ELECTROSTATIC PROBE PARADOX

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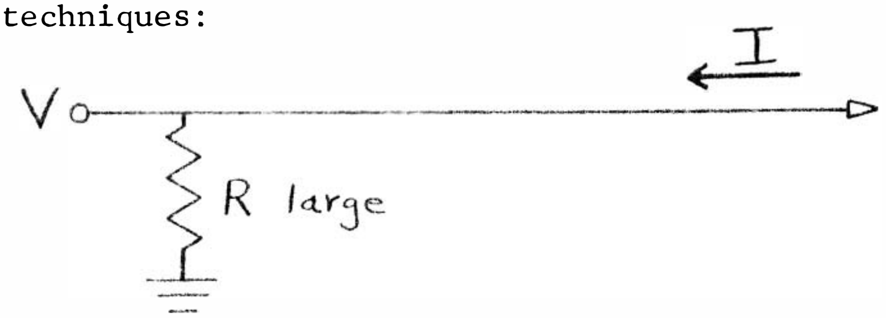
INTRODUCTION

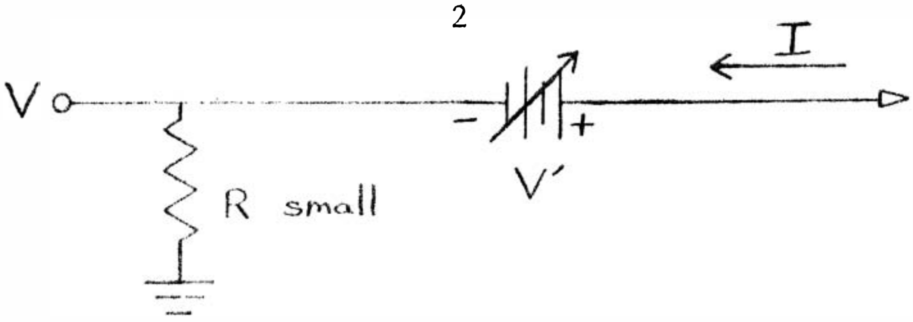
A yet unsolved paradox of Langmuir probe behavior involves the measurement of the floating potential during the quiescent confinement period in the Wisconsin toroidal octupole. The floating potential is defined as that potential assumed by an insulated conductor immersed in the plasma. Normally the floating potential V_f is negative with respect to the plasma potential V_p because the electron flux in the plasma exceeds the ion flux.

The floating potential can be measured by either of two techniques:



High Impedance Probe

As $R \longrightarrow \infty$, the current I to the probe goes to zero and $V \longrightarrow V_f$.



Low Impedance Probe

Adjust V' until V = 0. Then $I = V/R = 0$ and $V' = V_f$.

When V_f is measured by the above methods, the high-z probe gives $V \cong -7$ volts and the low-z probe gives $V_f \cong +15$ volts. This discrepancy of 22 volts has come to be called the "probe paradox".

The plasma on the minor axis in the toroidal octupole has the following desirable and perhaps unique properties:

1) Low density ($n \sim 10^9$ cm^{-3})

2) Collisionless ($\tau_{ee} \sim 1$ msec)

3) Isotropic and homogeneous

4) Reasonably Maxwellian

5) Long lived ($\tau \sim 1$ msec)

In addition,

6) The magnetic field is zero

7) Potential and density fluctuations are small ($\delta n/n$, $\delta V_f/V_f < 1\%$)

and 8) The floating potential is relatively constant for $\sim$ 1 msec.

Hence, an especially suitable plasma is available for studying probe phenomena.

PLASMA POTENTIAL MEASUREMENTS

We would prefer to believe the high-z rather than the low-z probe for two reasons:

1) All V_f measurements have been made that way.
2) The agreement with Erickson's plasma potential measurements using an electrostatic analyzer[1] are much better.

The relationship between V_p and V_f is given by[2]

$$V_p - V_f = V_e \ \ln \frac{\overline{v}_e}{\overline{v}_i} \tag{1}$$

where V_e is the electron temperature in eV, $\overline{v}_e$ is the mean electron velocity and $\overline{v}_i$ is the mean ion velocity. For the octupole,

$$V_p - V_f \cong 3.1\ V_e \cong 31 \text{ volts} \tag{2}$$

Using equation (2), the high impedance probe predicts $V_p = + 24$ volts and the low impedance probe predicts $V_p = + 46$ volts.

Erickson has attempted to measure V_p by electrostatically analyzing the energy spectrum of ions in the plasma.[1] With his analyzer grounded, he observes an energy spectrum which is nearly Maxwellian but displaced from zero by 7 eV. He interprets the sharp cutoff at + 7 eV as the presence of zero energy ions which acquire an energy of 7 eV upon falling through a potential drop of 7 volts between the plasma and the grounded analyzer. Hence he concludes that $V_p = + 7$ volts.

The accuracy of this measurement could be affected in several ways:

1) High energy incident ions could produce secondary low energy ions upon striking the inside wall of the extractor pipe.
2) The plasma could be depleted of zero energy ions.
3) The magnetic shielding might not be adequate to extract the lower energy particles.

