CONSIDERATIONS OF CYCLOTRON RESONANCE BREAKDOWN IN THE TOROIDAL OCTUPOLE

J. Clinton Sprott

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INTRODUCTION

As an alternative to injecting plasma into the octupole from an external gun, plasma can be produced within the field by filling the toroidal cavity with rf energy at microwave frequencies.[1] Preliminary tests indicate that this is a very simple and effective means of producing a well-behaved cold ion plasma with easily controlled properties.

The goal of this paper is to estimate theoretically the

1) spatial distribution
2) density
3) electron temperature, and
4) decay rate of the plasma which is produced for various values of
    a. microwave frequency
    b. power level
    c. pulse length
    d. magnetic field strength, and
    e. background gas pressure.

Because of the complicated geometry, however, exact calculations are generally too difficult and it will be necessary to resort to approximations and statistical arguments. In some cases the theoretical predictions will be compared with experimental results.

SPATIAL DISTRIBUTION

The rf power density required to ionize a volume of gas in the absence of any magnetic field can be estimated by equating the kinetic energy which an electron gains in a period of oscillation to the ionization energy of the gas:

$$\frac{1}{2}\ m\dot{x}_o{}^2 = eV_i$$

where $x_o = eE_o/m\ \omega$. Then the energy density is

$$U = \frac{1}{2}\ \varepsilon_o E_o{}^2 = \varepsilon_o \omega^2\ \frac{m}{e} V_i .$$

For a cavity with a Q of order unity the power density is given approximately by ωU or

$$\frac{dP}{dV} = \varepsilon_o\ \omega^3\ \frac{m}{e} V_i\ . \qquad (1)$$

For hydrogen, V_i = 15.8 eV and the power density required is

$$\frac{dP}{dV}\ (\text{watts/cm}^3) = 2 \times 10^{-26}\ f^3\ (\text{Hz}).$$

For microwave frequencies of 3 GHz (s band) and 10 GHz (x band) the required power densities are .54 and 20 kW/cm^3 respectively. Since the volume of the octupole is $2.5 \times 10^5 cm^3$, a total power of at least 135 MW would be required to produce a plasma in the absence of a magnetic field.

With a magnetic field, electrons can reach the ionization energy at much lower rf power levels provided the power is applied at the electron cyclotron frequency:

$$\omega_{ce} = \frac{eB}{m} \qquad (2)$$

or 2.8 MHz/gauss. For the power levels presently being contemplated ($\lesssim$ 1 MW), we can therefore rest assured that plasma will be produced only in those regions of the magnetic field where $\omega_{ce} = \omega$. Hence plasma is produced along surfaces of constant B in the octupole. A plot of lines of constant B in a cross sectional plane of the octupole is given in figure 1. B is normalized so that its value at the outside wall in the midplane is 1. At the normal operating field (2kV on the multipole capacitor bank) the numbers are approximately in units of kilogauss. Resonance occurs at ~1 kG for s band and ~ 3.5 kG for x band. Since B <7 kG everywhere in the field, only microwave frequencies less than 20 GHz can be used to produce plasma.

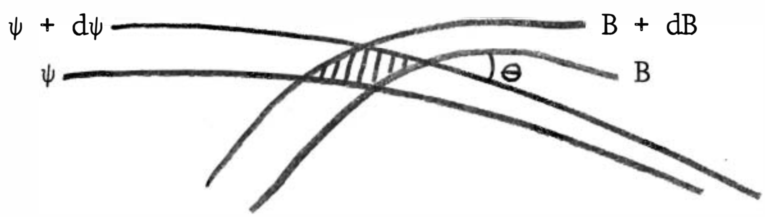
Of course plasma is not produced uniformly along a field line because the perpendicular component of the rf electric field varies along a field line. One would like to calculate the electric field distribution everywhere in the toroid. However, because of the complicated geometry and high mode number of the cavity, such a calculation is too difficult to perform. The mode number is given approximately by

$$N \cong \frac{V}{(\lambda/2)^3} = \frac{8f^3V}{c^3} \qquad (3)$$

or ~2000 for s band and ~74,000 for x band. Furthermore, the mode pattern changes drastically when the plasma density changes and when probes are moved.

Because of the complexity of the mode pattern it is possible to take a statistical approach and assume that these variations average out when integrated over an entire flux surface and that the amount of plasma produced on a flux surface is proportional to the fraction of the volume of the flux tube in which B is within dB of the resonance value.

Consider the intersection of the flux lines ψ and $\psi + d\psi$ with the constant $|\vec{B}|$ lines B and B + dB:



The area of the cross hatched region is given by

$$dA = \frac{d\psi dB}{|\nabla\psi x \nabla B|} \quad .$$

The volume is

$$dv = \frac{2\pi R d\psi dB}{|\nabla\psi x \nabla B|}$$

where R is the distance from the major axis to the point of intersection of B and ψ. In general, there will be several intersections over which a sum must be taken. The fraction of the flux surface which resonates is

$$\frac{dv}{dV} = \frac{1}{V'}\frac{dv}{d\psi} = \frac{2\pi R \; dB}{V'|\nabla\psi x \nabla B|} \tag{4}$$

where $V'(\psi) = \frac{dV}{d\psi} \propto \oint \frac{d\ell}{B}$ is a function of considerable importance in stability calculations. $V'(\psi)$ has been calculated for the octupole and is shown in figure 2.

If we assume that the density along a ψ-line becomes constant in a time short compared with any drifts or instabilities which lead to transport of plasma across the field, we expect the initial density distribution in ψ space to be proportional to the expression in equation (4):

$$n(\psi) \propto \frac{R}{V' \left| \nabla\psi x \nabla B \right|} \quad . \tag{5}$$

Two features of the density distribution are particularly worth noting:

1) The density should be zero on the separatrix since $V' \to \infty$ there.

2) The density should be very high wherever a ψ-line is tangential to the resonant constant B line since $\nabla\psi x \nabla B = 0$ there.

Computer obtained plots of $n(\psi)$ from equation (5) for various values of B (and hence ω) are shown in figures 3 - 7. The singularities have been removed by drawing a smooth curve which ignores peaks less than $\sim\frac{1}{4}$ of a ψ unit in width. Note that for low values of B, the plasma is deposited about equally on each side of the separatrix, but as the frequency increases, a greater proportion of the plasma is deposited near the hoops.

An experimentally obtained distribution is compared in figure 9 with the theoretically predicted distribution for f = 3250 MHz (1.16 kG). The agreement is good. Note particularly the minimum on the separatrix, the peaks at ψ = -3 and + 1.5, and the steep density gradient between

$\psi = -3$ and the hoops. Between $\psi = +3$ and the wall, the curves disagree, presumably because the plasma is unstable there and is rapidly lost to the walls.

We have seen that it should be possible to produce plasma with quite a wide range of spatial distributions by varying the microwave frequency (or more conveniently by varying B). Note that since the gyroradii of the electrons are quite small compared with the ion gyroradii from the gun, it is not necessary to run the magnetic field high to get long plasma lifetimes. It is possible to produce plasma only between the separatrix and the hoops, or to produce it only around the inside hoops. A distribution peaked on the separatrix, however, is not obtainable this way.

In general a distribution peaked off the separatrix would not be expected to be stable. Consequently, many of the distributions which can be produced would not persist for any appreciable time. It is possible, however, for the magnetic field to support a density distribution which is peaked off the separatrix provided the density gradient $n' = \frac{dn}{d\psi}$ is not too large. The condition for interchange stability including the finite pressure term is given by [2,3],

$$\frac{dE}{d\psi} = p'V'' + p\gamma \frac{(V'')^2}{V'}$$

$$= pV'' \frac{d}{d\psi} \ln (pV'^{\gamma}) < 0. \qquad (6)$$

The maximum stable inverted density gradient is obtained by setting $\frac{dE}{d\psi} = 0$ or

$$pV'^{\gamma} = \text{const.}$$

For a plasma without temperature gradients,

$$n(\psi) \propto V'^{-\gamma} \quad . \tag{7}$$

A plot of equation (7) for $\gamma = \frac{5}{3}$ and for $\gamma = 2$ is shown in figure 10.

The choice between the two values of γ depends upon whether the plasma should be considered two or three dimensional. The plasma is three dimensional wherever the average length of a field line between regions of good and bad curvature is short compared with the distance a particle travels during a half period of a fluctuation:

$$\ell < \frac{v}{2f} \tag{8}$$

For a 10 eV electron, $v \cong 2 \times 10^8$ cm/sec and ℓ is typically 10 cm. Hence for fluctuations below ~10 MHz the plasma behaves like a three dimensional gas with $\gamma = 5/3$.

From figure 10, therefore, it is apparent that most of the density distributions of figures 3 - 7 are unstable. It is experimentally observed that such distributions quickly readjust themselves until they satisfy the stability condition of equation (6).[1] In some cases the distribution overshoots the critical slope given by equation (7). The density distribution in figure 9, for example, becomes nearly flat after a very short time with only a slight peak at $\psi = -2$.

After the initial collapse, various density distributions can be obtained. Figure 11 shows a sample of the distributions which have been obtained experimentally for various frequencies and magnetic field strengths. Note that they all satisfy the stability condition of figure 10. All of these distributions decay more or less uniformly as the plasma decays.

DENSITY

The maximum plasma density which can be obtained from a microwave pulse of power P and duration τ is obtained from energy conservation:

$$U = \tau \frac{dP}{dV} = n(eV_i + kT_i + kT_e) \quad (8)$$

where V_i is the ionization potential, τ_i is the ion temperature and T_e is the electron temperature. For a zero temperature hydrogen plasma,

$$n(cm^{-3}) = 1.6 \times 10^9 \; P(kW) \; \tau(\mu sec)$$

In one experimental case, a magnetron input power of about 5 kW for 90 μsec produced a kT_e = 10 eV plasma with a density of about 10^9 cm^{-3} 500 μsec after breakdown. From the measured decay rate of density we can extrapolate back to get a density of ~2×10^9 cm^{-3} at t = 0. These numbers allow us to say that in this case about .25% of the dc input power to the magnetron actually appears as energy in the trapped plasma. That is,

$$\eta_M \, \eta_P \, \eta_T \approx 0.25\%$$

where η_M is the efficiency of the magnetron, η_P is the fraction of the rf energy leaving the magnetron which is absorbed by the plasma and η_T is the trapping efficiency of the produced plasma. This efficiency may seem low but it can be compared with the conical pinch gun which produces a 50 eV plasma (T_i ~40 eV, T_e ~10 eV) of about the same density at the expense of 1450 joules of dc stored capacitor energy, giving an efficiency of ~2.5×10^{-6}. Hence from a power standpoint, the microwave plasma is produced about 1000 times more efficiently than the gun plasma.

There are two additional effects which tend to limit the plasma density which can be obtained. When the density reaches the point where the electron plasma frequency is comparable to the microwave frequency, the rf does not penetrate the plasma. This occurs at

$$\omega_{Pe} = \sqrt{\frac{ne^2}{\varepsilon_o m}} \qquad (9)$$

or

$$f(Hz) = 9000 \sqrt{n(cm^{-3})} \; .$$

Hence the limiting density is ~10^{11} cm^{-3} for s band and ~10^{12} cm^{-3} for x band microwaves.

A second consideration is the number of neutral molecules available for ionization. The density of a gas is related to its pressure by

$$p = nkT. \qquad (10)$$

At T = 300°k,

$$n(cm^{-3}) = 3.2 \times 10^{16} \; p(Torr).$$

Hence a pressure of at least 1.7×10^{-7} Torr for s band or 1.7×10^{-6} Torr for x band is required to make full use of the microwave energy available. However, to avoid very long microwave pulses, considerably higher pressures are necessary. There is a limit to how high a background pressure can be tolerated. The lifetime of the gun plasma is seriously affected for $P \gtrsim 10^{-4}$ Torr. The cold ion microwave plasma should not be so seriously affected but for practical reasons high pressures are undesirable. Furthermore, at high background pressures, the breakdown can be inhibited.

Consider a model in which a single zero energy electron is present in a hydrogen gas of density n. If the electron is in the resonance region, its velocity will increase linearly as the microwaves are applied:

$$v = \frac{e}{m} \bar{E} t = at. \qquad (11)$$

The acceleration a is a constant related to the power density of microwaves by

$$a = \frac{e}{m} \bar{E} = \frac{e}{m} \sqrt{\frac{1}{\varepsilon_o \omega} \frac{dP}{dV}} \qquad (12)$$

Equation (12) is plotted in figure 12 for the octupole for the two frequencies of interest. The acceleration is typically 10^{15} cm/sec^2.

If an electron is to reach the ionization energy $\frac{1}{2} m v_o^2$, the acceleration must be fast enough that the electron does not elastically scatter off a hydrogen molecule in a time $\tau_o = v_o/a$. The scattering time τ_s is given by

$$\tau_s = \frac{1}{n \sigma_s v}$$

The scattering cross section[4] σ_s is nearly constant for low energies and is approximately 1.2×10^{-15} cm^2. Hence the breakdown condition is

$$\frac{n}{a} < \frac{1}{\sigma_s v_o^2} \quad . \qquad (13)$$

Since $v_o = 2.4 \times 10^8$ cm/sec, n must be less than about 1.4×10^{13} cm^{-3}, or $p < 4 \times 10^{-5}$ Torr. In practice, this requirement is probably somewhat too stringent because the decrease in the ionization time compensates for the losses through scattering. Breakdown seems to be most efficient at $P \sim 10^{-4}$ Torr.

