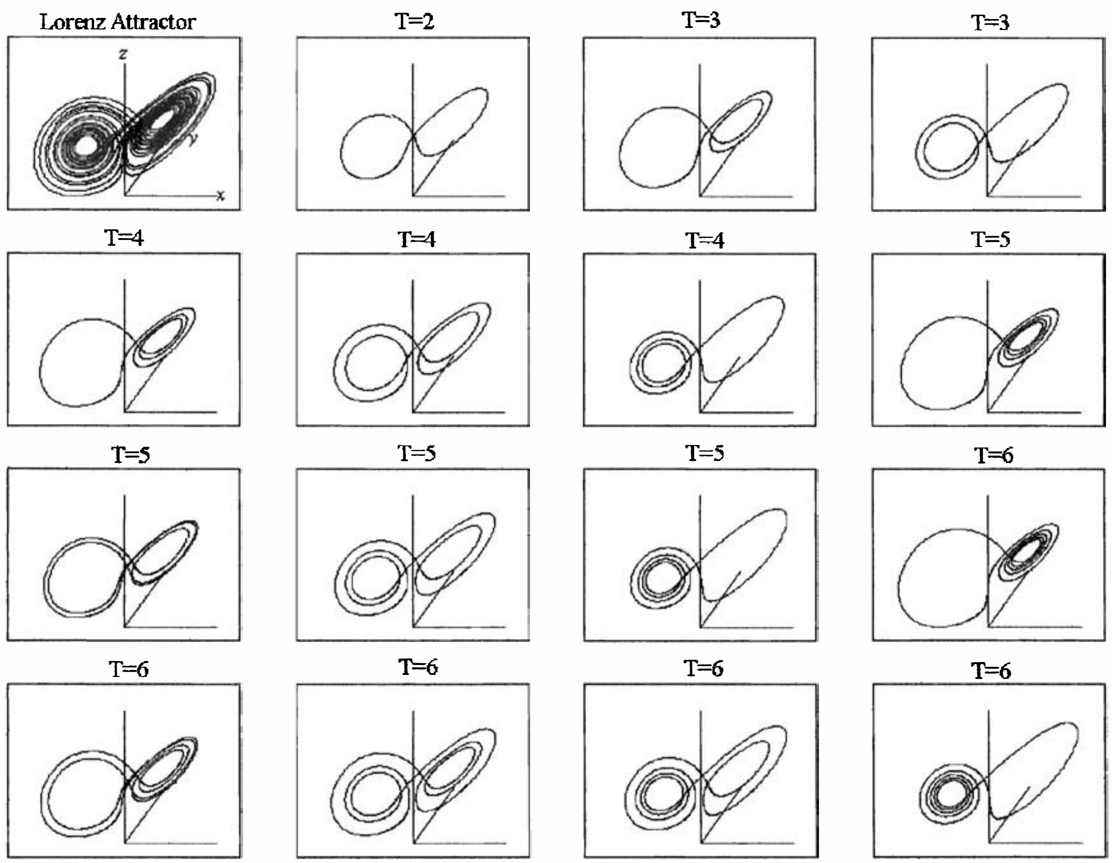
- UPO frequencies have been identified in MST.

- Attempts to control chaos in MST were limited by oscillator power.

2. Some Terminology

2.1. Unstable Periodic Orbits (UPOs)

- UPOs are organizing, unstable geometric structures underlying chaotic trajectories:



- UPOs are identified by measuring how long it takes for the system to return within an ε neighborhood of an initial point on the attractor.

2.2. Lyapunov Exponents and Dimension

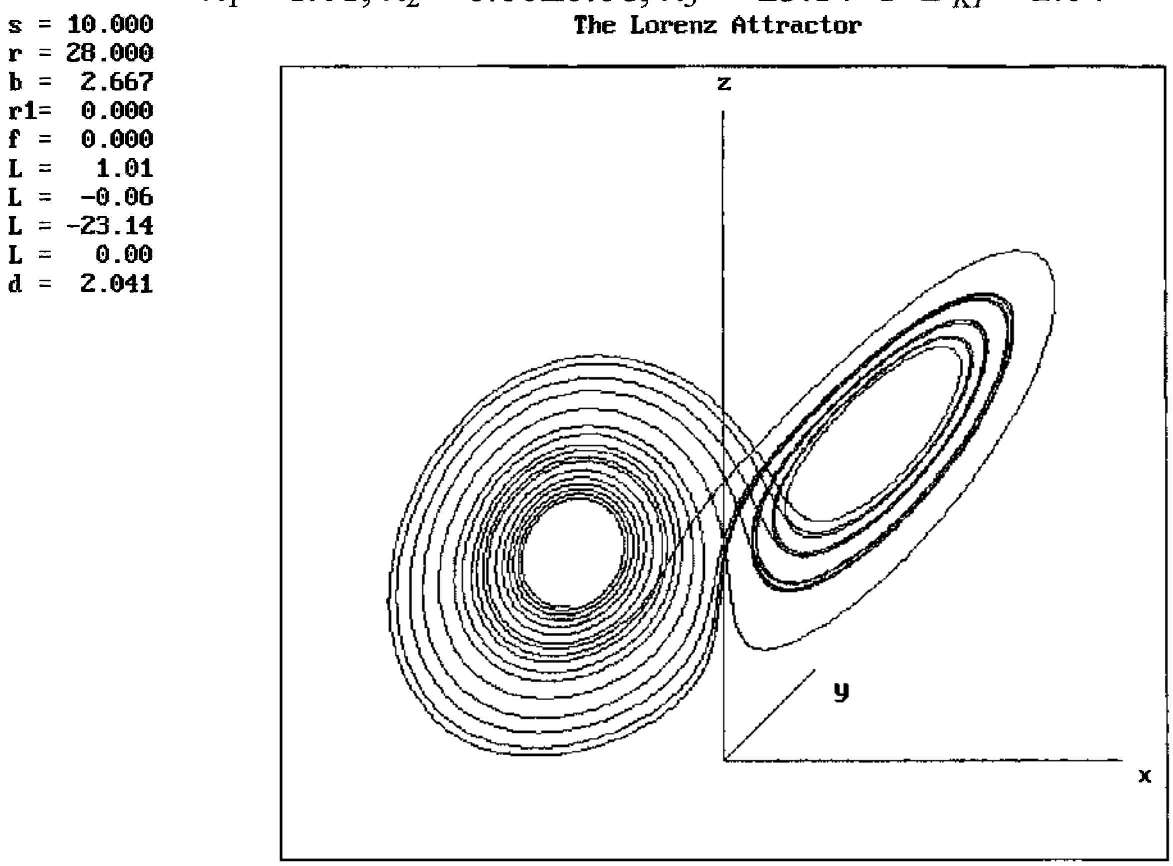
- Lyapunov exponents measure the local expansion/ contraction of an attractor

- Lyapunov exponents have been shown to be related to transport coefficients in certain systems.