The first effect would bring Erickson's result in closer agreement with probe results, while the second two would make the discrepancy worse.

If we believe the low impedance probe, the discrepancy is so bad (39 volts) that the results could be brought into agreement only by assuming that the ions are 10^4 times more energetic than the electrons.

The high impedance probe could be brought into agreement with Erickson's result if the electrons were:

1) Maxwellian with a temperature of 4.5 eV,

or 2) A 10 eV Maxwellian but truncated at ~ 18 eV.

These possibilities may not be too unreasonable especially if one considers that the only measurement of the electron distribution has been with the low impedance probe whose reliability at present seems to be in considerable doubt.

Another method of measuring the plasma potential involves the use of emissive (or emitting) probes.[3] Attempts have been made to measure V_p with such probes in the octupole, but their usefulness is limited by the presence of a double sheath.[4,5,6]

FURTHER CONSIDERATIONS

It is possible to change continuously from the low to the high impedance regime by reducing the bias to zero and increasing the load resistance. By keeping the load resistor small and varying the bias, the V-I characteristic in Figure 1 was plotted out. Note that the current is zero only when $V = + 15$ and that an appreciable current is flowing at $V = -7$ volts.

The slope of the curve at $I = 0$ is easily calculated[2] and is given by

$$\left.\frac{dI}{dV}\right|_{V_f} = \frac{I_{oi}}{V_e} \equiv \frac{1}{R_s} \qquad (3)$$

where I_{oi} is the ion saturation current to the probe. The slope has the dimensions of an inverse resistance called the sheath resistance R_s. Admittance probe tests in the octupole[7] have shown that the slope is indeed given by equation (3) over a range of densities. The magnitude of this sheath resistance vs density and temperature is given in Figure 2. For the octupole plasma $R_s \approx 100\ K\Omega$.

The electron temperature is obtained by plotting the log of the electron current vs probe voltage for $V < V_p$. The result is shown in Figure 3. Within experimental error, the points agree well with a 10 eV Maxwellian. If the electron distribution were truncated at 18 eV as

Erickson has suggested, the electron current would be zero for $V < 28$ volts.

According to equation (2) the plasma potential should be at 46 volts and the electron current should saturate above that value. The knee of the curve in figure 3 is not very pronounced but it appears to be nearer to 46 volts than to the 24 volts predicted by the high-z probe.

If we assume that R_s represents the resistance between the probe and the plasma, the following model is appropriate for a floating probe:

Such a probe can be considered floating only for $R \gg R_s \approx 100K$. As R increases from 0 to ∞, V should monotonically increase from 0 to V_f according to

$$V = \left(\frac{R}{R + R_s}\right) V_f \quad . \tag{4}$$

Since $V - V_f$ is often large compared with V_e, the linear approximation involved in calculating equation (4) is clearly inadequate. An exact result can be obtained by setting the current through the load resistor equal to the current to the probe:

$$\frac{V}{R} = I_{oi} - I_{oe}e^{(V - V_p)/V_e} \quad . \tag{5}$$

In terms of R_s, equation (5) becomes

$$V = V_e \frac{R}{R_s} [1 - e^{(V - V_f)/V_e}] \quad . \tag{6}$$

A plot of equations (4) and (6) is shown in Figure 4 assuming $V_f = +15$ and -7 volts. The difference between the two solutions is not great. A little thought shows that whenever the V-I characteristic monotonically increases, the V vs R curve must also monotonically increase (or decrease).

The experimentally observed V vs R curve is shown in Figure 5. Note that V is positive for small R and negative for large R. V(R) has a single extremum and crosses zero at $R \approx R_s$.

Similar curves have been obtained for:

1) Different times (300-800 μsec after injection)
2) Different ports
3) Different probe materials
4) Different probe radii
5) Different places in the magnetic field
6) Different plasmas (μwave and gun)
7) Different observers

The curves always have the same form although the polarity is sometimes reversed.

It is possible to fit to the experimental data a curve of the form

$$V = \frac{RV_1}{R + R_1} + \frac{RV_2}{R + R_2} \qquad (7)$$

(note the similarity to equation (4)). The solid curve in Figure 5 is such a fit with

$V_1 = 1.4$ volts $\qquad R_1 = 34K$

$V_2 = -6.4$ volts $\qquad R_2 = 450K.$

The significance of these numbers has not yet been determined.

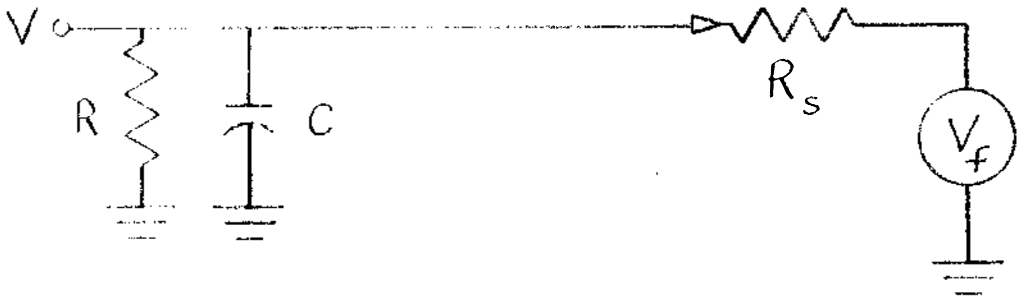
The fact that the probe voltage is zero for some R means that the floating potential is zero at that value of load resistor. In fact it is clear that as R goes from 0 to ∞ , the value of V_f goes continuously from + 22 to -7 volts.