Now we are in a position to consider the microwaves pulse length required to raise the density of electrons in a gas up to the point where $\omega_{Pe} = \omega$. We will assume that the background gas pressure is high enough to supply the required electrons but not so high that elastic scattering becomes important (i.e.: $1.7 \times 10^{-7} \leq P \lesssim 4 \times 10^{-5}$ Torr). Again consider an electron starting from rest uniformly accelerated with acceleration a. The probability that such an electron will undergo an ionizing collision in a time dt is

$$dP = n\sigma(v)\ v\ dt.$$

In one collision time τ_i, the probability of a collision is $1 - \frac{1}{e}$. Therefore

$$1 - \frac{1}{e} = \int_0^{\tau_i} n\sigma(v)\ v\ dt \tag{14}$$

where $v = at$ and $\sigma(v)$ is given by the standard form[5]:

$$\sigma(v) = 2\sigma_o \left(\frac{v_o^2}{v^2} - \frac{v_o^4}{v^4} \right) \ln\,(1.12\ v/v_o)$$

where $\sigma_o = 9.4 \times 10^{-16}\ cm^2$ for electrons on hydrogen. Inserting this cross section into equation (14) gives

$$\frac{n}{a} = 0.061\ \left[2\left(\ln \frac{1.12\tau_i}{\tau_o}\right)^2 + \frac{\tau_o^2}{\tau_i^2}\left(2\ln \frac{1.12\tau_i}{\tau_o} + 1\right) - 1.25\right]^{-1}. \tag{15}$$

where $\tau_o = v_o/a$.

Equation (15) gives the ionization time τ_i in terms of the density n. A graph of equation (15) is given in figure 13. Figure 12 and figure 13 together allow one to predict the ionization time in terms of the microwave frequency and power level and the background gas density. As an example, consider a 100 kW s band rf source. This provides an acceleration of $\sim 10^{15}$ cm/sec^2. At a pressure of 2×10^{-5} Torr, $a\tau_i \approx 2 \times 10^9$ or $\tau_i \approx 2$ μsec.

The number of electrons after a time t can be obtained from τ_i by assuming that the neutral density does not change appreciably during the time t. Then

$$N = e^{t/\tau_i} . \qquad (16)$$

Or in terms of N,

$$t = \tau_i \ln N = \tau_i \ln(nV) \qquad (17)$$

If, for example, one wishes to build up a density of 10^{10} cm^{-3} in the toroid ($V = 2.5 \times 10^5$ cm^3), the time required would be ~ 70 μsec using the previous numerical example. These predictions agree amazingly well with what is observed experimentally.

If sufficiently long microwave pulses cannot be obtained, high densities can still be achieved by introducing some preionization by injecting a low density gun plasma or by producing electrons from a hot filament. The hot filament has the additional advantage that electrons can be put only on certain flux surfaces and hence the plasma could presumably be produced with essentially a delta function distribution in ψ space.

In practice, it has been possible to build up electron densities of $\sim 10^6$ cm^{-3} in the octupole using a hot filament. If the microwave pulse is applied in the presence of a background density n_o of electrons, the density at time t is

$$n = n_o e^{t/\tau_i} \tag{18}$$

and the time required to achieve a density n is

$$t = \tau_i \ \ell n \frac{n}{n_o} . \tag{19}$$

In the previous example, a 10^{10} cm^{-3} plasma could be produced in ~ 10 μsec, if one started with 10^6 electrons/cm^3. This is a considerable improvement over the 70 μsec required with no preionization. If there is no preionization off the field line where one wishes to produce the delta function distribution, the density should be only $\sim 6 \times 10^{-2}$ cm^{-3}. This treatment is clearly rather idealized and it is doubtful that such a sharply peaked function could be experimentally produced. However, a distribution which even remotely approximated this ideal would be of tremendous value in studying the diffusion process.

In practice, it is usually not possible to build up the density everywhere at once. Plasma is produced only in the resonance zones and then must collapse by instability to fill the volume. If the microwave pulse is long enough to build the density up to the value where $\omega_{Pe} = \omega$ but not so long that appreciable spatial redistribution takes place, the final density that one obtains is proportional to the fraction of the cavity in which resonance occurs. This fraction can be calculated by integrating equation (4) over ψ space:

$$\frac{dv}{V} = \frac{2\pi \ dB}{V} \int \frac{R}{|\nabla\psi x \nabla B|} d\psi \qquad (20)$$

The quantity $\frac{dv}{dB}$ in equation (20) has been calculated by computer by integrating between $\psi = -5$ and $\psi = +5$. The result is shown in figure 14. Since the plasma between $\psi = +3$ and $\psi = +5$ is flute unstable and is rapidly lost, a more realistic estimate of dv/dB.is obtained by integrating from $\psi = -5$ to $\psi = +3$. This function is also shown in figure 14. Note that the production efficiency is high for f = 3GHz (s band) and f = 10 GHz (x band) and that it falls to zero at f = 0 (zero frequency) and at f $\approx$ 20 GHz. Qualitative agreement with this prediction has been obtained by keeping the rf frequency constant and varying the magnetic field.

ELECTRON TEMPERATURE

We now turn our attention to the more difficult problem of estimating the kinetic energy of the electrons which have been cyclotron heated by the rf pulse. We will first estimate the electron energy at a time corresponding to the end of the microwave pulse and then conclude by examining the way in which the electron temperature decays after the rf is turned off.

The simplest possible model would ignore collisions and energy loss mechanisms and would require that the electron be continously accelerated with acceleration a given by equation (12) for a time τ equal to the duration of the microwave pulse. This approximation would lead to an energy

$$E = \frac{1}{2} m a^2 \tau^2. \tag{21}$$

For a typical acceleration of 10^{15} cm/sec^2 and a pulse length of 10 μsec, we obtain E = 30 keV.

A more realistic treatment would include the fact that the magnetic field is non-uniform and hence when the particle energy increases, the gyroradius increases and the gyrofrequency gets out of tune with the driving force. The equation of motion of an electron in a non-uniform magnetic field is

$$m\ddot{\vec{r}} = e \dot{\vec{r}} \times \vec{B}(\vec{r}) + \vec{F}_o \cos \omega t$$

where $\vec{F}_o$ is the driving force ($=e \vec{E}_o$). If we assume that the driving force is perpendicular to $\vec{B}$, we can separate the equation of motion into components:

$$m\ddot{x} = e\dot{y}B + F_o \cos \omega t$$

$$m\ddot{y} = e\dot{x}B.$$

Now assume that B can be expanded in the form

$$B(x) = B_o + x\nabla B + \frac{1}{2} x^2 \nabla^2 B + \ldots\ldots\ldots .$$

Substituting this form of B into the above equations and eliminating y leads to

$$\ddot{x} + \omega_o^2 x + \frac{3}{2} \omega_o^2 (\frac{\nabla B}{B}) x^2 = \frac{F_o}{m} \cos \omega t \qquad (22)$$

where terms of order $\frac{x^3 \nabla^2 B}{B^3}$ have been neglected. Equation (22) is the equation of a non-linear oscillator with an unsymmetric restoring force. It can be solved [6] by assuming a solution of the form

$$x = \rho \cos \omega t + \rho_o$$

and adjusting ρ and ρ_o in such a way that equation (22) is satisfied. If higher harmonic terms (cos 2 ωt) are ignored, the solution reduces to the simple form:

$$\frac{9}{4} \omega_o^2 \rho^3 (\frac{\nabla B}{B})^2 + (\omega^2 - \omega_o^2) \rho - a = 0 \qquad (23)$$

where a is the acceleration $(\frac{eE_o}{m})$ given by equation (12).

Equation (23) allows us to determine the gyroradius ρ and hence the energy of a particle at a point in the field where the cyclotron frequency is ω_o. Because higher harmonic terms have been neglected, equation (23) is valid only for $\omega^2 - \omega_o^2 << \omega_o^2$. In particular, equation (23) allows us to calculate the gyroradius of a particle which is exactly in resonance

$(\omega^2 = \omega_o^2)$. Its value is

$$\rho = [\frac{4a}{9\omega_o^2} (\frac{B}{\nabla B})^2]^{\frac{1}{3}} . \qquad (24)$$

Consider as an example an electron that happens to be at the point where the 1 KG B surface intersects the midplane in the octupole at $\psi = 0.5$ in figure 1. Such an electron would be trapped in a mirror field and would gain energy until its gyroradius was equal to the value in equation (24). At this point in the field $\frac{B}{\nabla B} \approx 5$ cm. Consider an s-band rf source with $a = 10^{15}$ cm/sec ^{2}. The resulting gyroradius would be 0.32 mm. The small gyroradius justifies neglecting terms of order $x^3 \nabla^2 B/B^2$.

In the non-relativistic limit the particle energy is related to the gyroradius by

$$E = \frac{1}{2} m \omega_o^2 \rho^2$$

and so the energy of a particle in the resonance zone will be

$$E = 0.29 \ m \ [a\omega_o (\frac{B}{\nabla B})^2]^{\frac{2}{3}} . \qquad (25)$$

For the previous numerical example, equation (25) predicts an electron energy of only ~100 eV.

Note that equation (25) is a steady state solution and hence would not be valid for very short pulses. If the pulse is so short that the energy calculated from equation (21) is less than that calculated from equation (25), an equilibrium will not be reached and the electron energy will be given by equation (21). Equation (25) should be valid for

$$\tau > \frac{0.76}{a} [a \omega_o (\frac{B}{\nabla B})^2]^{\frac{1}{3}} \qquad (26)$$

or about 0.5 μsec in the present example.

Equation (25) illustrates the futility of trying to produce a high temperature plasma by microwave heating in the toroidal octupole. Since the electron energy varies inversely with the $\frac{4}{3}$ power of ∇B, and since the toroidal octupole was purposely designed to have a deep magnetic well, electron cyclotron heating is exceedingly difficult. Furthermore, the situation is further aggrevated by the fact that a large field gradient leads to narrow resonance bands and hence the fraction of the volume in which heating takes place is quite small. This situation is in direct contrast to magnetic mirror geometries where the field gradients are small and electron cyclotron heating is extremely effective.[7]

It might be hoped that cyclotron heating would take place at those points in the field where $\vec{B}$ is parallel to ∇B (i.e. where a constant B line intersects a ψ line at right angles in figure 1) since the particle could have a very large gyroradius without getting out of tune with the driving field. Unfortunately a particle in such a field is accelerated parallel to the field with an acceleration

$$\dot{v}_z = \frac{1}{2} v_\perp^2 \frac{\nabla B}{B} \qquad (27)$$

where $v_\perp$ is the velocity perpendicular to $\vec{B}$ given by $v_\perp = at$. Equation (27) can be integrated twice to give

$$z = \frac{1}{24} a^2 \frac{\nabla B}{B} t^4 .$$

A particle which has drifted a distance z is out of resonance by an amount

$$\Delta\omega = \omega_o z \frac{\nabla B}{B}$$

and its amplitude will wow with a period $T = 2\pi/\Delta\omega$. The particle will cease to gain energy from the electric field when $1/\Delta\omega$ is about equal to the duration of the pulse. Combining the above equations with $\Delta\omega = \frac{1}{t}$ gives

$$t = \left[\frac{24}{\omega_o a^2} \left(\frac{B}{\nabla B}\right)^2\right]^{\frac{1}{5}} \quad . \tag{28}$$

Taking $a = 10^{15}$ cm/sec^2 and $\frac{B}{\nabla B} = 5$ cm as in the previous case, we obtain for s band microwaves the result that the particle gains energy only for a time $t = 3.2 \times 10^{-8}$ sec. In this time the electron will have acquired an energy of $E \simeq 0.3$ eV. Therefore heating is especially ineffective where $\vec{B}$ is parallel to ∇B.

The preceding discussion assumed that the electron did not scatter off other particles during the time in which the acceleration was taking place. Equation (26) showed that equilibrium was reached in times of the order of 0.5 μsec. Hence equation (25) should be valid only when

$$\tau_s > \frac{0.76}{a} \left[a\omega_o \left(\frac{B}{\nabla B}\right)^2\right]^{\frac{1}{3}} \tag{29}$$

where τ_s is given by

$$\tau_s = \frac{1}{n\sigma_s v} \quad .$$

In the range above a few eV, σ_s is proportional to $1/v$ and τ_s is a function only of the background gas pressure:[4]

$$\tau_s(\text{sec}) = \frac{2.5 \times 10^{-10}}{p(\text{torr})} . \tag{30}$$

Hence scattering becomes important only at pressures above about 5×10^{-5} torr. At higher pressures we can probably assume that the electron accelerates uniformly between successive scatterings and that its effective energy is reduced to zero after a time τ_s since its velocity becomes uncorrelated with the driving electric field in that time. The probability that a particle will have velocity v in dv is then

$$P(v)\ dv = \frac{dt}{\tau_s} .$$

The mean square velocity can then be calculated:

$$\overline{v^2} = \int_0^{v_s} v^2\ P(v)\ dv = \frac{1}{\tau_s} \int_0^{\tau_s} v^2\ (t)\ dt.$$

Substituting v = at gives

$$\overline{v^2} = \frac{a^2}{\tau_s} \int_0^{\tau_s} t^2\ dt = \frac{1}{3} a^2\ \tau_s^{\ 2}.$$

The electron energy corresponding to this velocity is

$$E = \frac{1}{2}\ m\overline{v^2} = \frac{1}{6}\, ma^2\ \tau_s^{\ 2} \tag{31}$$

or, numerically,

$$E(\text{eV}) \cong 2.5 \times 10^{-39} \left(\frac{a}{p} \right)^2 .$$

Hence it is extremely difficult to obtain high temperature plasmas in the presence of high background pressures by electron cyclotron heating. Since gas breakdown requires high pressures as previously shown, high temperature plasmas are most easily obtained by using a low background pressure and some preionization.

The only place in the octupole field which is free of gradients in all directions is the zero field axis. However, it is impossible to cyclotron heat plasma at a point where the field vanishes. With a superimposed toroidal (B_θ) field, however, there would be a point in the field where $\nabla B = 0$.

If the B_θ field were uniform, this point would be on the minor axis. However, the B_θ field in the toroidal octupole has the form

$$\vec{B}_\theta = \frac{A_1}{R}\,\hat{\theta}$$

where R is distance to the toroid's major axis. Near the minor axis, the octupole field increases like r^3 and in the midplane is given by

$$\vec{B}_\phi = \frac{A_2}{R}(R - R_o)^3 \;\hat{\phi}$$

where R_o = 43.2 cm. The total magnetic field in the midplane has a magnitude

$$B = \sqrt{\left(\frac{A_1}{R}\right)^2 + \left(\frac{A_2}{R}\right)^2 (R - R_o)^6}\;. \tag{32}$$

The requirement that $\nabla B = 0$ leads to the fifth order equation,

$$R_o(R - R_o)^5 = \frac{1}{3}\left(\frac{A_1}{A_2}\right)^2 \;. \tag{33}$$

Note that R depends only on the ratio A_1/A_2 (i.e.: only on the relative strength of the two fields at some reference point). In our experiment this ratio can be varied from zero up to some maximum value. The values of B_ϕ, B_θ (for maximum A_1/A_2), and B in the midplane of the octupole are plotted in figure 15. Note that $\nabla B = 0$ at a point about 5 cm from the minor axis toward the outside wall. The value of B at this point is about 335 gauss at the normal operating amplitude, but it can be adjusted to any value between zero and 335 gauss. To resonate in this region would require a microwave source with a frequency of 940 MHz or less.