- The time-averaged Lyapunov exponent spectrum can be used to calculate the dimension of a chaotic system (Kaplan & Yorke):

$$D_{KY} = j + \frac{\sum_{i=1}^{j} \lambda_i}{\left|\lambda_{j+1}\right|}, \text{ where } \sum_{i=1}^{j} \lambda_i > 0, \text{ and } \sum_{i=1}^{j+1} \lambda_i < 0.$$

- Application to the Lorenz equations:
  $\lambda_1 = 1.01$, $\lambda_2 = 0.00 \pm 0.06$, $\lambda_3 = -23.14$ ➔ $D_{KY} = 2.04$



3. Periodic Perturbations Applied to Numerical Systems

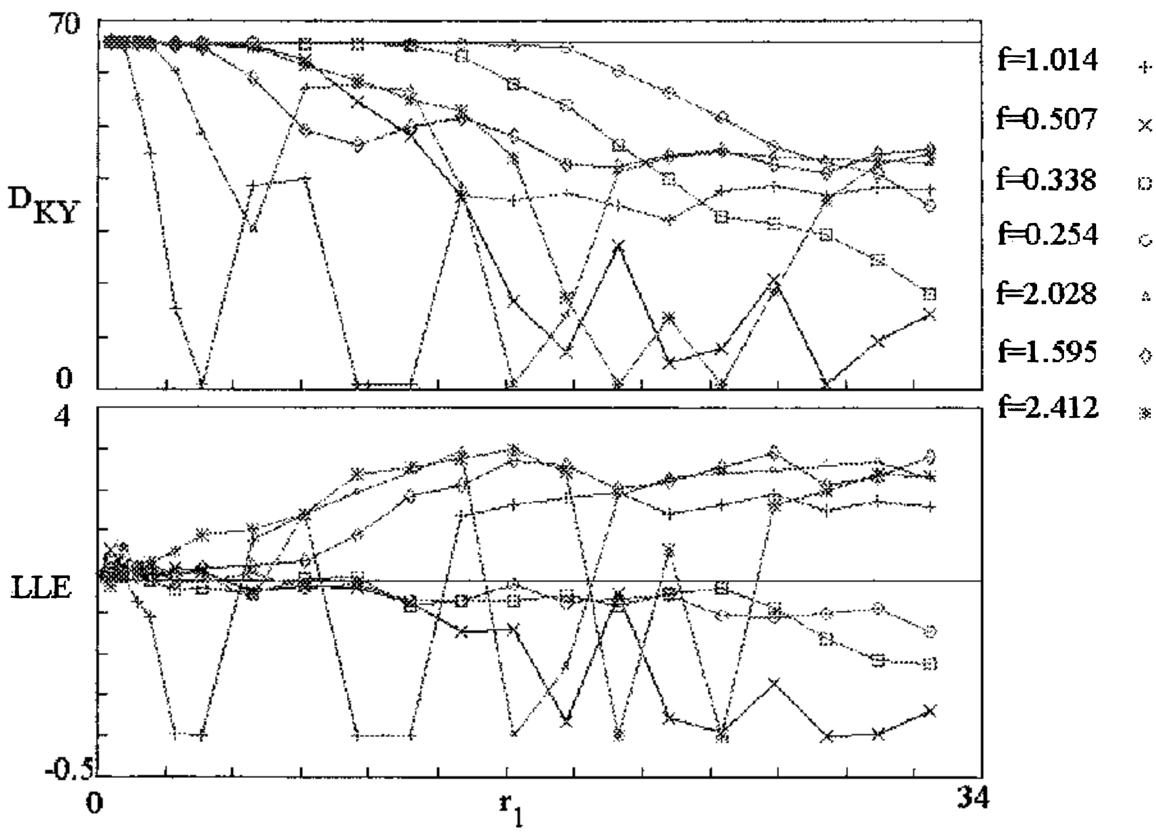
3.1. Coupled Lorenz Equations

- The perturbed equations:

$$\dot{x}_i = \sigma(y_i - x_i) - \mu(x_{i+1} + x_{i-1} + 2x_i)$$
$$\dot{y}_i = x_i(r_0 + r_1\sin(\phi) - z_i) - y_I - \kappa(y_{i+1} + y_{i-1} + 2y_i)$$
$$\dot{z}_i = x_i y_i - bz_i$$
$$\dot{\phi} = \omega$$

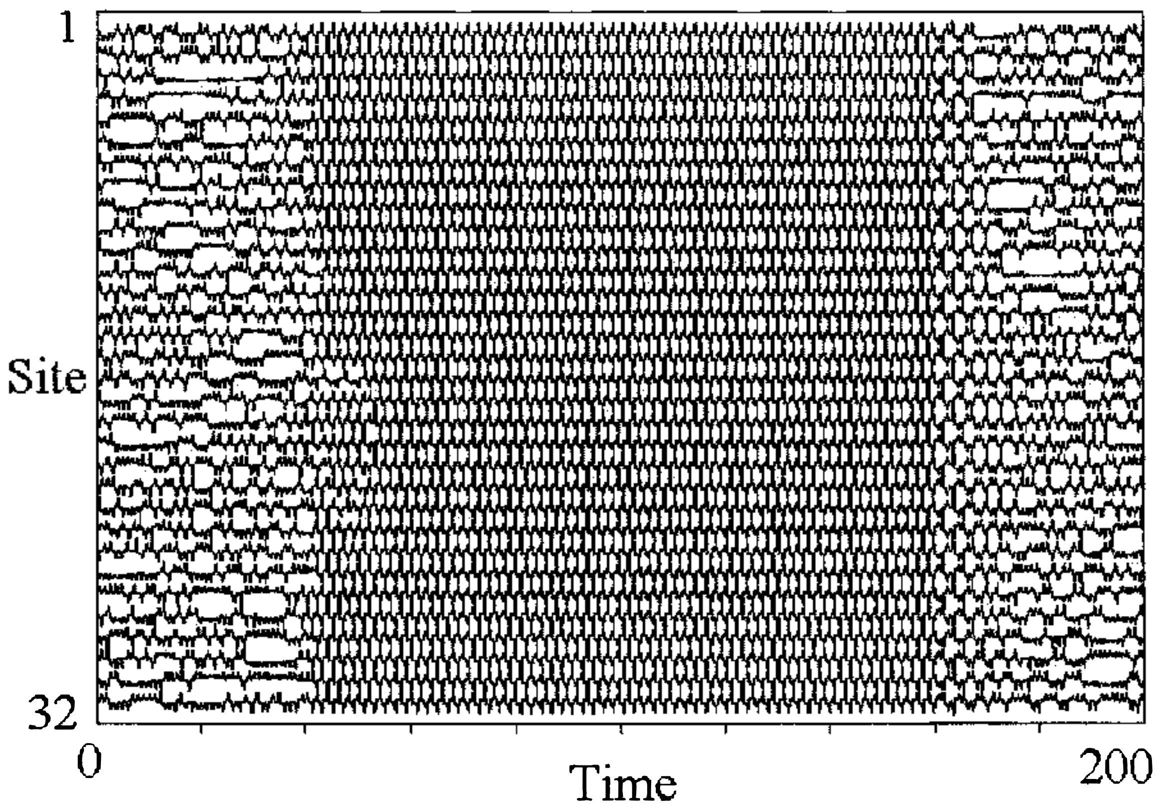
- The period-one UPO has frequency f=1.014 ±0.050 Hz. This is one-half the peak power spectral frequency.