TIME DEPENDENT EFFECTS

It is very hard to imagine how the plasma could possibly distinguish between a probe connected to an infinite resistance and a probe biased to draw zero current unless the probe has a memory of its past history. When plasma is first injected into the octupole its density is a few x 10^{10} and potentials are often 100 volts or higher.

Figure 6 shows oscilloscope traces of the probe voltage for two different load resistors. During the first 50 μsec, the low-z probe draws a negative (electron) current of ~ 30 ma. The high-z probe also goes negative to a potential of -50 volts. The high-z probe remains negative thereafter but the low-z probe goes positive and decays as if it were collecting ion saturation current. The fact that a grounded probe draws positive current while a floating probe reads a negative voltage is another statement of the probe paradox.

The simplest time dependent model would simply include a finite probe capacitance:



The capacitance C is typically $\sim$ 100 pF. The low-z probe responds in a time $\tau \approx RC$ which is very short. The high-z probe is slower and responds in a time $\tau \approx R_s C \approx 10$ μsec.

A more pessimistic res

$$Q = CV = I_{oi} \tau \ . \tag{8}$$

Taking V = 100 volts gives $\tau \approx 10\ R_s C \approx 100$ μsec. This time is not quite long enough to explain the experimental result. Furthermore by using high-z probes with the 10 MΩ resistor at the probe tip, C can be reduced from 100 pF to $\sim$ 1 pF, but the results are the same.

It is possible that the low-z probe draws such a high current from the plasma that the potential is changed by 22 volts. The charge of electrons removed by the low-z probe is $\sim$ 1 μcoulomb. Poisson's equation predicts that the voltage would change by $\sim$ 10 KV if isolation from the walls and hoops were perfect. That this does not happen is easily verified by monitoring V_f with high-z probe before and after a low-z probe is inserted into the plasma. No difference is seen. In fact, a high-z and a low-z probe less than 1 cm apart still read vastly different floating potentials.

Sheath effects can also be ruled out. The capacitance of the sheath is negligibly small. Hot ions traverse the sheath in times of the order of the period of an ion plasma oscillation or about 0.1 μsec. Even a room temperature tungsten molecule travels a Debye length in only 5 μsec.

Time changes in the magnetic field would at most produce potentials of $\sim$ 10 volts. The experiment is performed when $\dot{B} \approx 0$ and is unaffected by operating with $\dot{B} < 0$ and $\dot{B} > 0$.

POSSIBLE EXPLANATIONS

An interesting possibility is the trapping of ions in orbits around the probe. The theory for this is complicated. Bernstein and Rabinowitz[8] work it out for $V_e \gg V_i$ and Lam[9] treats the arbitrary temperature case but assumes a probe radius much larger than a Debye length. Neither paper treats the time dependent case. Chen[6] remarks that there is no evidence that this is an important effect.

The possibility of high frequency fluctuations shifting the probe operating point through a rectifying action caused by the non-linear V-I probe characteristic has been treated by several authors.[10,11,12,13] A 22 volt D.C. shift would require fluctuation amplitudes of comparable magnitude. No such fluctuations have been observed in the range 0 - 1 GHZ, however.

Perhaps the most promising explanation is that during the initial blast of plasma, something physical might happen to the probe such as the deposition of inpurities or an insulating layer of hydrogen or some compound of hydrogen which then slowly recovers between shots of plasma (~ 1 minute).

The electron and ion energy of particles striking a probe at potential V is:

$$\left.\begin{array}{l} E_i = V_i + V_p - V \\ \\ E_e = V_e - V_p + V \end{array}\right\} \qquad (9)$$

During transport, $V_f \approx -50$ volts, so the energy of particles striking the probes is:

low-z: $E_i = 20$ eV, $E_e = 30$ eV

high-z: $E_i = 70$ eV, $E_e = -20$ eV.

Both probes receive the same ion flux, but the mean energy of incident ions to the high-z probe is considerably higher than to the low-z probe. The high-z probe collects very few electrons (only those with energy greater than ~ 30 volts), while the grounded probe collects a high flux of 30 volt electrons.

Various authors[14,15,16,17] report variations in probe potential due to probe contamination but these effects are invariably in steady state plasmas and have time scales of several seconds to several hours. We know that if our probes are suffering contamination, they recover in times much less than 1 minute because the same probe can be used as a high impedance probe on one shot and a low impedance probe on the next shot and still give contradictory results.

In summary, we can say that every test we have made on the low-z probe indicates that it is operating correctly and every test we have made on the high-z probe indicates that it is working properly. But the two still give floating potentials which differ by about 22 volts.