To estimate the energy to which a particle at that point in the field would be raised, we perform a calculation similar to the one which yielded equation (25). Assume a B field of the form

$$B(x) = B_o + \frac{1}{2} x^2 \nabla^2 B$$

with $\nabla^2 B$ = const. Substituting this form of B into the equation of motion gives

$$\ddot{x} + \omega_o^2 x + \frac{3}{4} \omega_o^2 \left(\frac{\nabla^2 B}{B} \right) x^3 = \frac{F_o}{m} \cos \omega t. \qquad (34)$$

This is a standard non-linear equation known in the literature as Duffing's equation. It can be solved by a perturbation method[6] similar to that used to obtain equation (23). The result is

$$\frac{9}{16} \omega_o^2 \rho^3 \left(\frac{\nabla^2 B}{B}\right) + (\omega^2 - \omega_o^2) \rho - a = 0. \qquad (35)$$

A particle on resonance will therefore have a gyroradius

$$\rho = \left[\frac{16a}{9 \omega_o^2} \left(\frac{B}{\nabla^2 B}\right) \right]^{\frac{1}{3}} \qquad (36)$$

and an energy

$$E = 0.73\ m\ [a\omega_o\ (\frac{B}{\nabla^2 B})]^{\frac{2}{3}} \quad . \tag{37}$$

The value of $\frac{B}{\nabla^2 B}$ can be obtained by differentiating equation (32) twice and imposing the condition that $\nabla B = 0$ (equation (33)). The result is

$$\frac{B}{\nabla^2 B} = [\frac{3R_o^3 + 7R_o^2\ r + 5R_o r^2 + r^3}{15R_o^2 + 27R_o r + 7r^2}]\ r \doteq \frac{1}{5} R_o r \tag{38}$$

where the latter approximation is valid for $r << R_o$ where $r = R - R_o$. In the case under consideration, $R_o = 43.2$ cm and $r = 5$ cm. If we take $\omega/2\pi = 1$GHz and $a = 10^{15}$ cm/sec^2 we obtain from equation (37) an energy of about 200 eV. Hence, even in this case, cyclotron heating would not be too effective.

DECAY RATE

In this final section we consider the question of what happens to the plasma after the microwave energy is turned off. We expect the plasma to be 1) non-Maxwellian, 2) anisotropic (since all the energy has been put into motion perpendicular to B), and 3) unstable (since plasma is not produced on the separatrix). The time required for a non-Maxwellian distribution to thermalize and the time required for any anisotropy to disappear should be about equal to each other and to the electron-electron collision time. The electron-electron collision time is given by Spitzer[8]:

$$\tau_{ee} = \frac{0.266\ T_e^{3/2}}{n\ \ell n\ \Lambda} \quad . \tag{39}$$

For 10eV electrons,

$$\tau_{ee} = \frac{4.6 \times 10^{15}}{n} \quad .$$

For a density of $n = 10^{11}\ cm^{-3}$, $\tau_{ee} = 4.6\ \mu sec$. Hence the plasma should become Maxwellian very quickly after the microwaves are turned off.

The rate at which the plasma achieves a stable configuration is more difficult to estimate. From the diffusion equation

$$n\vec{v}_D = -D\nabla n, \tag{40}$$

we can express the diffusion velocity v_D in terms of the diffusion coefficient D:

$$\vec{v}_D = -D\frac{\nabla n}{n} \quad .$$

If we take for D the collisional diffusion rate, then $v_D \simeq 1$ cm/sec, and the distribution would not change appreciably for the duration of the experiment ($\sim$1 msec). A more realistic estimate would assume that D is equal to the Bohm diffusion coefficient:

$$D = \frac{kT_e}{16eB} . \tag{41}$$

Most experiments in which hydromagnetic instabilities are present (including the octupole outside ψ_{crit}) have diffusion which is close to that given by equation (41).

For 10 eV electrons and a typical magnetic field of 500 gauss, the diffusion velocity is $\sim 10^4$ cm/sec for $n/\nabla n = 10$ cm. Hence Bohm diffusion should stabilize the density distribution in a time of $\sim$1 msec. In practice, however, the plasma generates potential fluctuations an order of magnitude higher than kT_e/e and collapses to a stable configuration in 50 - 100 μsec.[1] We will not investigate the turbulent period during which the plasma readjusts itself, but will consider what happens to the density and temperature after the plasma has become Maxwellian, and achieved a stable spatial distribution.

We first consider hanger loss since we consider this to be the main loss mechanism for the gun injected plasma. The continuity equation is

$$\nabla \cdot \vec{j} + \frac{\partial \rho}{\partial t} = 0 \tag{42}$$

where the current j to any object in the plasma is given by

$$j = \frac{1}{4} n e\bar{v}.$$

Applying the divergence theorem gives

$$jA = \oint \vec{j} \cdot d\vec{s} = - \int_V \frac{\partial \rho}{\partial t} dV = - eV \frac{dn}{dt} .$$

Substitution for j gives a differential equation for the density n(t):

$$\frac{dn}{dt} + \frac{1}{4}\bar{v}\left(\frac{A}{V}\right) n = 0,$$

which has as a solution

$$n(t) = n_o e^{-t/\tau}. \tag{43}$$

Hence we obtain the result that the density n of particles of velocity v should decay exponentially with a lifetime τ given by

$$\tau = \frac{4}{\bar{v}}\left(\frac{V}{A}\right) . \tag{44}$$

V is the containment volume and A is the area of the hangers in contact with the plasma.

The above treatment has neglected the presence of any magnetic field. The magnetic field has two effects: 1) it changes the effective collecting area, and 2) it makes the lifetime a function of ψ:

$$\tau(\psi) = \frac{4}{\bar{v}} \frac{dV}{dA} = \frac{4}{\bar{v}} V'(\psi) / \frac{dA}{d\psi} \tag{45}$$

Note particularly that $\tau \to \infty$ on the separatrix since $V'(\psi) \to \infty$ there and $dA/d\psi$ is finite (and approximately constant).

When the finite gyroradius of the ions is taken into account, the lifetime on the separatrix will no longer be infinite but it should be appreciably greater than the lifetime off the separatrix. The fact that this effect is not observed[9] either with the gun plasma or with the microwave plasma (which has very small gyroradii) leads to the conclusion that diffusion must play a major role in plasma loss from the separatrix. This fact is not too surprising since both Bohm and collisional diffusion rates go to infinity when B = 0. Since the decay rate seems to be fairly independent of ψ we will use the lifetime given by equation (44) where 4V/A is an effective mean free path to be determined experimentally.

Taken at its face value, equation (44) gives a lifetime of 15 msec for 40 eV ions. The actual decay rate observed with an electrostatic energy analyzer[10] has about 600 μsec. Hence the magnetic field apparently plays a large role in determining/the effective mean free path.

When the electron temperature exceeds the ion temperature as in the case of the microwave plasma, ions are collected at a rate faster than that given by their thermal velocity.[11] In this case, the sheath criterion[12] requires that the ions be collected with an effective velocity of approximately

$$v \cong \sqrt{\frac{kT_e}{m_i}} = \sqrt{\frac{m_e}{2\, m_i}}\; v_e . \qquad (46)$$

From equation (46) it is easily shown that a zero temperature ion plasma should have a lifetime only $2\sqrt{2}$ times longer than a 40 eV plasma with the same electron temperature (10 eV) provided we neglect the effect of the magnetic field on the effective hanger collection area. In fact, the microwave plasma appears to have a lifetime only about 10 - 20% longer than the gun plasma.

We will now assume that the mean free path of the electrons λ is a constant so that the rate of plasma loss of particles of velocity v is proportional to v/λ. We will further assume that the thermalization time is short so that the electron velocity distribution function f(v) never departs significantly from a Maxwellian:

$$f(v) = \frac{4N\ v^2}{\sqrt{\pi}} \left(\frac{m}{2kT}\right)^{3/2} e^{-\frac{mv^2}{2kT}} \tag{47}$$

Following a derivation by Erickson[10], we write the total rate of particle loss as

$$\frac{dN}{dt} = - \int_0^\infty \frac{v}{\lambda} f(v)\ dv = - \frac{2N}{\lambda} \sqrt{\frac{2kT}{\pi m}} \tag{48}$$

where the temperature T is also time dependent. Similarly the energy loss rate is

$$\frac{dE}{dt} = - \int_0^\infty \frac{1}{2} E_o \frac{v^3}{\lambda}\ f(v)\ dv = - \frac{4NE_o kT}{\lambda m} \sqrt{\frac{2kT}{m}}$$

Since the energy is related to the temperature by

$$E = \frac{3}{2} N\ kT,$$

the above two equations can be combined to give

$$\frac{N}{N_o} = \left(\frac{E}{E_o}\right)^3 = \left(\frac{T}{T_o}\right)^3 .$$

Substituting this relation into equation (48) gives the solutions

$$T = \frac{T_o}{(1 + .188\ \bar{v}_o t/\lambda)^2} \tag{49}$$

$$N = \frac{N_o}{(1 + .188\ \bar{v}_o\ t/\lambda)^6} . \tag{50}$$

Also of interest is the ion saturation current drawn by a Langmuir probe which is proportional to dN /dt or

$$I = \frac{I_0}{(1 + .188\ \bar{v}_0 t/\lambda)^7} \tag{51}$$

where $\bar{v}_0$ is the average electron velocity at t = 0.

These three quantities are plotted in figure 16. The temperature decays considerably more slowly than does the density, and the ion saturation current is very nearly exponential up to times of the order of $\lambda/\bar{v}_0$. This kind of departure from an exponential decay is frequently observed for microwave plasmas but sheath expansion at low densities would have the same effect so it cannot be said that this behavior has been experimentally observed. Of particular interest is the fact that this derivation predicts that the electron temperature should fall ~45% during the time that it takes the density to decay to 1/e of its original value. In fact the electron temperature does decay about 40% in this time provided the background pressure is not too high. Also note the actual lifetime of the density is 15-20% longer than the lifetime of the ion saturation current read by a Langmuir probe.

At the high background pressures used with microwave plasmas, we expect ionization and excitation losses to be appreciable. We can calculate the density change by evaluating the rate coefficient in a manner similar to the above derivation:

$$\frac{dN}{dt} = n \int_0^\infty \sigma(v)\ v\ f(v)\ dv \tag{52}$$

where f(v) is a Maxwellian and $\sigma(v)$ is given by

$$\sigma(v) = 2\sigma_o\left(\frac{v_o^2}{v^2} - \frac{v_o^4}{v^4}\right) \ell n(1.12\ v/v_o)$$

for $v > v_o$ where v_o is the velocity of a 15.8 eV electron. Equation (52) cannot be evaluated in closed form, but it can be expressed as

$$\frac{dN}{dt} = 1.85 \times 10^{10}\ u^{\frac{1}{2}}\ \psi(u)\ Np \qquad (53)$$

where $u = E_i/E$.

The function $\psi(u)$ has been calculated by computer and is available in tabulated form.[13] $\psi(u)$ can be approximated by

$$\psi(u) \cong \left[\frac{e^{-u}}{1 + u}\right] \left[\frac{1}{20 + u} + \ell n(1.25\{1 + 1/u\})\right].$$

The quantity $u^{\frac{1}{2}}\ \psi(u)$ has been plotted vs electron temperature in figure 17. Note that at high temperatures the rate coefficient is large and fairly constant but that as the temperature falls below the ionization energy (15.8 eV) the rate decreases sharply. This is physically what would be expected since at low temperatures only a small fraction of the Maxwellian tail participates in ionization loss. The plasma density then should increase like

$$N = N_o\ e^{pt/\tau} \qquad (54)$$

where τ is itself time dependent and is given by

$$\tau = \frac{1}{1.85 \times 10^{10}\ u^{\frac{1}{2}}\ \psi(u)} \qquad (55)$$

At the same time, the temperature should decrease like

$$T = T_o e^{-pt/\tau} - 23 \frac{pt}{\tau} \text{ (eV)}. \qquad (56)$$

The first term is due to the fact that in every ionizing collision, the incident electron's energy is shared between two electrons and the second term is the energy lost in the ionization process.

Equations (54) and (56) can be solved simultaneously by using figure 17 if we make a stepwise linear approximation equation (54). To do this we start with some temperature, say 100 eV,
change slightly with a constant τ given by equation (55) evaluated at T_e = 100 eV. The new temperature is then used to find a new rate coefficient and the process continued until the temperature and density no longer decay appreciably. A graph of the time evolution of N and T_e obtained in this way is given in figure 18. The decay can be calculated starting at any other temperature by simply shifting the time axis horizontally. Note that the abscissa of figure 18 is in terms of pt where p is the background pressure in Torr. This figure should be applicable to any hydrogen plasma including that from the gun. For example, if we start at t = 0 with T_e = 10 eV and assume a background pressure of 5×10^{-7} Torr, we conclude that in a time of 1 msec the temperature should decay to 3.5 eV. The actual decay rate is much slower than this by about two orders of magnitude. This discrepancy is most perplexing and leads to the conclusion that either the electron temperature is considerably less than has been measured by Langmuir probes or that the electrons are non-Maxwellian as proposed by Erickson.[10] If excitation losses are included, the decay rate should be even faster. At high background pressures where ionization losses exceed hanger losses, the pressure dependence of the temperature decay has been ex-

perimentally verified. The slight density increase at low temperatures is usually masked by the decrease due to hanger losses, but it has been possible to generate a plasma with microwaves whose increase in density due to ionization nearly cancels the hanger loss giving a density which does not decay appreciably in time.

