A Model for the Microwave Plasma Interaction

by

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Introduction

When producing or heating a plasma with microwaves, the goal is usually to transform as much electromagnetic wave energy into plasma energy as possible. To determine the optimum parameters for energy transfer, a theoretical model is useful.

An electromagnetic wave can give up its energy to a plasma in three basic ways:

1) collisional resistivity
2) cyclotron resonance acceleration
3) Landau damping

In addition, there is a wide variety of nonlinear effects whereby an electromagnetic wave can excite various forms of plasma waves.

Landau damping is an important mechanism whenever the phase velocity of the wave (ω/Rek) is comparable to the particle velocity. In the absence of a magnetic field, the phase velocity is always greater than c and hence Landau damping is not possible for a transverse wave. With a magnetic field, the phase velocity tends to zero at the electron cyclotron frequency and Landau damping can occur. However, a simple calculation shows that Landau damping is probably less important than cyclotron resonance absorption which occurs under the same conditions, and hence it will not be treated here.

The other two mechanisms will be considered in the following sections.

If the scale of distance over which ω_p and ν change is large compared with the wavelength of the radiation, the power absorbed by the plasma is

$$P = \frac{\bar{K}}{\bar{K} + K_c} P_o \qquad (5)$$

where $\bar{K}$ is the spatial average of K, K_c is the absorption of the cavity in the absence of plasma and P_o is the input microwave power to the cavity.

In order to evaluate K_c, it is useful to relate K_c to the Q of the cavity. The power remaining in the wave after traveling a distance z is

$$P = P_o \, e^{-2K_c z}.$$

The electromagnetic wave energy stored in the cavity is

$$U = \int P dt = \frac{1}{c}\int_0^\infty P dz = P_o/2cK_c.$$

From the definition of Q we obtain

$$Q = \frac{\omega U}{P_o} = \frac{\omega}{2cK_c}. \qquad (6)$$

The Q of a cavity depends on the distribution of electric fields inside the cavity. However, to within a geometrical factor of order unity, the Q of all cavity modes is given by

$$Q \cong \frac{1}{2\pi}\frac{V}{A\delta} \qquad (7)$$

where V is the volume, A is the surface area and δ is the skin depth of the cavity walls. Applying equation (7) to the toroidal octupole gives Q $\approx$ 10,500 at a frequency of 9GHz. This is surely an overestimate since it neglects such factors as wall contamination and port holes.

A more realistic Q was determined experimentally. A 9GHz klystron was used to feed power into the cavity through the same window as used for the high power magnetron. A ferrite isolator was used to decouple the klystron from the cavity. The cavity modes were observed by coupling a small amount of power out of the cavity through a window into a calibrated crystal detector. By sweeping the klystron frequency with a saw-tooth voltage, the mode pattern of Fig. 1 was obtained. The three prominent peaks in Fig. 1 have an average Q of 3300. From older data at 24 GHz, a Q of 8700 was calculated. Since $Q \propto \sqrt{\omega}$, this would imply a Q of 5400 at 9 GHz. Even earlier data of Meade (PLP 51) imply a Q of about 4000. Since professors are never wrong, a Q of 4000 will be assumed. From equation (6) we conclude that at 9 GHz, $K_c \cong 2.7 \times 10^{-4}$ Np/cm.

If we require that at least half the injected microwave power be absorbed by the plasma, equation (5) gives

$$\bar{K} = \frac{\omega_p^2 \nu}{2c\omega^2} > K_c = \frac{\omega}{2cQ} = 2.4 \times 10^{-4}$$

or

$$\omega_p^2 \nu > 4.5 \times 10^{28}/\text{sec}^3 \qquad (8)$$

Substituting for ω_p and ν gives

$$n\, n_n\, \sigma_s\, v > 1.4 \times 10^{19}\ \text{cm}^{-3}\ \text{sec}^{-1}$$

where n_n is the neutral particle density, σ_s is the scattering cross section, and v is the electron velocity. Hence, for effective collisional absorption, we need high preionization and high background gas pressure. In a real case, we might, for example, wish to use the gun plasma ($n \sim 10^9$ cm^{-3}) for preionization. Taking a typical value of $\sigma_s v \sim 10^{-7}$ cm^3/sec, we conclude that good efficiency would require a neutral density of $\sim 10^{17}$cm^{-3} or a background pressure of several torr. Hence, resistive absorption does not appear to be an efficient process with the present apparatus.

In spite of this limitation, resistive absorption is of some academic interest because the theory is sufficiently simple to allow quantitative predictions which can be checked by experiment. In the usual case of $\bar{K} \ll K_c$, the power absorbed by the plasma is

$$P = P_o \bar{K}/K_c = P_o Q \frac{\omega_p^2 \nu}{\omega^3} \tag{9}$$

The energy in the plasma is

$$U = nV(E_i + kT_e)$$

where E_i is the ionization energy, T_e, the electron temperature. Since $P = dU/dt$, we obtain the differential equation

$$V(E_i + kT_e)\frac{dn}{dt} + nVk\frac{dT_e}{dt} = P_o \frac{4\pi e^2}{m\omega^2} \frac{\nu}{\omega} Qn. \tag{10}$$

If we neglect all sources of particle loss, the rate of density increase is given in terms of the ionization cross section σ_i by

$$\frac{dn}{dt} = \frac{n}{\tau_i} = n\, n_n\, \sigma_i v \qquad (11)$$

The two equations can be combined to give:

$$(E_i + kT_e)\, n_n\, \sigma_i v + k\,\frac{dT_e}{dt} = \frac{P_o}{V}\,\frac{4\pi e^2}{m\omega^2}\,\frac{\nu}{\omega}\, Q \qquad (12)$$

Equation (12) can be used to find $T_e(t)$ and then equation (11) used to find n(t) if the energy dependence of ν and σ_i are known. The required cross sections are plotted in Fig. 2 using previously published data.