- Kaplan-Yorke dimension and LLE for six different f:



- Limit cycles were achieved for period one, two, and twice peroid one UPO frequencies.

- Spatiotemporal plot of coupled Lorenz equations:



3.2. The Yoshida Equations

- The Yoshida equations are a model for nonlinear interactions of magnetic islands in a low-beta tokamak:

$$\dot{b}_{r,i} = -\eta/\mu_0 \lambda_i^2 b_{r,i} - \mathrm{V}\lambda_i b_{p,i}$$

$$\dot{b}_{p,i} = -\eta/\mu_0 \lambda_i^2 b_{p,i} + \mathrm{V}\lambda_i b_{r,i}$$

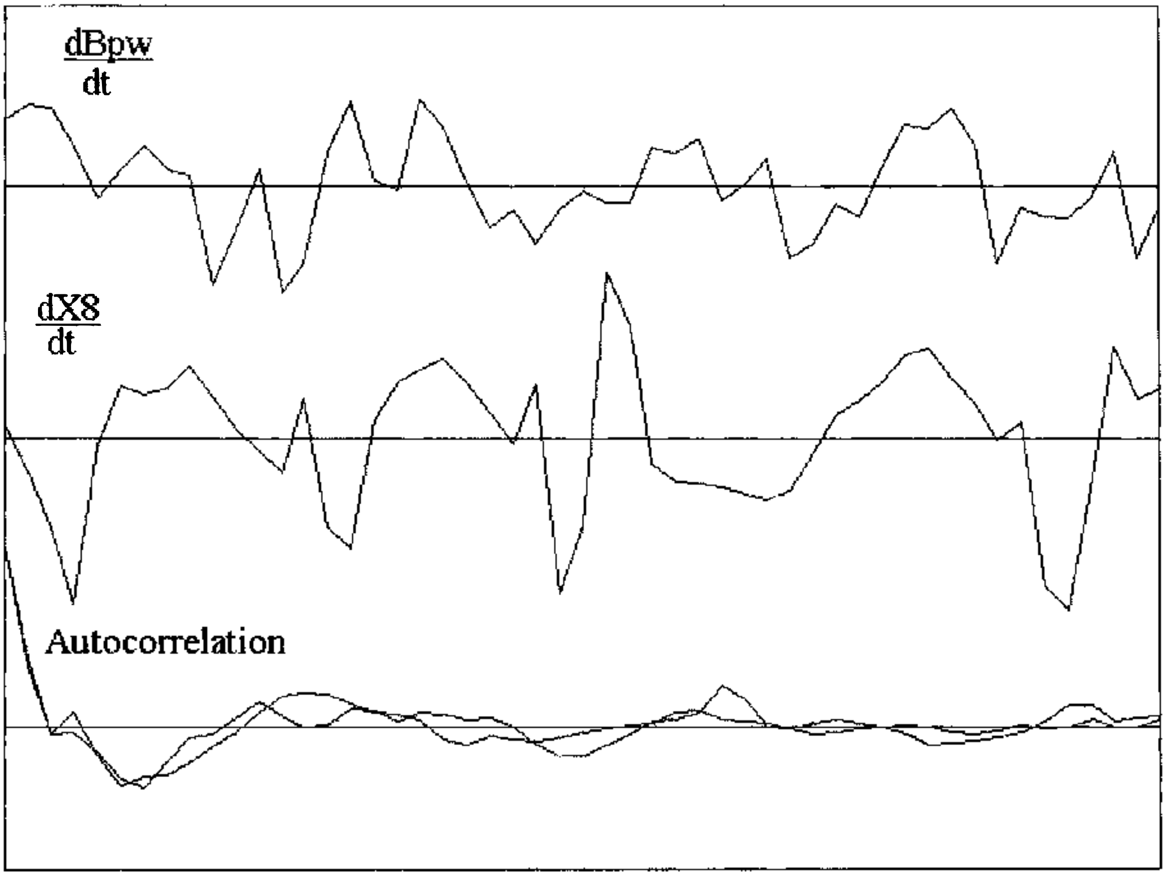
$$+\frac{2C}{B_0}\frac{\langle b_{r,i+1}\rangle^2 - \langle b_{r,i-1}\rangle^2}{\Delta_n}\frac{\lambda_{i+1}+\lambda_{i-1}-2\lambda_i}{\Delta_n^{\,2}}$$

$$\dot{\lambda}_i = \lambda_n^{\,2}\left(\frac{C}{B_0^{\,2}}\langle b_{r,i}\rangle^2\left(\frac{\lambda_{i+1}+\lambda_{i-1}-2\lambda_i}{\Delta_n^{\,2}}\right)+\frac{E_h}{B_0}\right)$$