So far we have restricted our discussion to the electron component of the plasma. If we assume that the ions are produced at time $t = 0$ with a temperature $T_{io} << T_e$, we can proceed to calculate the rate of temperature rise of the ions due to their interaction with the hot electrons. According to Spitzer,[8] the rate at which the ion temperature changes in time is given by

$$\frac{dT_i}{dt} = \frac{T_e - T_i}{\tau_{eq}} \cong \frac{T_e}{\tau_{eq}} \qquad (57)$$

where the equipartition time τ_{eq} is given by Spitzer[8] as

$$\tau_{eq}(\text{sec}) = \frac{14\,[T_e(°k)]^{3/2}}{n(\text{cm}^{-3})} \qquad (58)$$

If we assume that the density decays exponentially with time,

$$n = n_o e^{-t/\tau},$$

and that the electron temperature is constant, equation (57) becomes

$$\frac{dT_i}{dt} = \frac{n_o}{14\,T_e^{1/2}} \quad e^{-t/\tau} \quad .$$

Solving for T_i leads to

$$T_i(t) = T_{oi} + \frac{n_o \tau}{14\, T_e^{1/2}} (1 - e^{-t/\tau}). \tag{59}$$

Taking $n_o = 2 \times 10^9$ cm^{-3} and $T_e = 10^5$ ^{o}k (~10eV), with $\tau = 1$ msec, the ion temperature should rise only ~500 ^{o}k. The actual ion temperature rise is probably somewhat larger because, when the plasma is first created, its density is considerably higher than is given by the exponential law.

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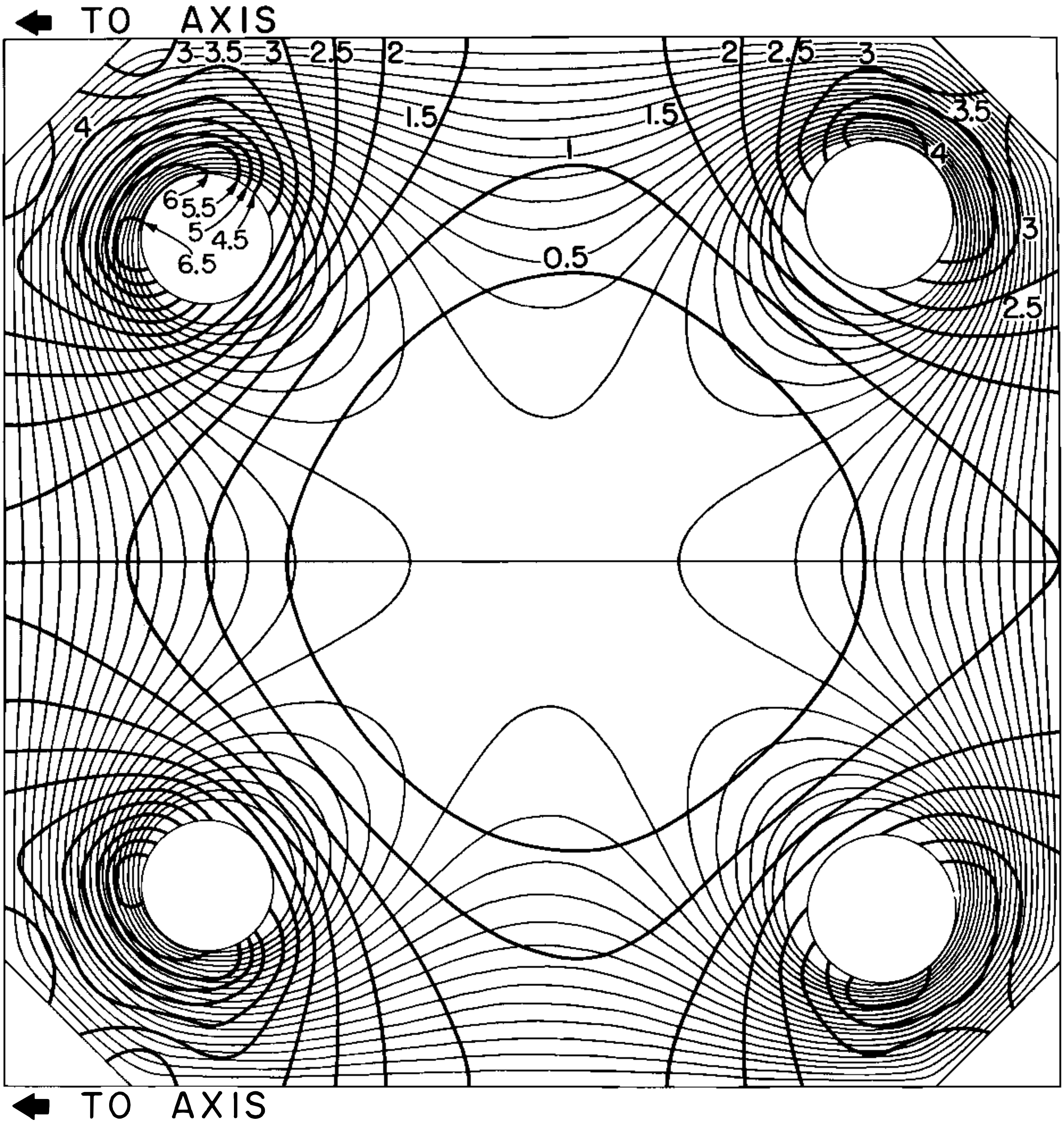
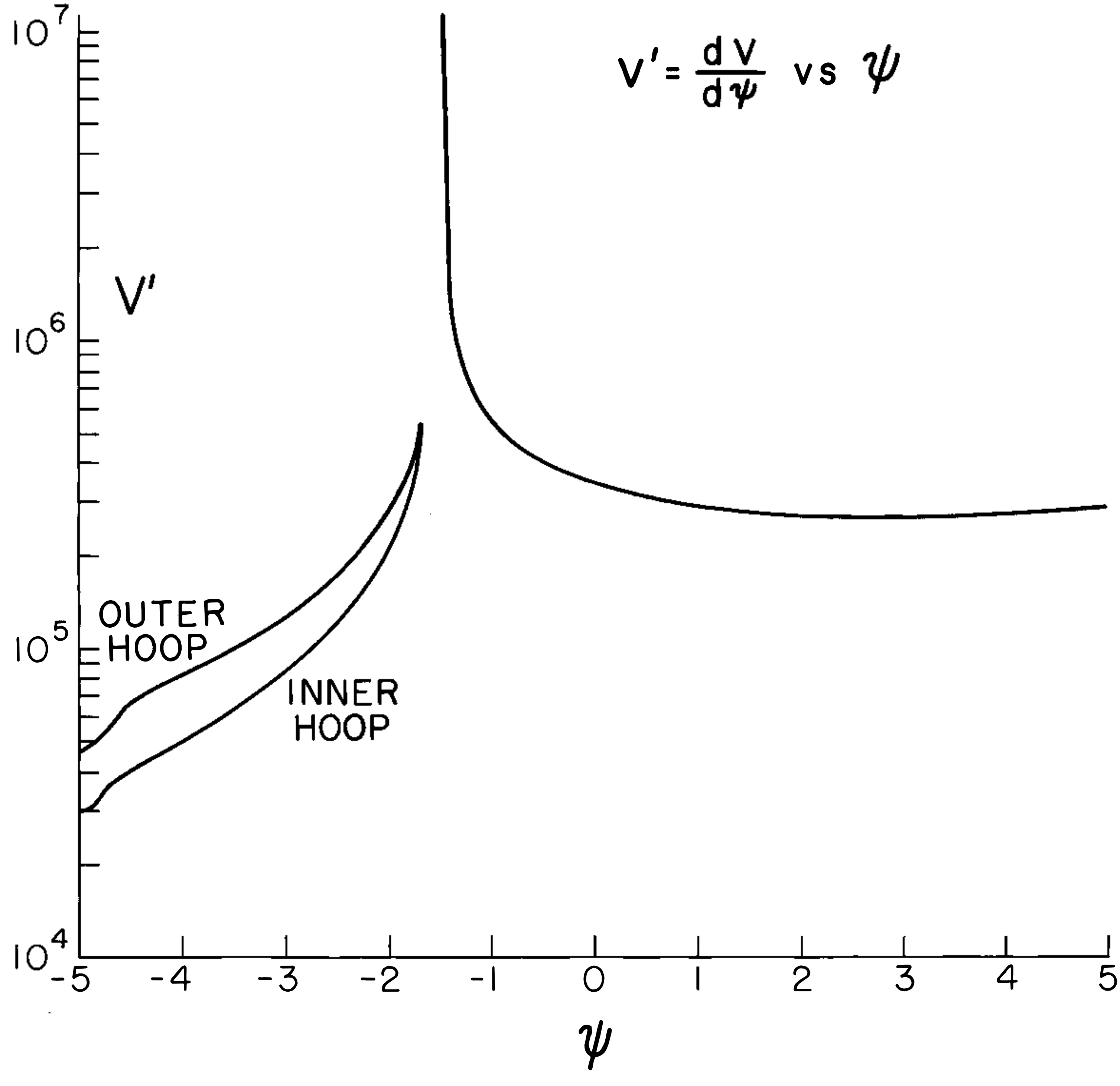
FIGURES



CONSTANT B SURFACES IN THE WISCONSIN TOROIDAL OCTUPOLE

Figure 1



$V' = \frac{dV}{d\psi}$ vs ψ
V'
OUTER HOOP
INNER HOOP
10^7
10^6
10^5
10^4
-5
-4
-3
-2
-1
0
1
2
3
4
5
ψ