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TABLE OF RESULTS

Measurement	V_f	V_p
High-z probe	-7	(+24)
Low-z probe	+15	+46
Electrostatic Analyzer	(-24)	+7

( ) indicates values calculated from $V_p - V_f = 31$ volts

DEFINITION OF SYMBOLS

V	probe potential
V_f	floating potential
V_p	plasma potential
V_e	electron temperature (eV)
V_i	ion temperature (eV)
I	current to probe
I_{oe}	electron saturation current
I_{oi}	ion saturation current
R_s	$\equiv V_e/I_{oi}$
R	probe load resistor

FIGURES

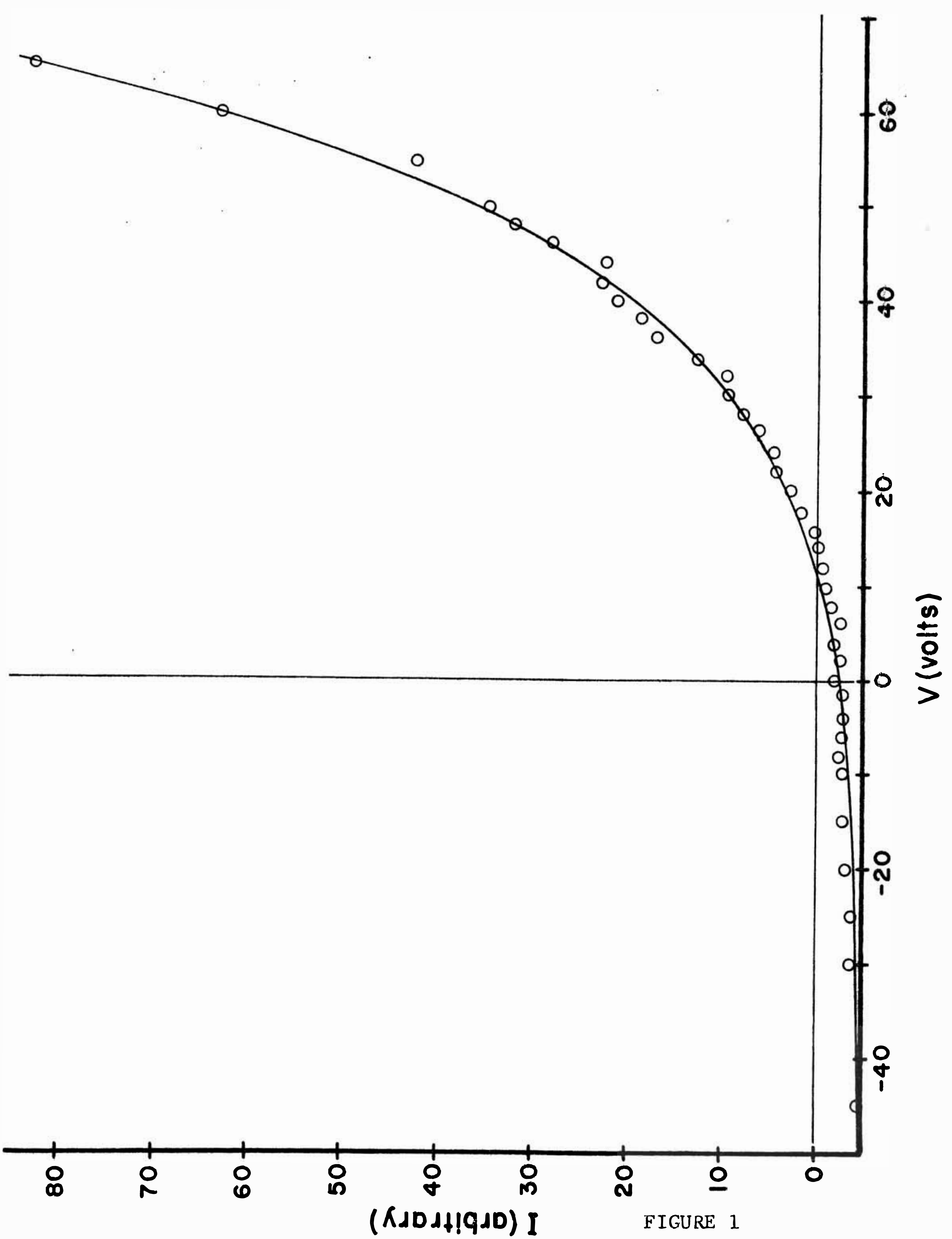
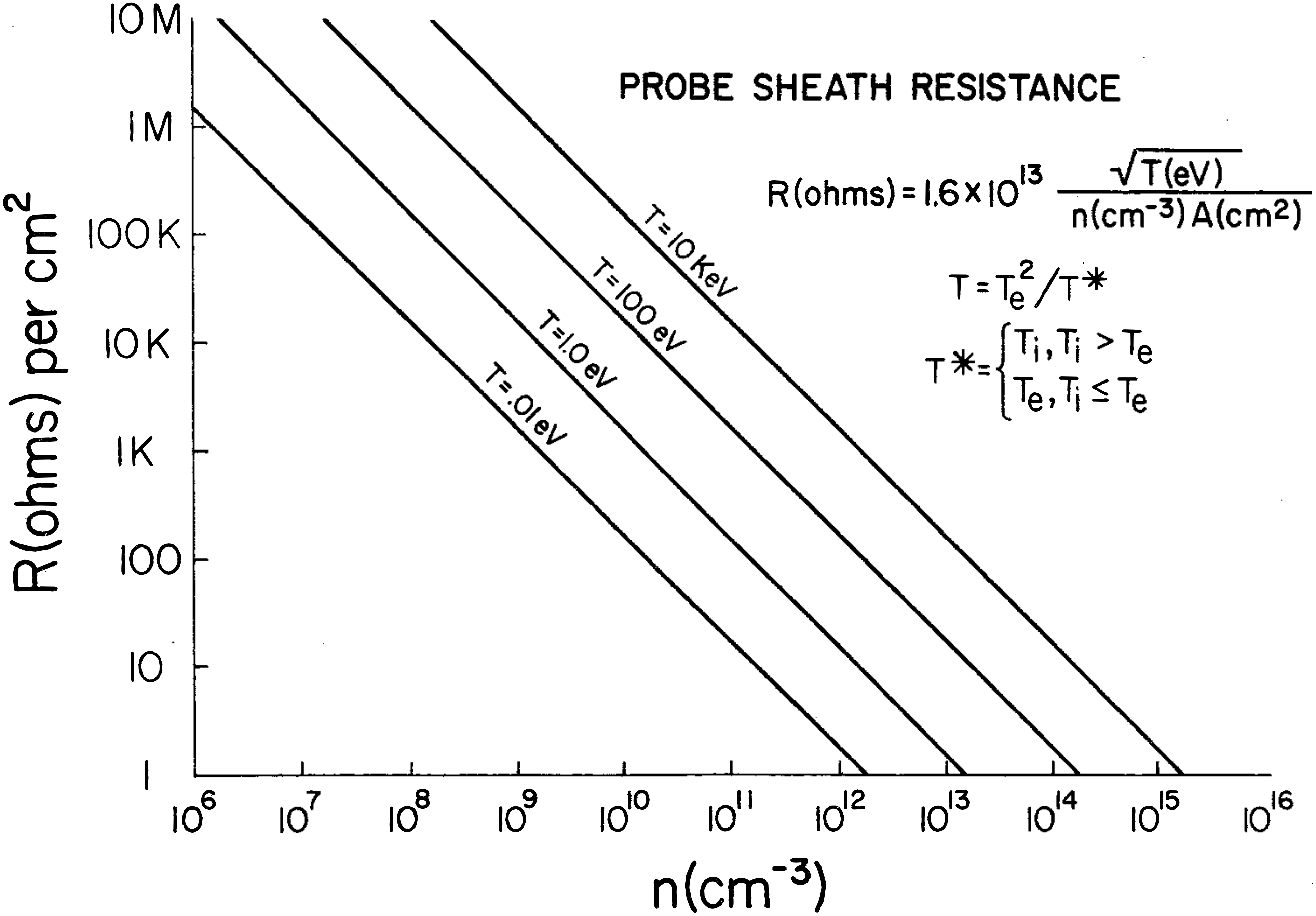