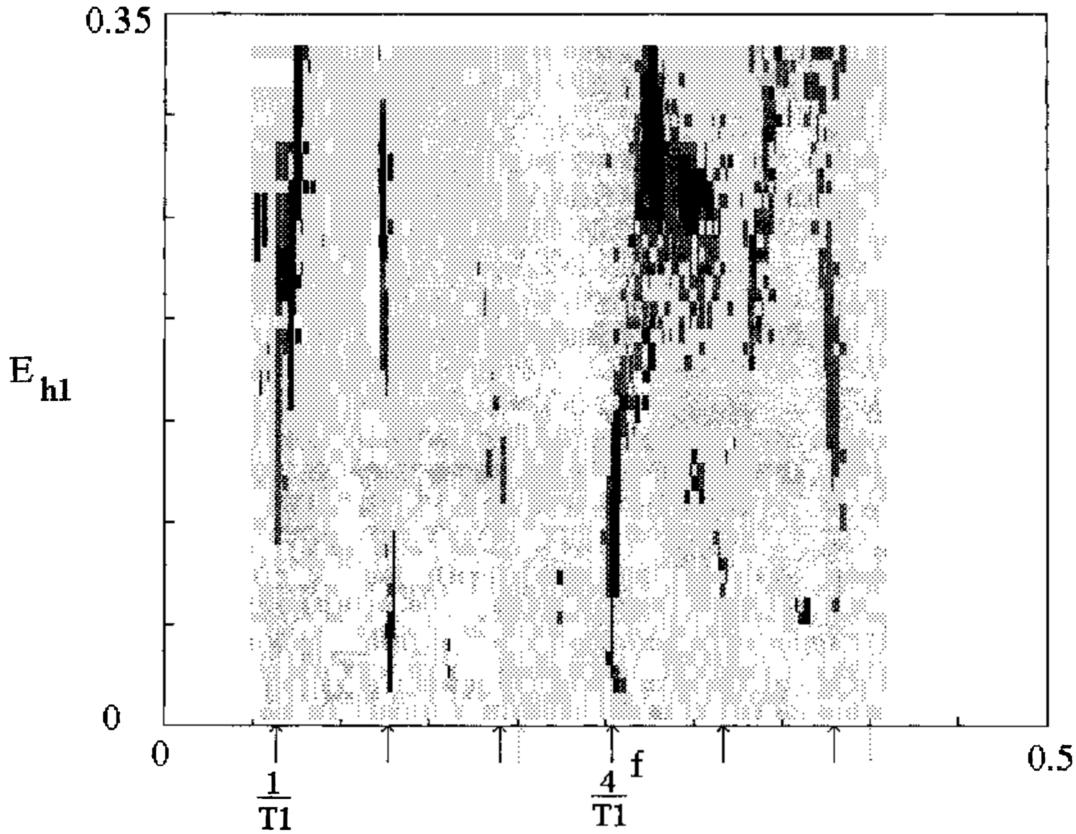
- The parameter E_h was perturbed because it corresponds to the ohmic heating transformer induction.

- The period-one UPO has frequency f=12.7±0.4 kHz (in MST time). This is one-third the peak power spectral frequency.

- Comparison of signals for an *N*=3 MST-like case:



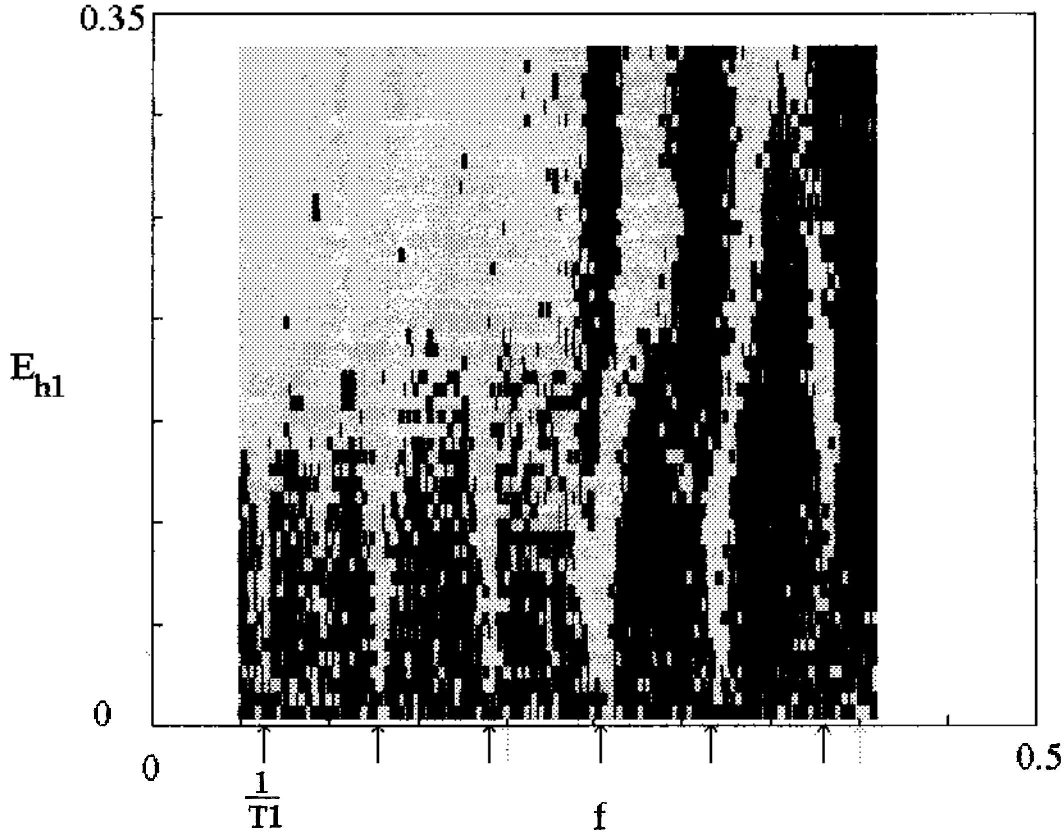
- Results of 10,000 N=3 perturbation cases:



- The fluctuation amplitudes are important in this model because they are related to electron heat diffusion (Rechester & Rosenbluth):

$$D \propto \sum_{i=1}^{N} b_{r,i}^{2} .$$

- Normalized diffusion coefficient for perturbed cases:

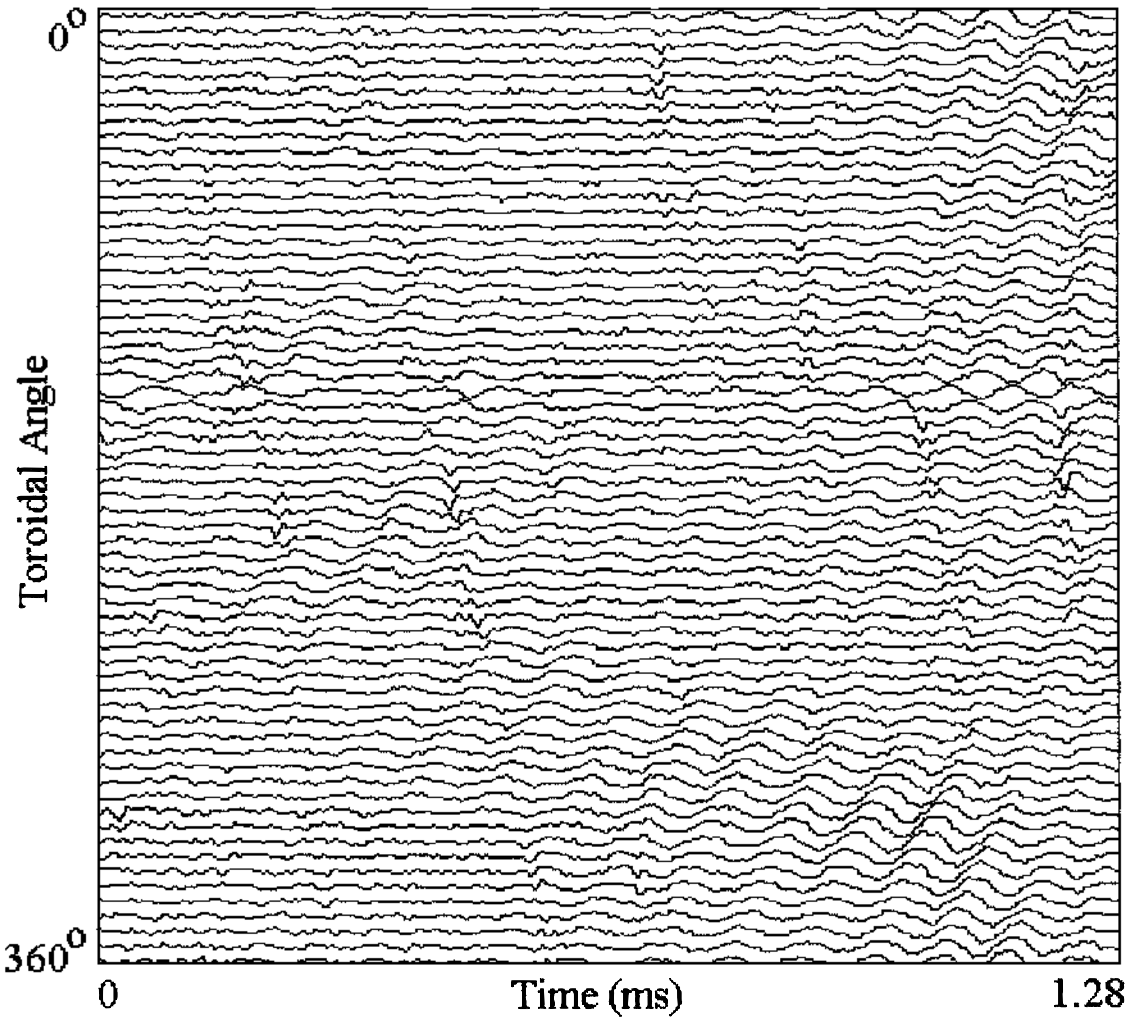


3.3. Neural Net Model for MST

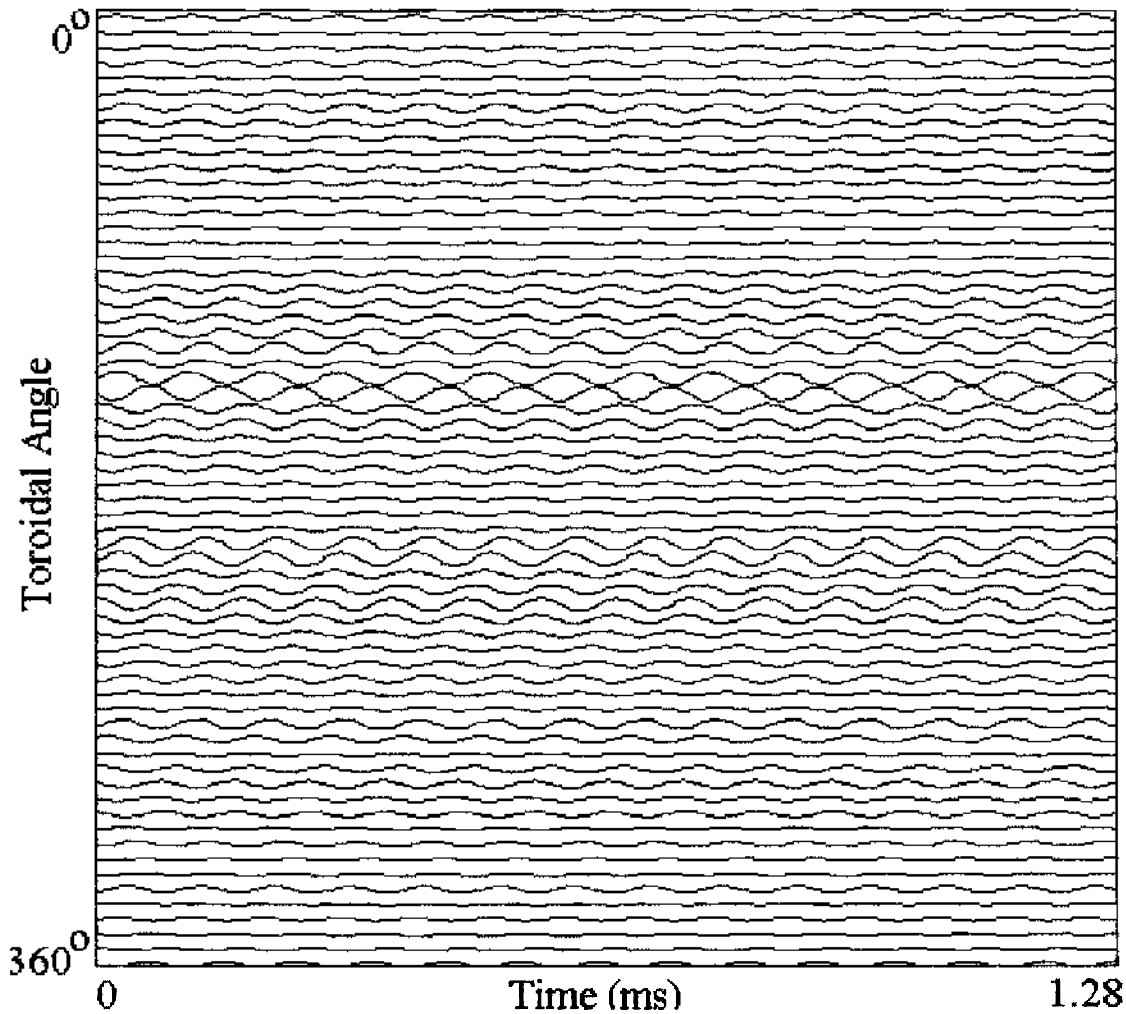
- Poloidal field fluctuations at the wall can be modeled by a neural network:

$$x_i(t+1) = \tanh\left(s \sum_{j=1}^{N} W_{ij} x_j(t) \right)$$

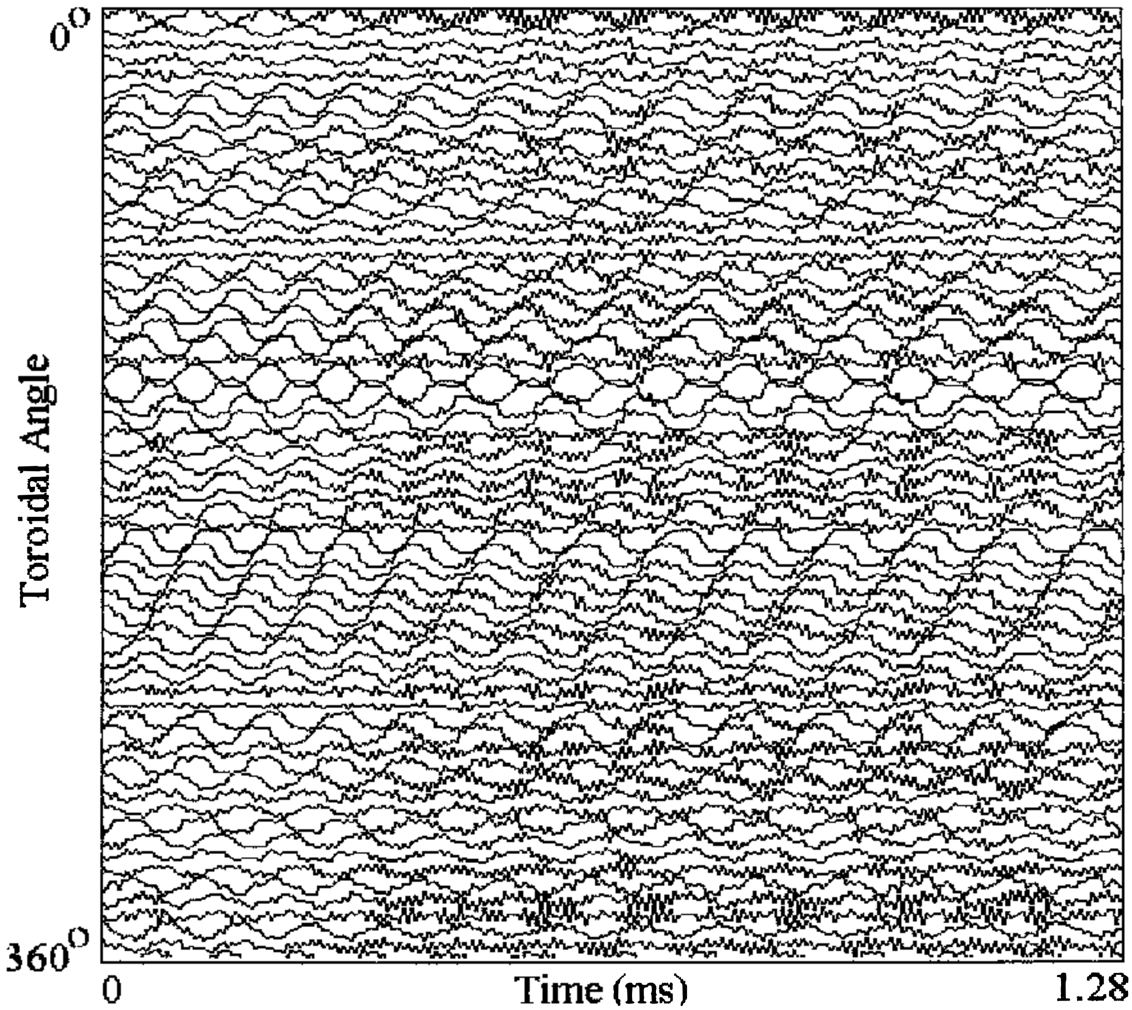
- Spatiotemporal plot of poloidal field fluctuations in MST:



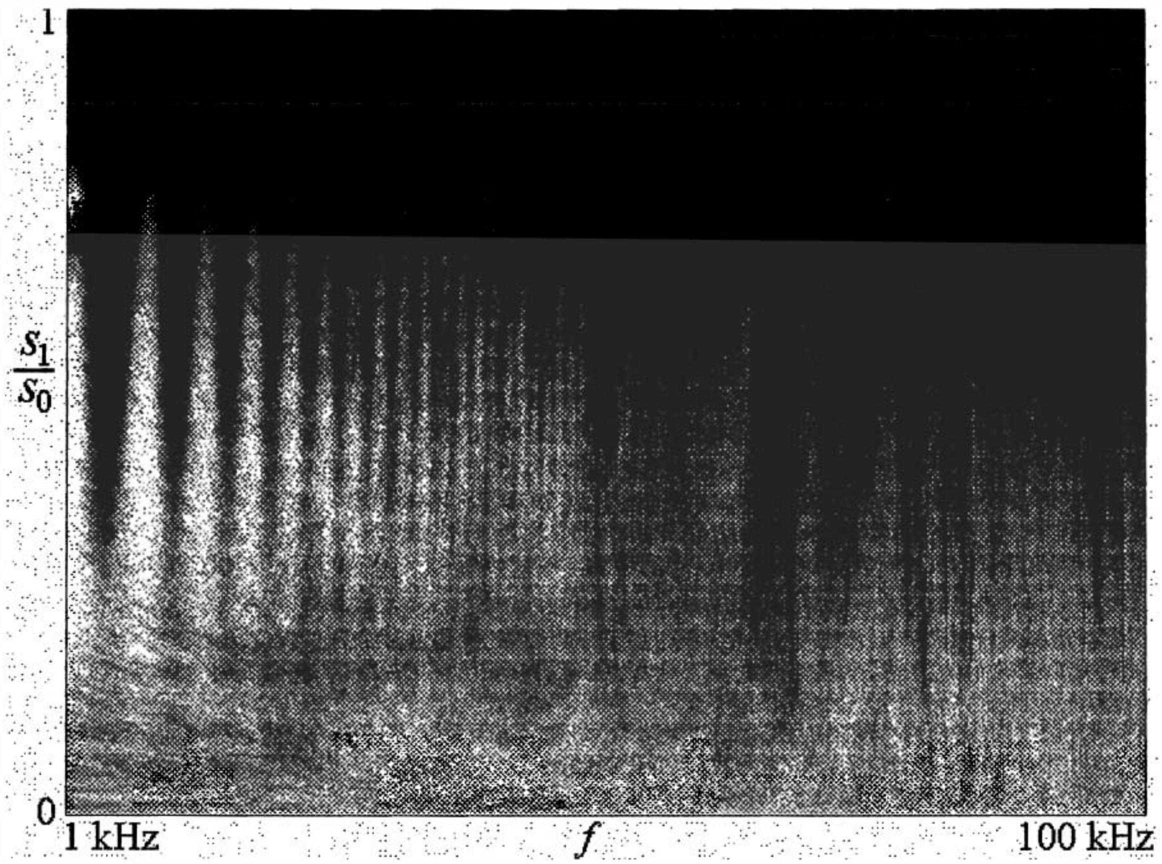
- Spatiotemporal plot of neural network reproduction of MST fluctuations:



- Spatiotemporal plot of chaotic fluctuations generated by neural network:



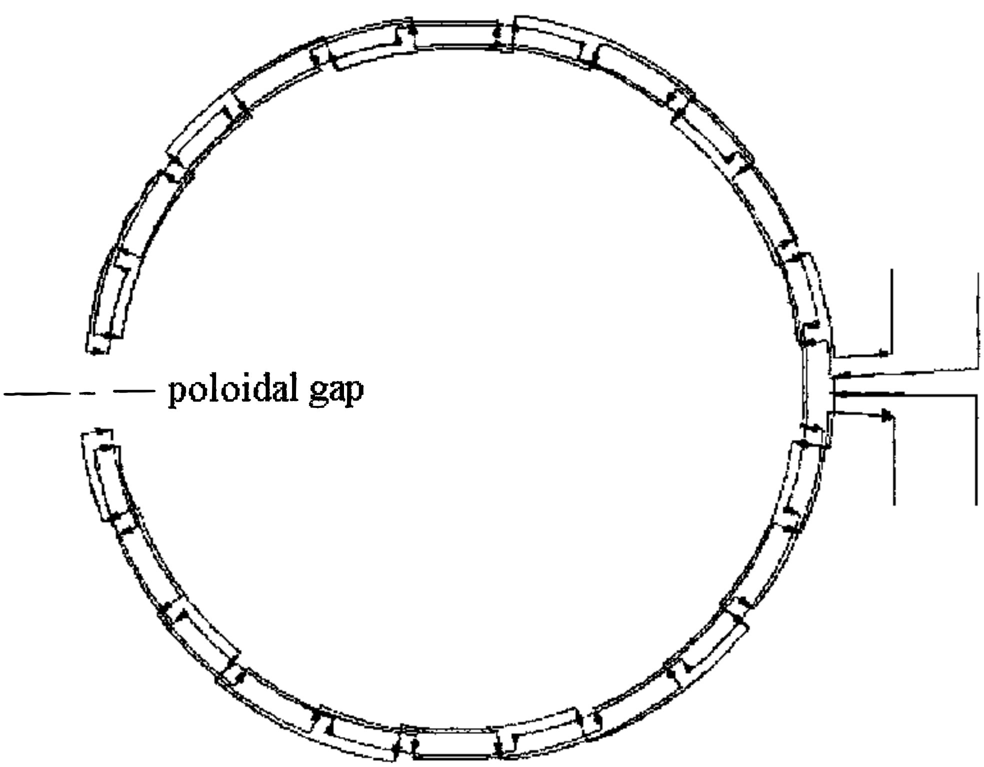
- LLE results for perturbing the parameter s:



4. Periodic Perturbations Applied to MST

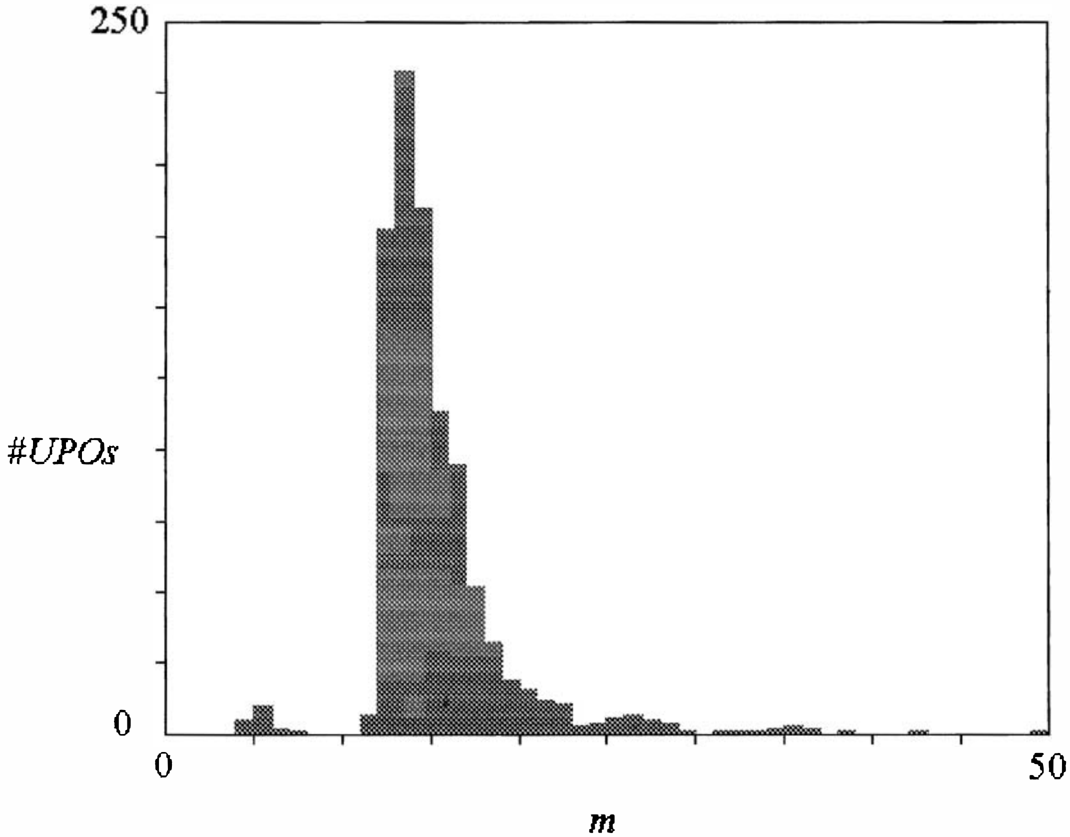
4.1. Experimental Apparatus

- Two sets of n=1 and n=6 spatially structured coils applied radial magnetic field perturbations through the toroidal gap:
- n=6 coils:

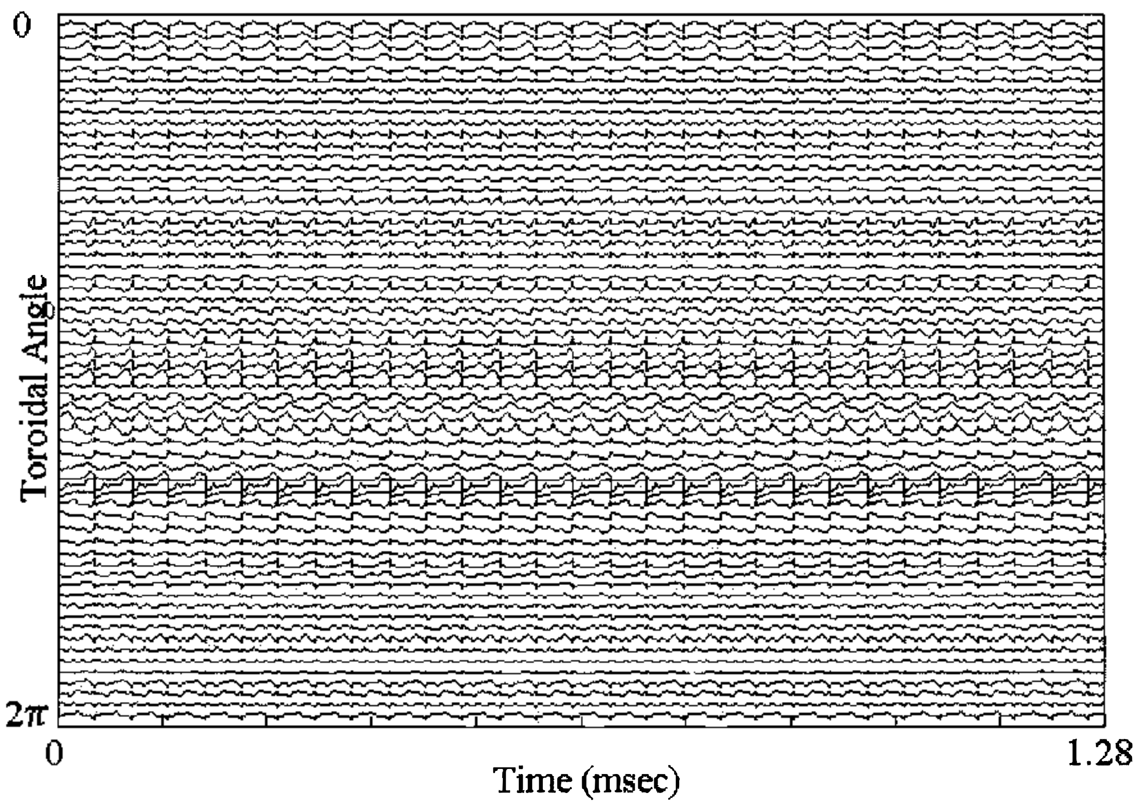


4.2. Unstable Periodic Orbits

- Period one UPOs were identified at 7.1±0.7 kHz, roughly one-half of the tearing mode frequency, using a 64-dimensional embedding space provided by the toroidal array magnetic field pick up coils:



- Spatiotemporal plot of a period one UPO:



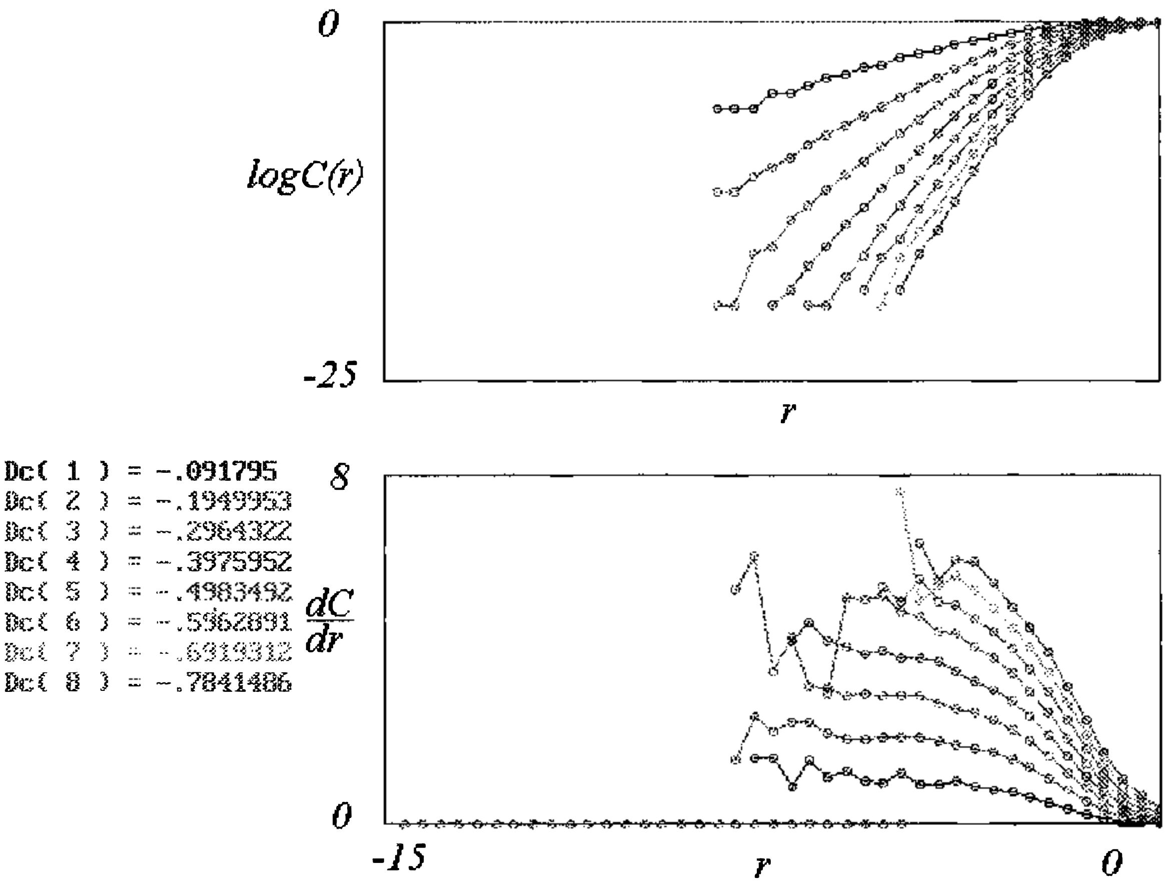
4.3. Correlation Dimension

- The correlation dimension is defined in terms of the correlation sum:

$$C(R) = \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{\substack{j=1, \\ j \neq i}}^{N} \Theta\left(R - \left|x_i - x_j\right|\right)$$

$$D_c = \lim_{R \to 0} \frac{\log C(R)}{\log R}$$

- Correlation dimension of perturbed shots does not show indications of low dimensionality:



5. Conclusions and Future work

- Periodic perturbations can control chaos in a wide variety of systems.
- This can yield a wide variety of behaviors.
- The most effective frequencies are at the frequencies of the UPOs in the system.
- UPO frequencies do not necessarily correspond to peak power spectral frequencies.
- Perturbations can greatly decrease the dimension of high dimensional systems.
- Perturbations may be able to control or at least reduce fluctuations in MST at frequencies of ~7 kHZ or multiples thereof.
- These perturbations could be applied with an upgraded rotating magnetic perturbation, or to the poloidal field / toroidal field transformers.