Figure 2

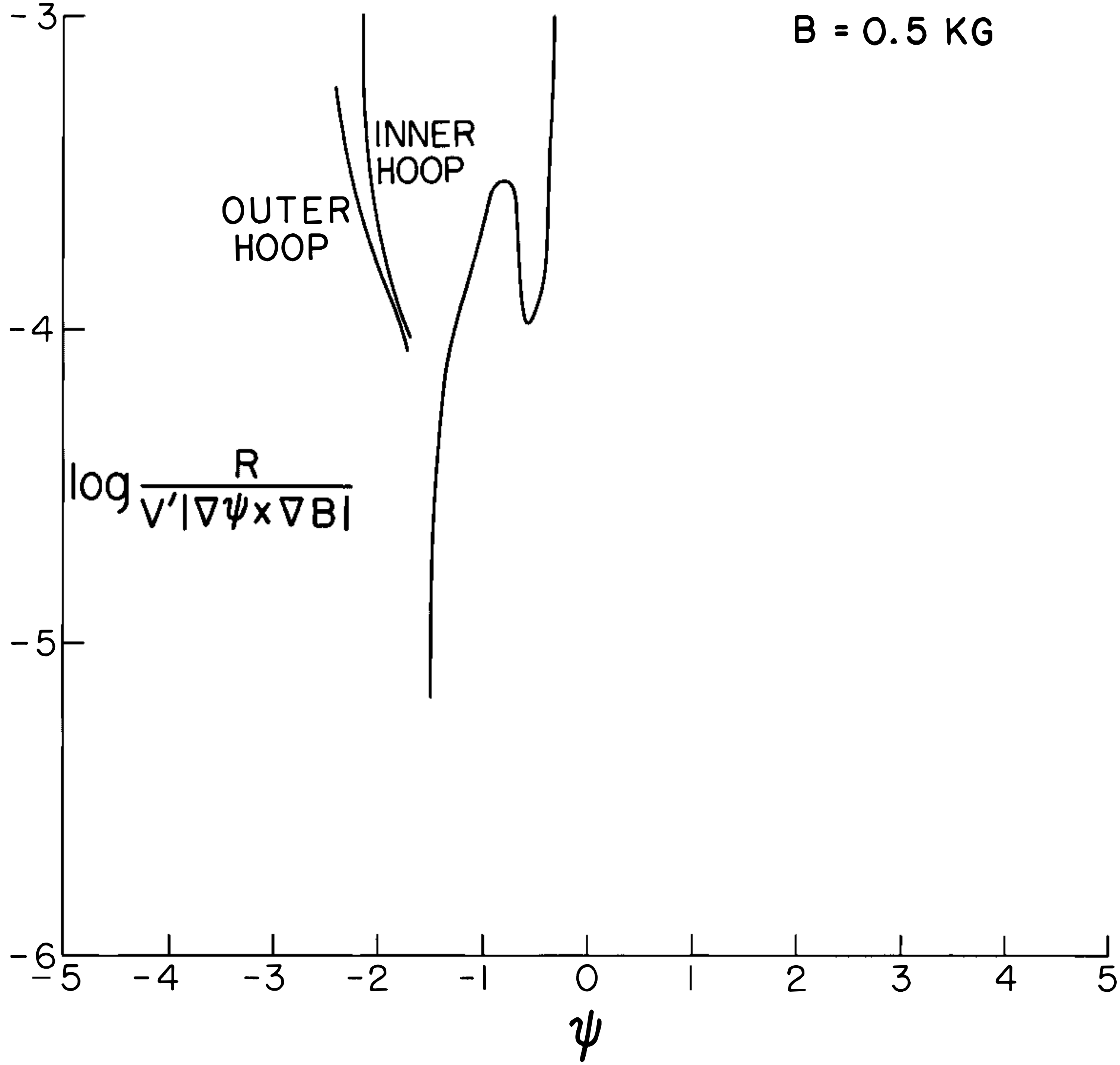
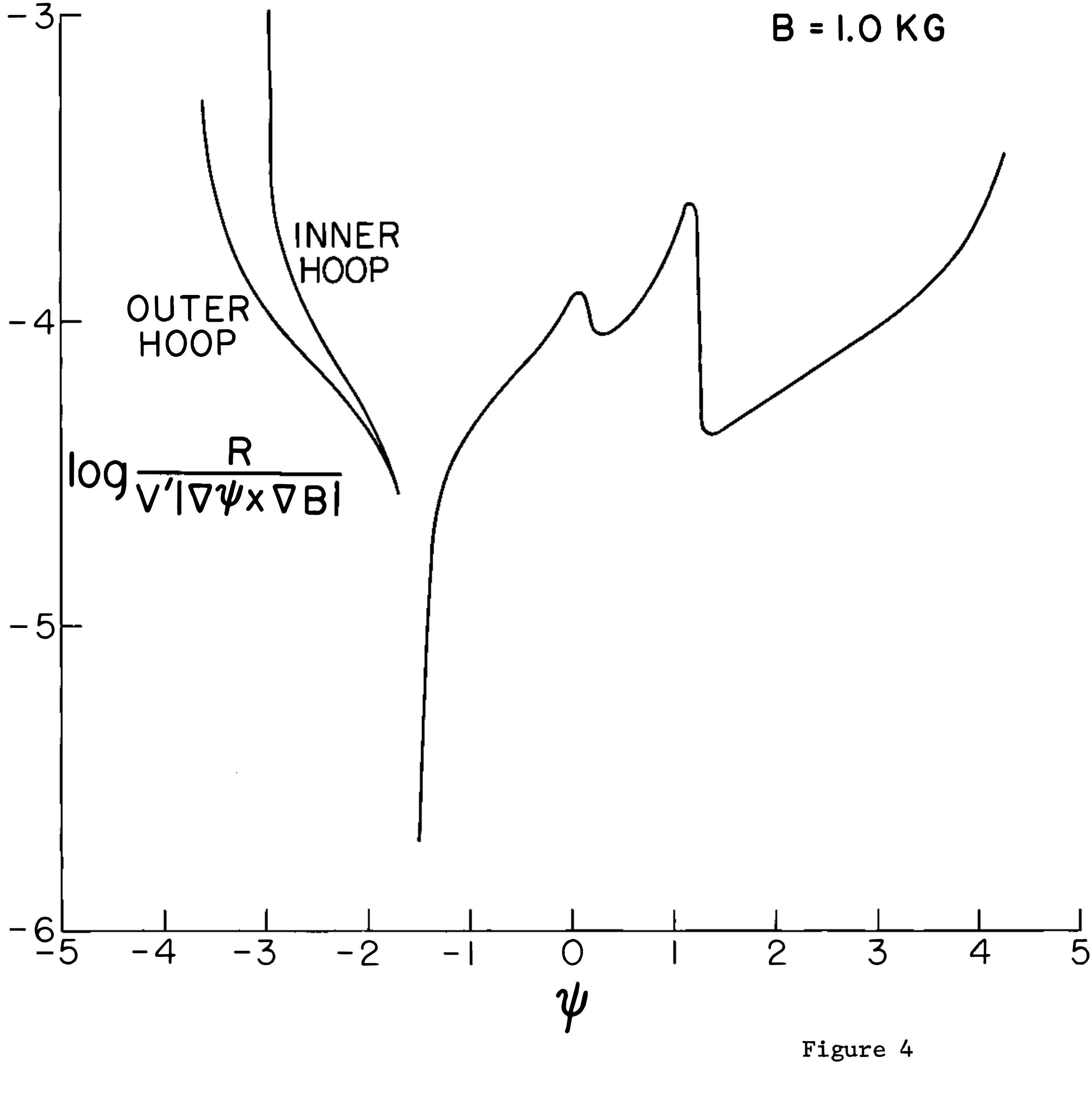
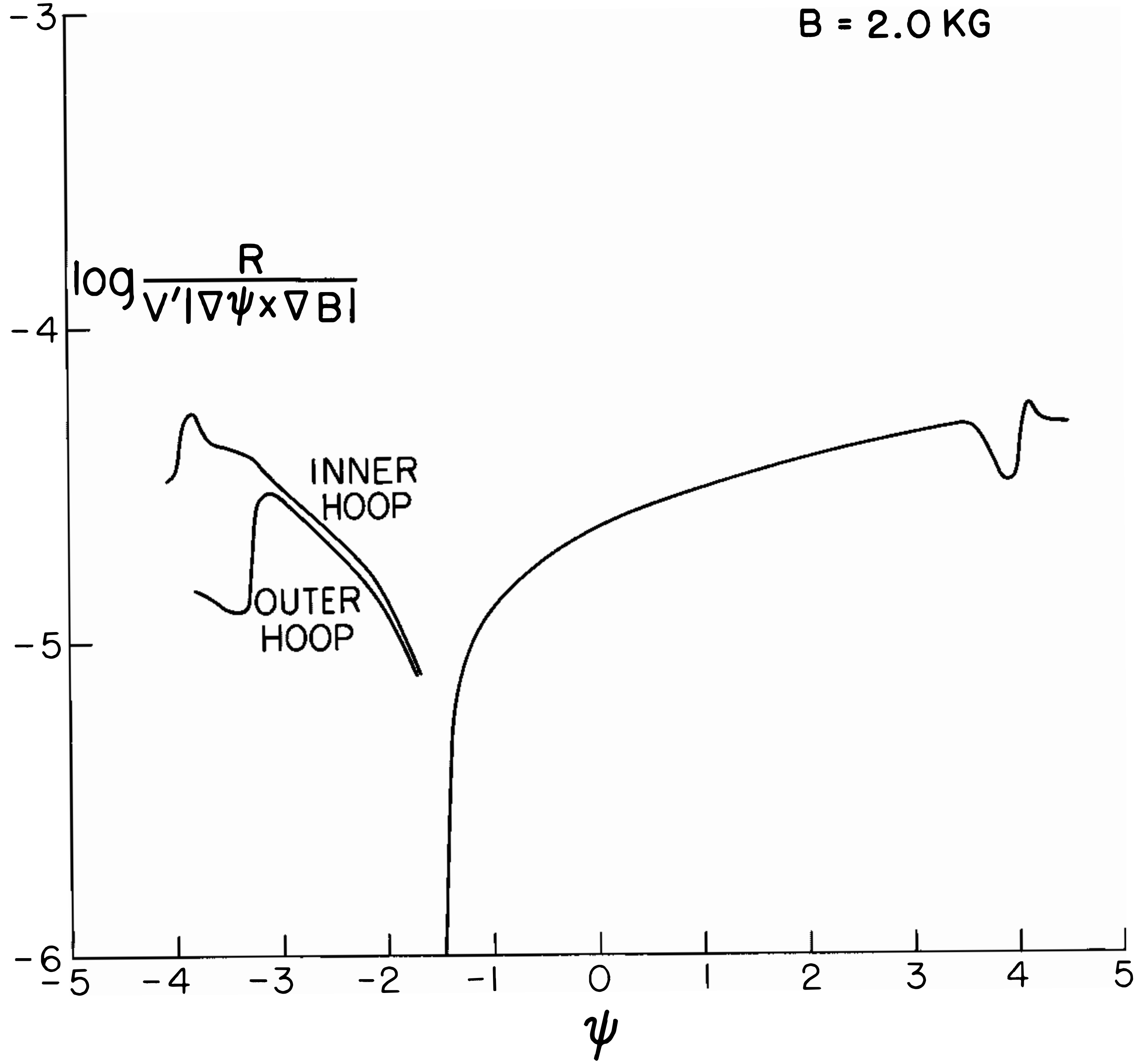

B = 0.5 KG
INNER HOOP
OUTER HOOP
$\log \frac{R}{V'|\nabla\psi \times \nabla B|}$
-3
-4
-5
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
ψ

Figure 3



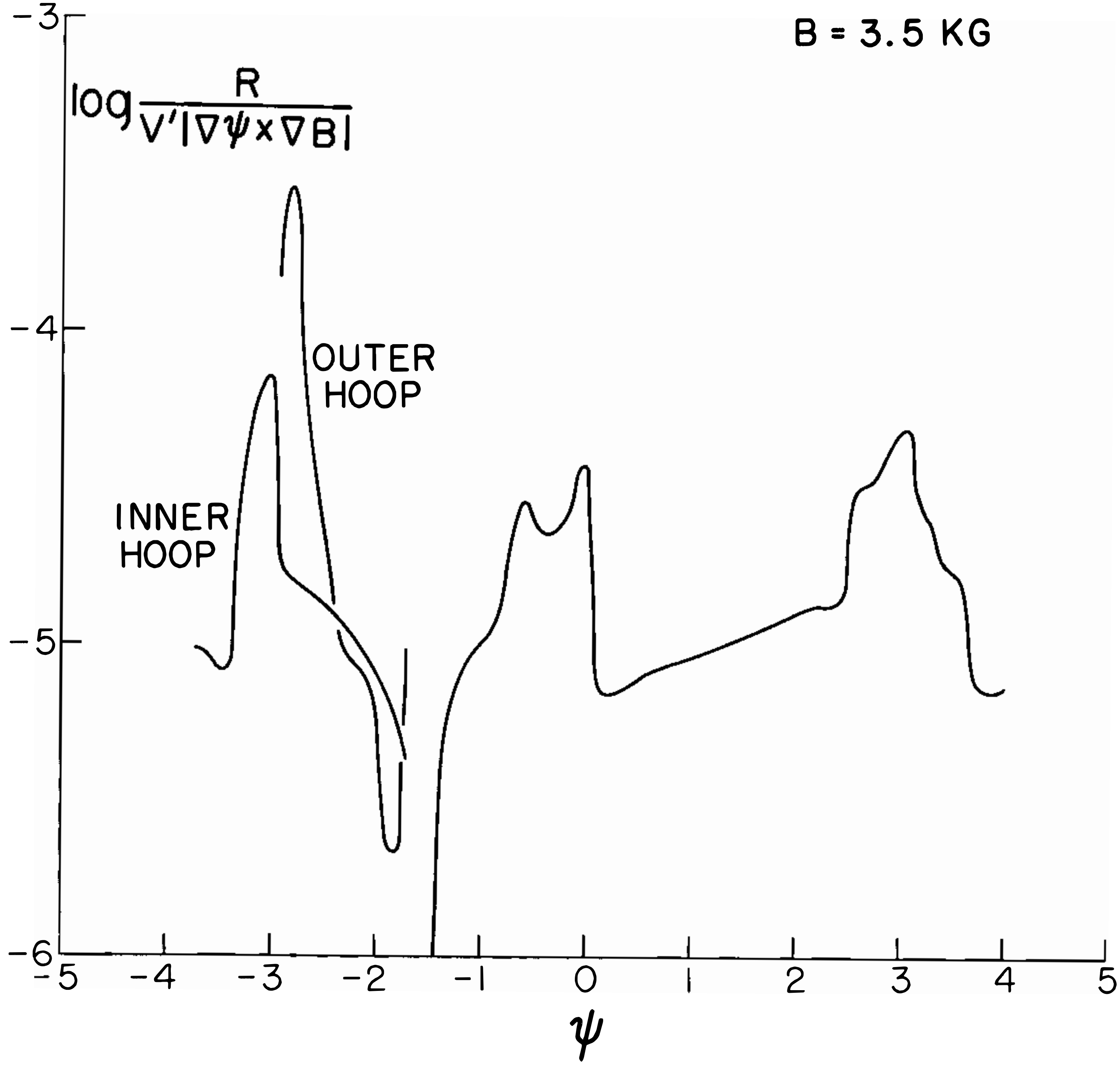
B = 1.0 KG
INNER HOOP
OUTER HOOP
$\log \frac{R}{V'|\nabla\psi \times \nabla B|}$
-3
-4
-5
-6
-5 -4 -3 -2 -1 0 1 2 3 4 5
ψ

Figure 4



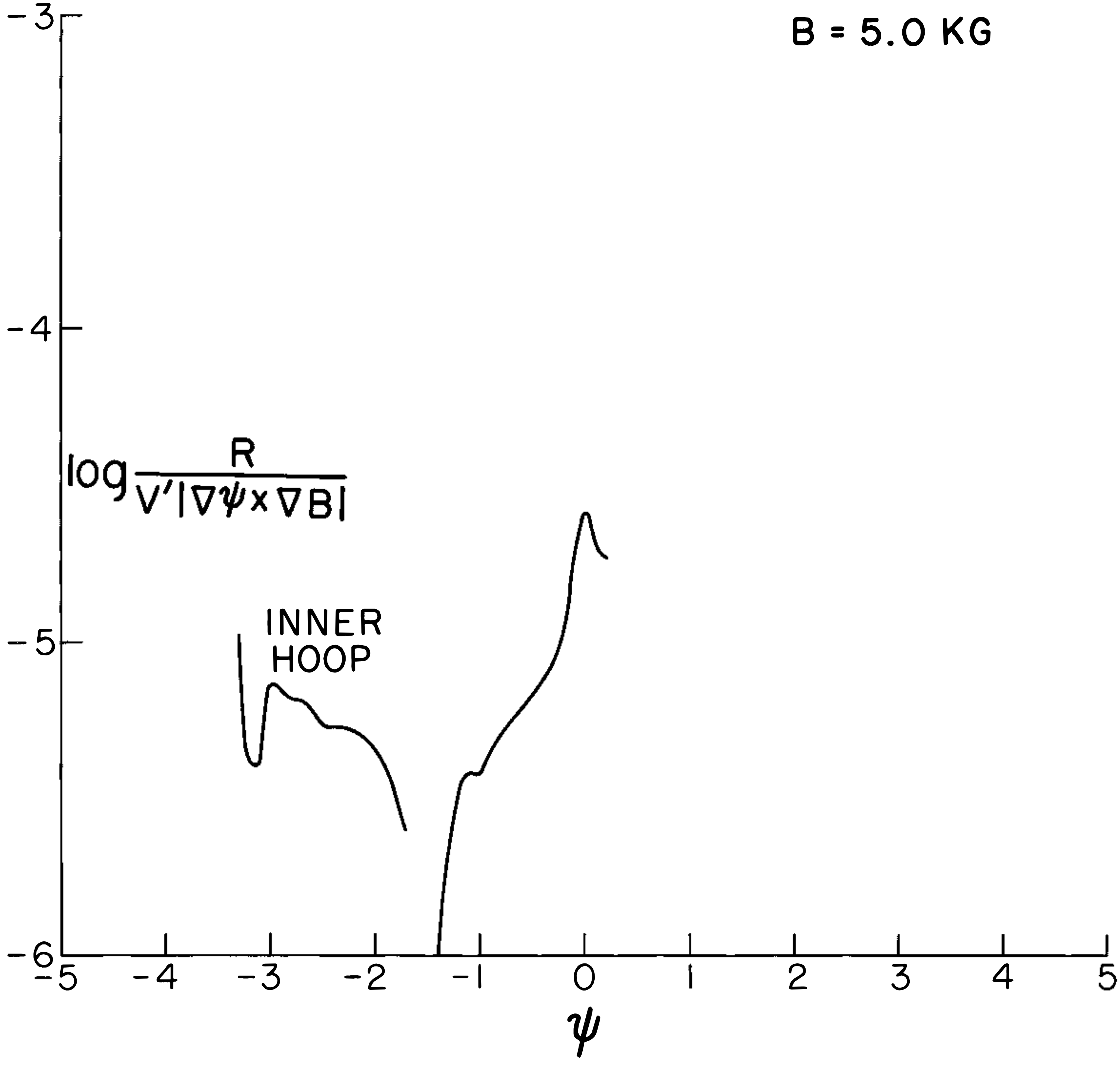
B = 2.0 KG
$\log \frac{R}{V'|\nabla\psi \times \nabla B|}$
INNER HOOP
OUTER HOOP
-3
-4
-5
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
ψ