FIGURE 1

FIGURE 2


PROBE SHEATH RESISTANCE
R(ohms) = 1.6 × 10^13 √T(eV) / n(cm^-3) A(cm^2)
T = Te^2/T*
T* = {Ti, Ti > Te; Te, Ti ≤ Te}
T=10KeV
T=100eV
T=1.0eV
T=.01eV
R(ohms) per cm^2
10M
1M
100K
10K
1K
100
10
1
n(cm^-3)
10^6
10^7
10^8
10^9
10^10
10^11
10^12
10^13
10^14
10^15
10^16

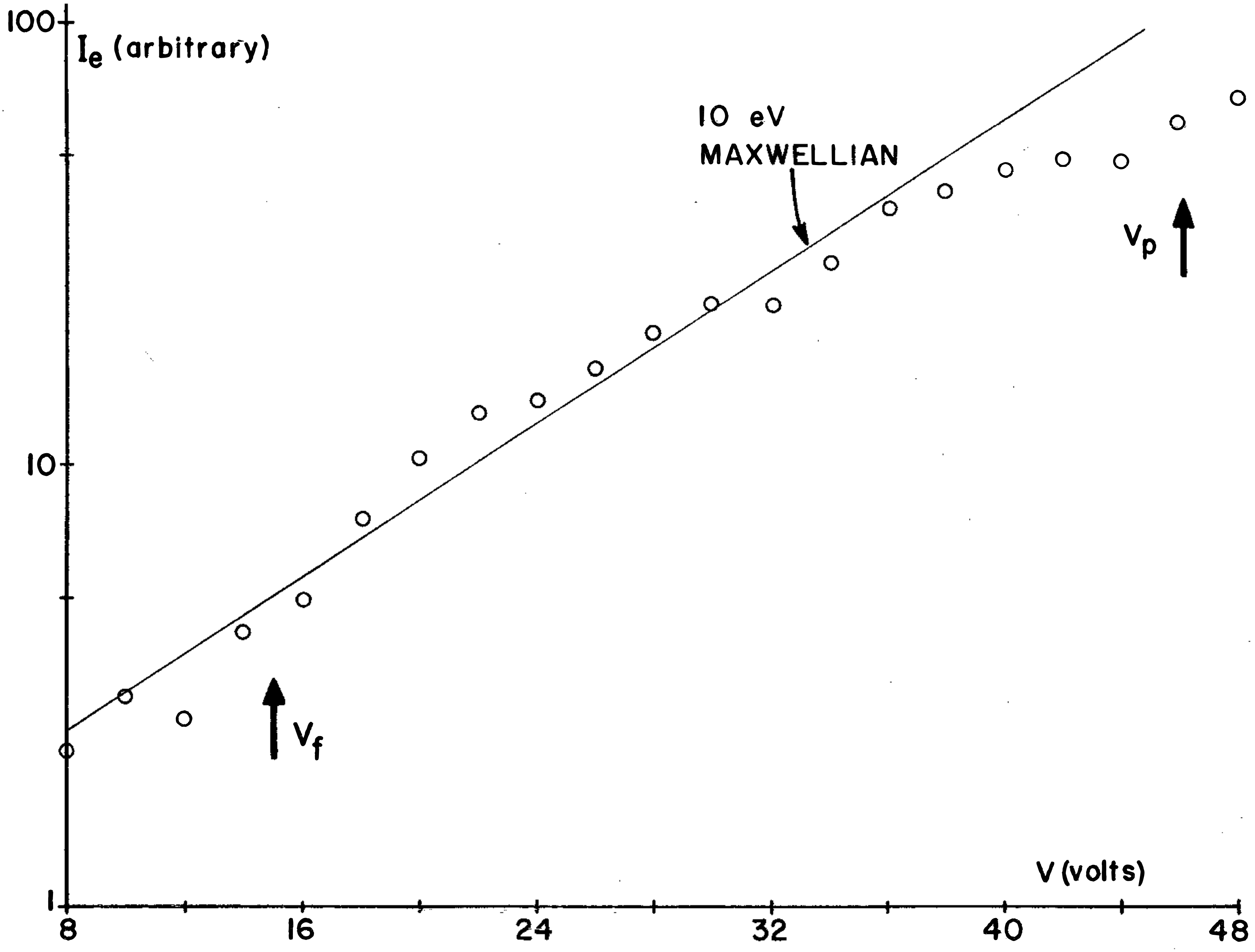
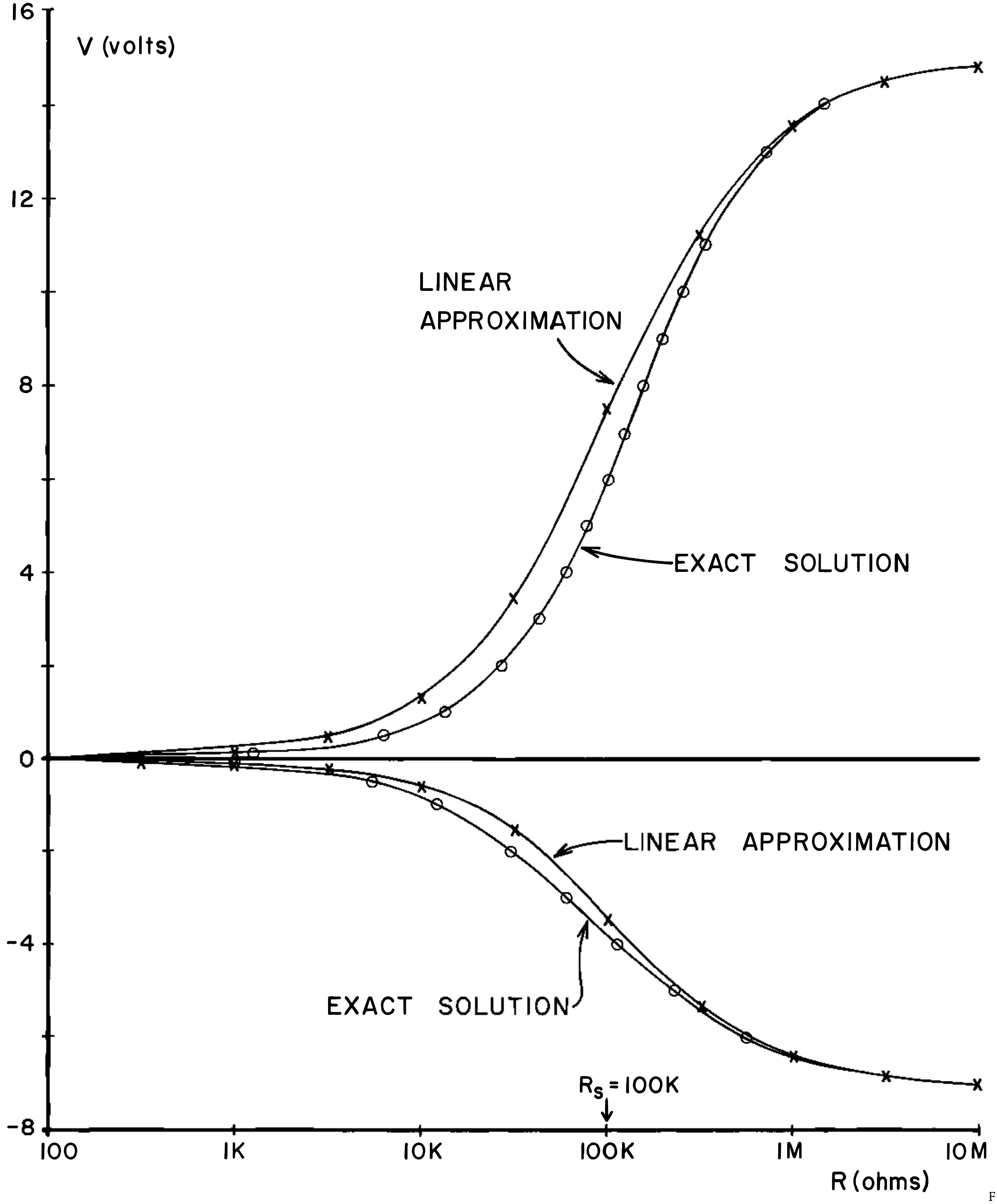