Figure 5



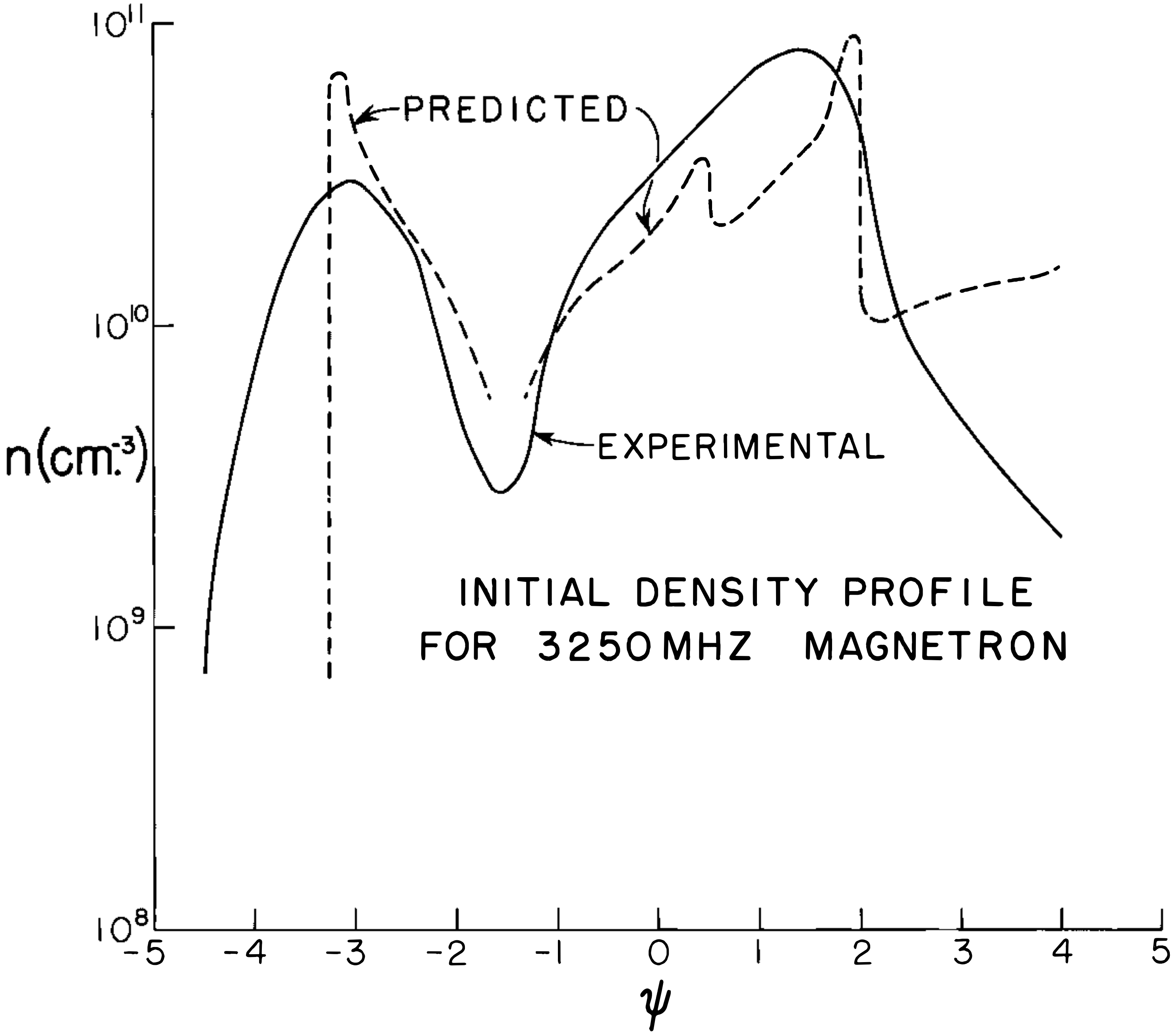
B = 3.5 KG
$\log \frac{R}{V' |\nabla\psi \times \nabla B|}$
OUTER HOOP
INNER HOOP
-3
-4
-5
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
ψ

Figure 6



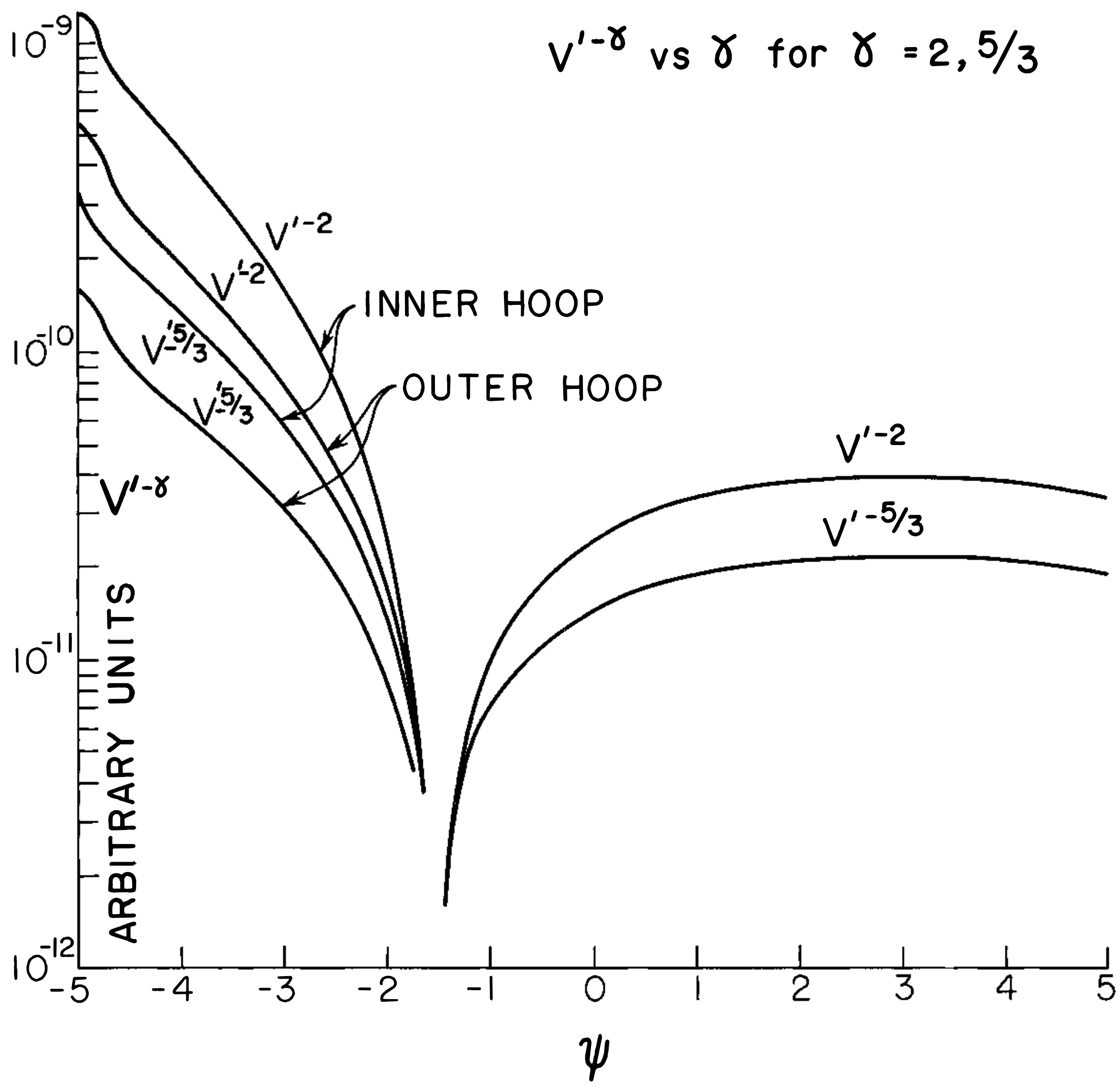
B = 5.0 KG
-3
-4
-5
-6
log $\frac{R}{V'|\nabla\psi \times \nabla B|}$
INNER
HOOP
-5
-4
-3
-2
-1
0
1
2
3
4
5
ψ

Figure 7



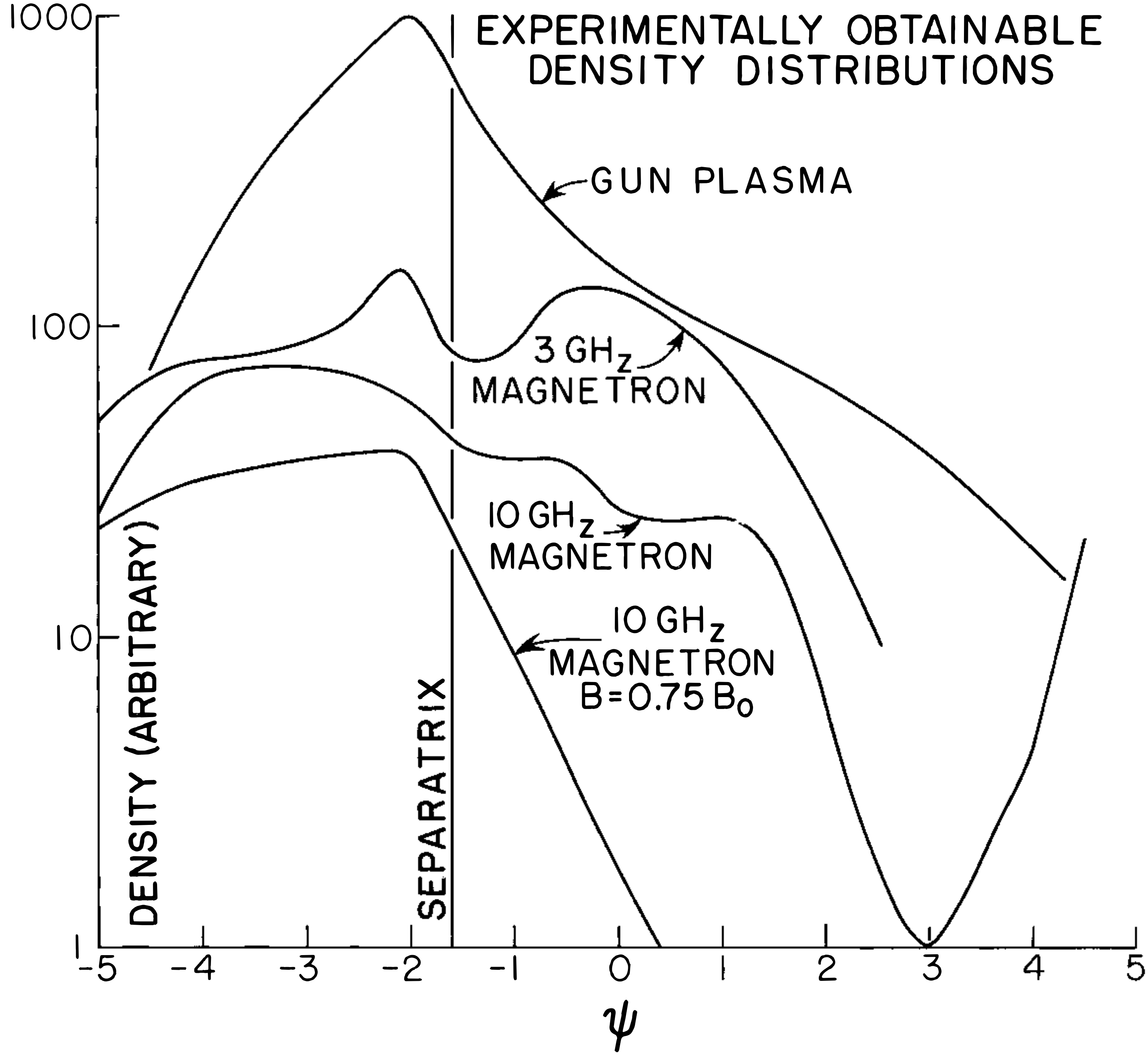
10^{11}
10^{10}
10^{9}
10^{8}
n(cm⁻³)
PREDICTED
EXPERIMENTAL
INITIAL DENSITY PROFILE
FOR 3250MHZ MAGNETRON
-5 -4 -3 -2 -1 0 1 2 3 4 5
ψ