100
I_e (arbitrary)
10 eV
MAXWELLIAN
V_p
10
V_f
1
V (volts)
8
16
24
32
40
48

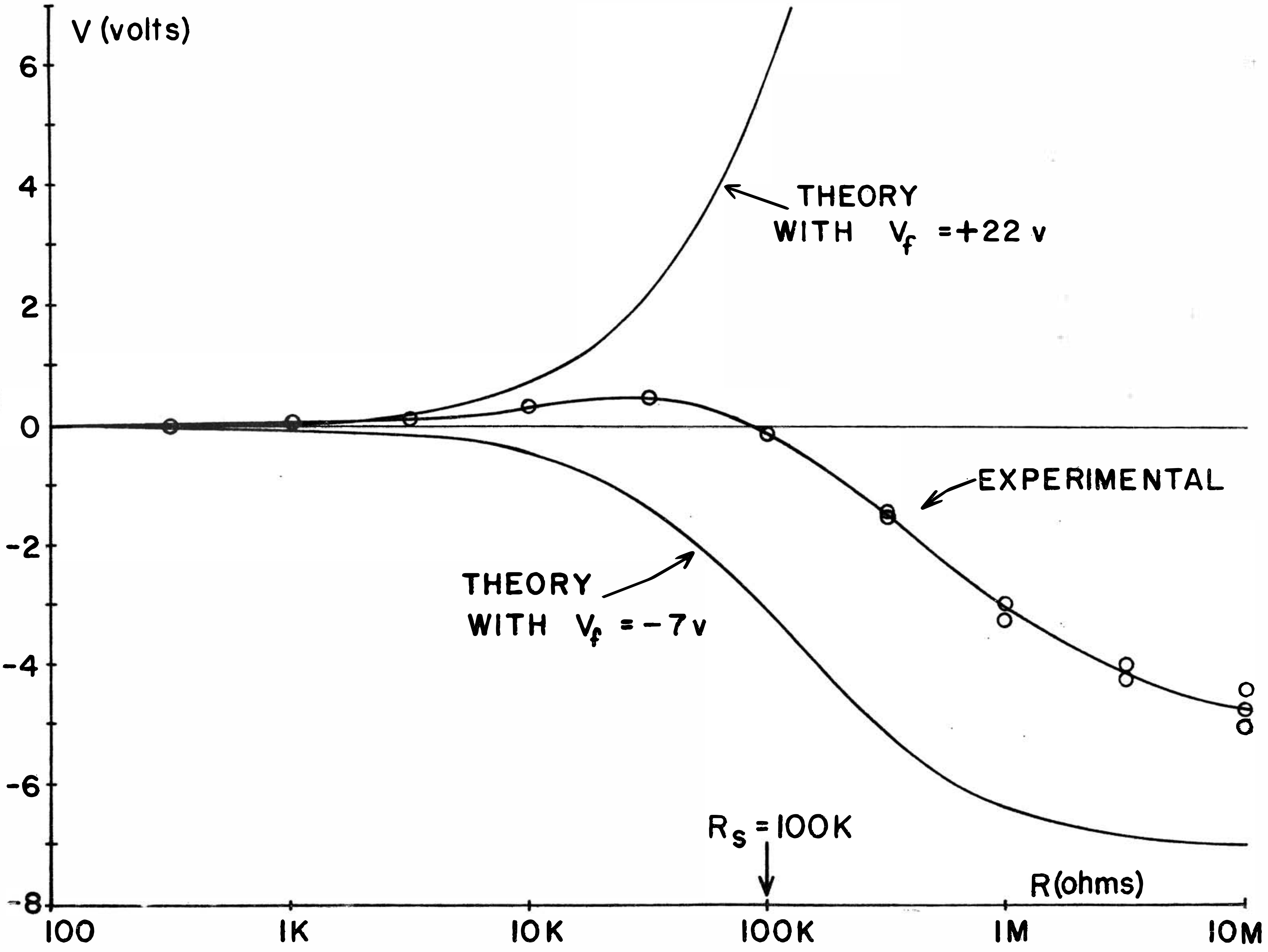
FIGURE 3



16
V (volts)
12
8
4
0
-4
-8
LINEAR
APPROXIMATION
EXACT SOLUTION
LINEAR APPROXIMATION
EXACT SOLUTION
$R_S = 100K$
100
1K