Figure 9



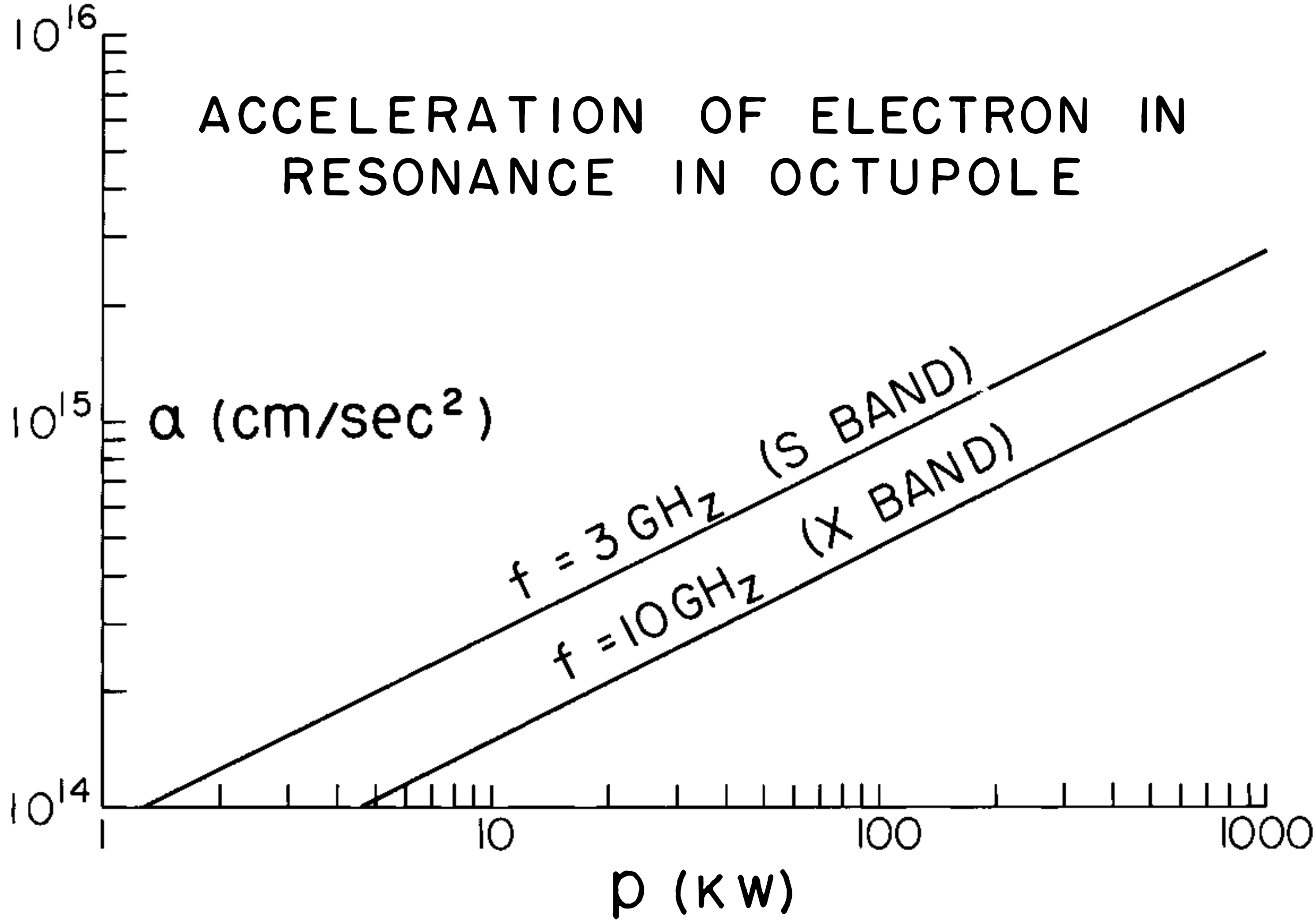
V'-γ vs γ for γ = 2, 5/3
V'-2
V'-2
INNER HOOP
V'-5/3
OUTER HOOP
V'-5/3
V'-γ
V'-2
V'-5/3
ARBITRARY UNITS
10-9
10-10
10-11
10-12
-5 -4 -3 -2 -1 0 1 2 3 4 5
ψ

Figure 10



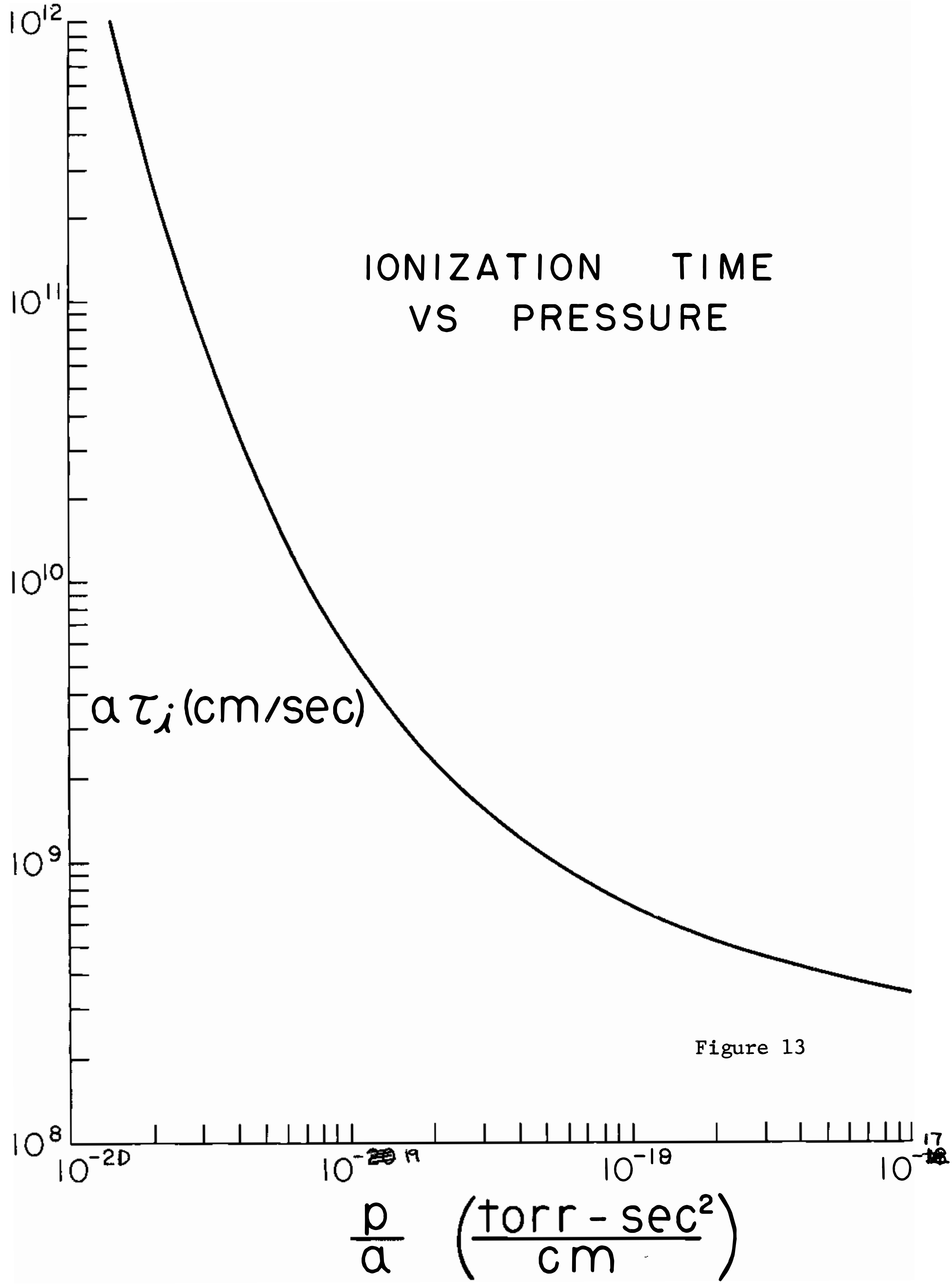
EXPERIMENTALLY OBTAINABLE DENSITY DISTRIBUTIONS
GUN PLASMA
3 GHz MAGNETRON
10 GHz MAGNETRON
10 GHz MAGNETRON B=0.75 B0
SEPARATRIX
DENSITY (ARBITRARY)
1000
100
10
1
-5
-4
-3
-2
-1
0
1
2
3
4
5
ψ

Figure 11



ACCELERATION OF ELECTRON IN RESONANCE IN OCTUPOLE
10^{16}
10^{15}
10^{14}
a (cm/sec^2)
f = 3 GHz (S BAND)
f = 10 GHz (X BAND)
1
10
100
1000
p (KW)

Figure 12



IONIZATION TIME
VS PRESSURE
$a\tau_i$ (cm/sec)
10^12
10^11
10^10
10^9
10^8
10^-20
10^-19
10^-18
10^-17
$\frac{p}{a}$ $\left(\frac{\text{torr-sec}^2}{\text{cm}}\right)$

Figure 13

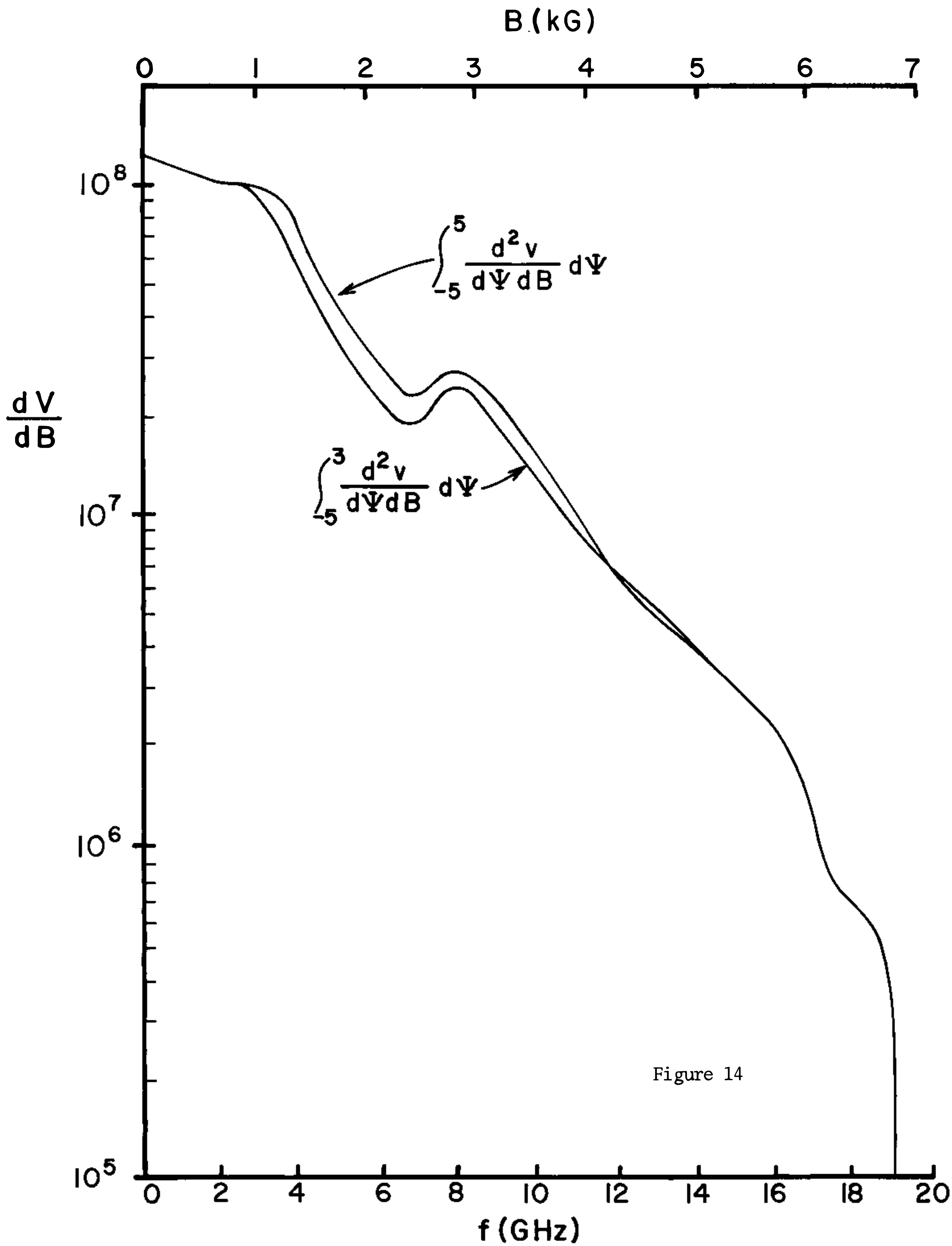
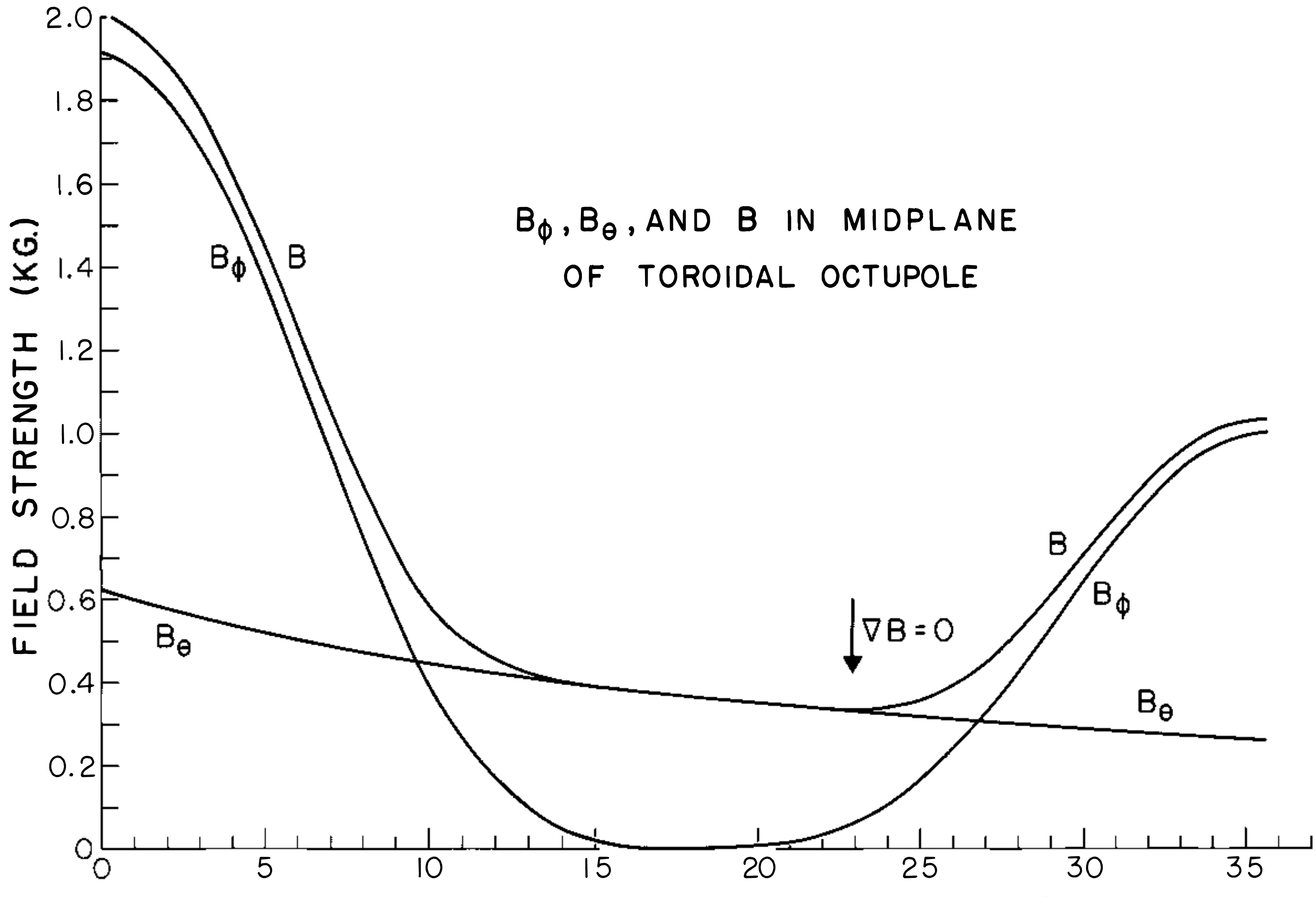

B (kG)
0
1
2
3
4
5
6
7
10^8
10^7
10^6
10^5
$\frac{dV}{dB}$
$\int_{-5}^{5} \frac{d^2 v}{d\Psi dB} d\Psi$
$\int_{-5}^{3} \frac{d^2 v}{d\Psi dB} d\Psi$
0
2
4
6
8
10
12
14
16
18
20
f (GHz)

Figure 14



Bφ, Bθ, AND B IN MIDPLANE
OF TOROIDAL OCTUPOLE
FIELD STRENGTH (KG.)
DISTANCE ALONG MIDPLANE (cm.)
Bφ
B
Bθ
∇B = O
B
Bφ
Bθ
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0
5
10
15
20
25
30
35

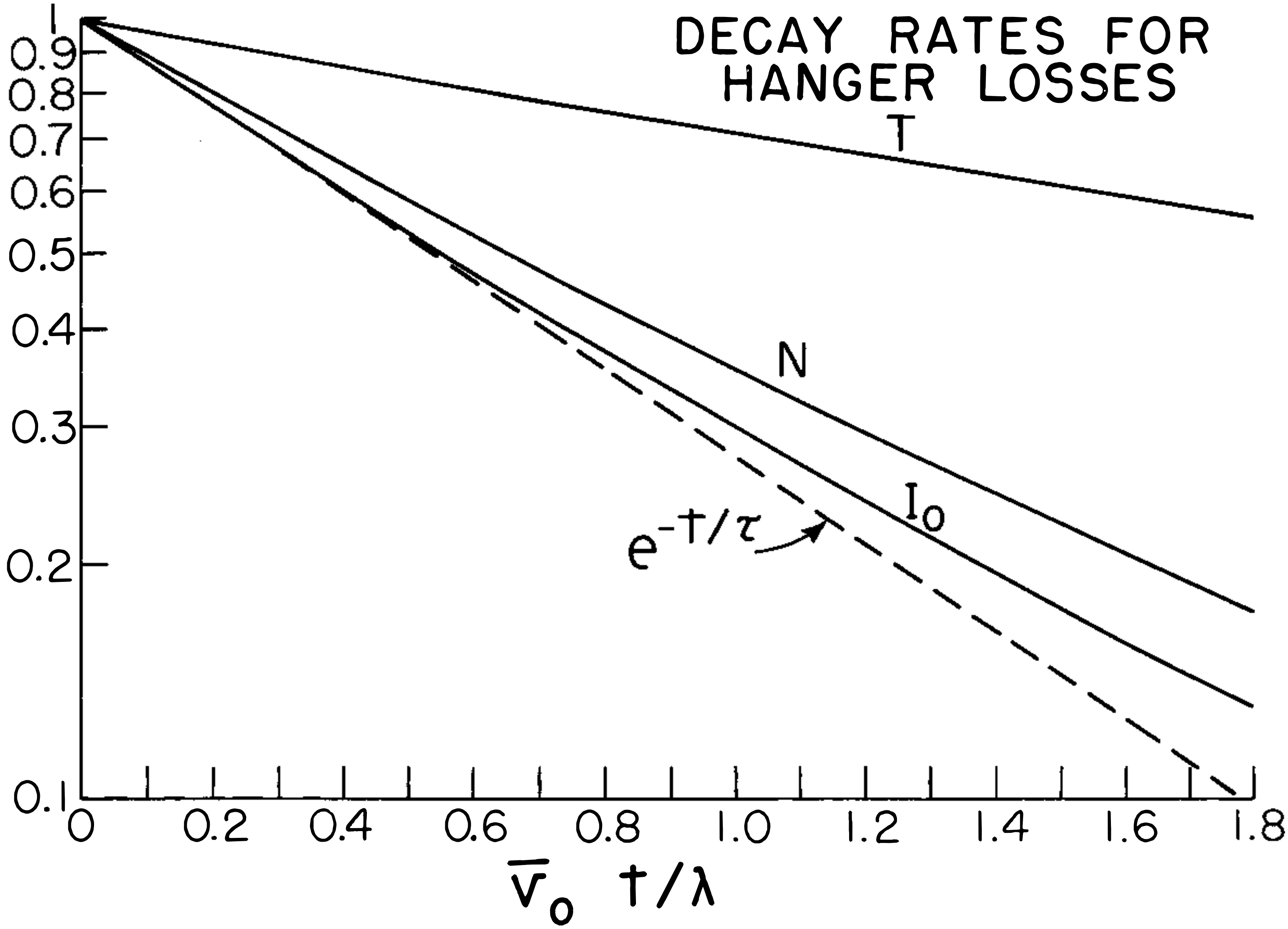

DECAY RATES FOR HANGER LOSSES
T
N
I_0
$e^{-t/\tau}$
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
$\bar{v}_0 t/\lambda$

Figure 16

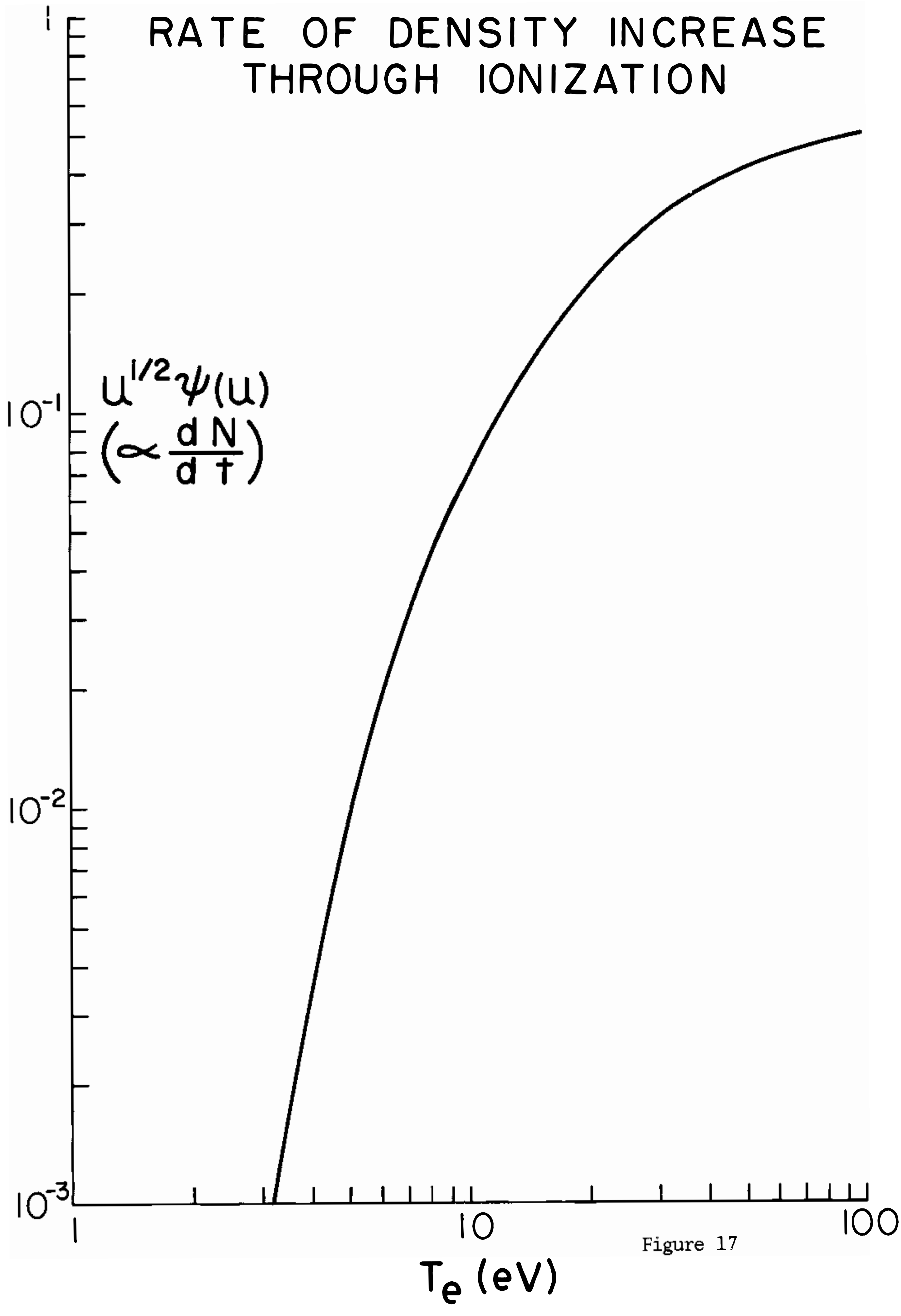
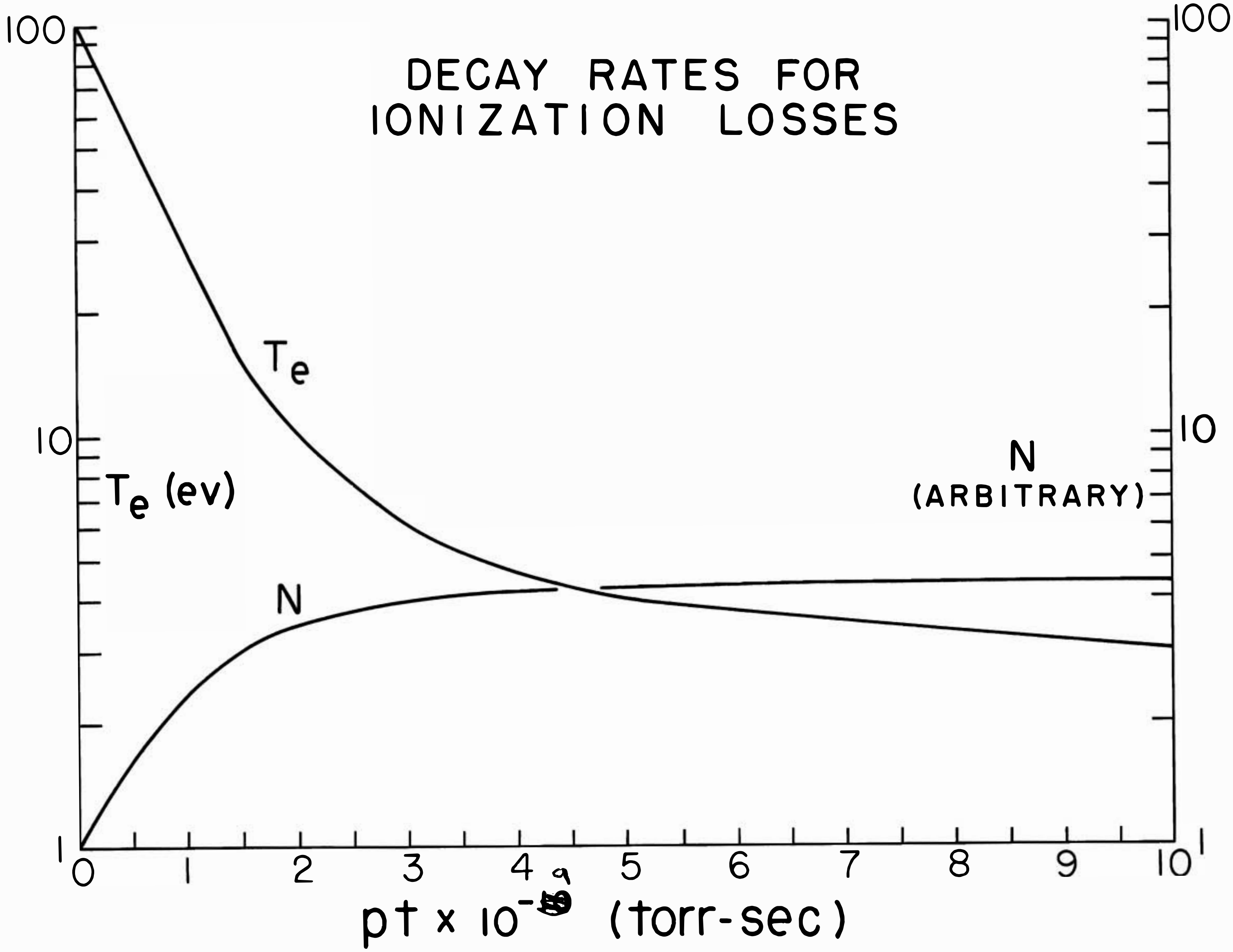

RATE OF DENSITY INCREASE
THROUGH IONIZATION
$u^{1/2}\psi(u)$
$\left(\propto \frac{dN}{dt}\right)$
1
10^{-1}
10^{-2}
10^{-3}
1
10
100
T_e (eV)

Figure 17



DECAY RATES FOR IONIZATION LOSSES
Te
Te (ev)
N
N
(ARBITRARY)
100
10
1
0
1
2
3
4
5
6
7
8
9
10
pt x 10-9 (torr-sec)