10K
100K
1M
10M
R (ohms)

FIGURE 4

FIGURE 5



V (volts)
6
4
2
0
-2
-4
-6
-8
THEORY WITH V_f = +22 v
EXPERIMENTAL
THEORY WITH V_f = −7 v
R_s = 100K
R(ohms)
100
1K
10K
100K
1M
10M

PORT 2 $\rho = 0$

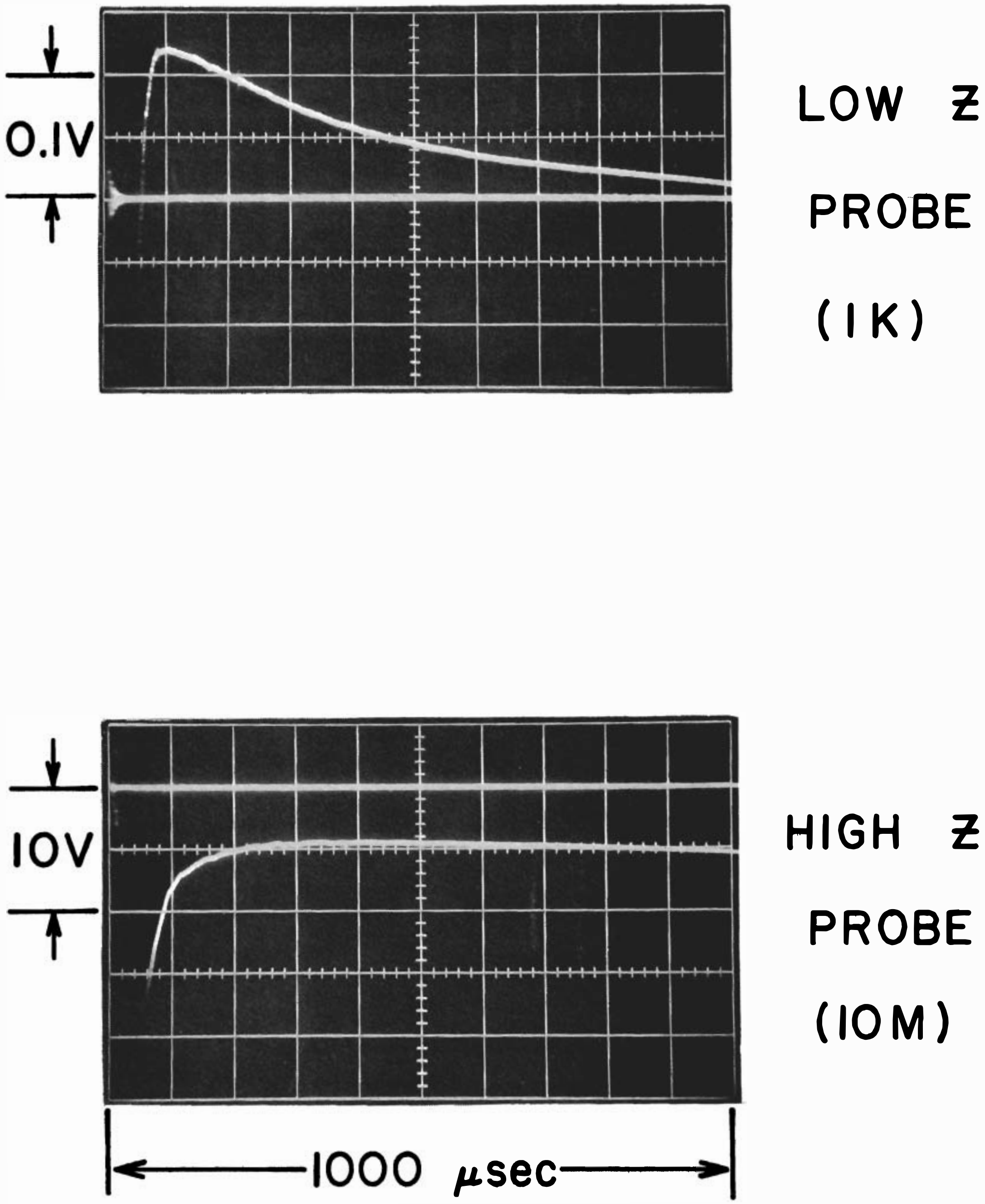


FIGURE 6