Two cases are sufficiently simple to allow exact analytic solutions: 1) weakly ionized plasmas ($n \ll n_n$), and 2) $>$ n

be considered separately.

Case I (weakly ionized plasma):

For a weakly ionized plasma, $\nu = n_n \sigma_s v$ and the second term of equation (11) vanishes. An equilibrium temperature is quickly reached and is given by

$$E_i + kT_e = \alpha_1 P_o \frac{\sigma_s}{\sigma_i} \qquad (13)$$

where $$\alpha_1 = \frac{Q}{V}\,\frac{4\pi e^2}{m\omega^2} = 2.3 \times 10^{-2} \text{ sec}$$

for 9GHz radiation in the octupole. The temperature predicted by equation (13) is plotted vs. microwave power in Fig. 3. For low power levels, the energy reaches 15.8 eV whereupon ionization prevents further increase. Only at power levels of several hundred kilowatts does resistive heating become effective.

The previous derivation is somewhat crude because it neglects the following effects:

1) The electrons are not monoenergetic. It was assumed that all electrons have an energy of kT_e. A broadened distribution would cause the curve to fall below 15.8 eV at low powers.

2) Other losses such as excitation and diffusion to the wall have been neglected. These effects would reduce the temperature obtainable at a given power level.

3) The ionization potential was assumed to be 15.8 V (ionization potential of hydrogen atom + 1/2 dissociation potential of hydrogen molecule). Multiple excitations lower the actual threshold to 10.2 eV.

Having calculated the equilibrium temperature, it is then a simple matter to obtain the density from equation (11):

$$n(t) = n_o e^{t/\tau_i}$$

where n_o is the initial electron density and

$$\tau_i = \frac{1}{n_n \sigma_i v} = \frac{E_i + kT_e}{\alpha_1 P_o n_n \sigma_s v} .$$

For low powers, $kT_e \approx 15.8$ eV and

$$\tau_i = 1.66 \times 10^{14} \frac{1}{P_o(\text{watts})n_n(\text{cm}^{-3})}.$$

The energy required to increase the density of a plasma by a factor of e is plotted vs. background pressure for resistive heating in the octupole in Fig. 4. With presently available energy sources ($\sim$ 10 J at 9 GHz or 1 J at 3 GHz), pressures in excess of 10^{-4} torr are required to significantly increase the density.

Some preliminary measurements of resistive heating have been made with the toroidal octupole by injecting a gun plasma, raising the background pressure, and applying a pulse of 9 GHz microwaves with a very weak magnetic field (strong enough to confine the plasma but weak enough to avoid electron cyclotron resonance). The exponential dependance of density increase on pressure has been verified and the observed value of n/n_o checked with the theory to within about a factor of two.

Case II (strongly ionized plasma):

For a strongly ionized plasma, the dominant collisions are electron-ion and the density increase dn/dt is zero. In this case, equation (12) becomes

$$k \frac{dT_e}{dt} = \frac{P_o}{V} \frac{4\pi e^2}{m\omega^2} \frac{n\sigma_s v}{\omega} Q = \alpha_2 P_o n(kT_e)^{-3/2} \qquad (14)$$

where $\alpha_2 = 2 \times 10^{-45}$ erg$^{3/2}$cm^3 for 9 GHz. Equation (13) can be solved to obtain $T_e(t)$:

$$kT_e(t) = 1.45[(kT_e)_o^{5/2} + \alpha_2 P_o nt]^{2/5}. \qquad (15)$$

As a numerical example, a 1 msec, 100 KW, 9 GHz microwave pulse and a plasma density of $10^{11}cm^{-3}$ would be required in order to heat the plasma to 100 eV. Hence, resistive heating of a fully ionized plasma in the octupole is quite difficult. If higher plasma densities were available, the original assumption that $\omega \gg \omega_p$ would be invalid and a more exact calculation would be necessary.

As the plasma density increases, the absorption increases and the distance traveled by the wave before damping decreases. For $\omega_p > \omega$ and $\nu \ll \omega$ equation (3) gives a penetration depth of

$$\delta = \frac{1}{K} = \frac{1}{\mathrm{Im}\ k} = \frac{c}{\sqrt{\omega_p^2 - \omega^2}} .$$

As ω_p increases, δ decreases until δ becomes comparable to the wavelength divided by 2π, at which point the reflected power becomes comparable to the absorbed power and the efficiency falls off rapidly. This condition occurs for

$$\frac{c}{\omega} = \delta = \frac{c}{\sqrt{\omega_p^2 - \omega^2}}$$

or

$$\omega_p = \sqrt{2}\ \omega .$$

Hence, for a given microwave frequency (ω) the density that can be achieved is limited. For 3 GHz, the maximum density is about $2 \times 10^{11}cm^{-3}$ and for 9 GHz it is about $2 \times 10^{12}cm^{-3}$.

Cyclotron Resonance Acceleration

When the static magnetic field is taken into account, a dispersion relation similar to equation (3) is obtained. This relation is called the Appleton equation and appears as follows:

$$k^2 = \frac{\omega^2}{c^2} - \frac{\omega_p^2}{c^2\left[1 + i\frac{\nu}{\omega} - \left(\frac{(\omega_c^2 \sin^2\theta)/\omega^2}{2\left(1+i\frac{\nu}{\omega}-\frac{\omega_p^2}{\omega^2}\right)}\right) \pm \left(\frac{(\omega_c^4 \sin^4\theta)/\omega^4}{4\left(1+i\frac{\nu}{\omega}-\frac{\omega_p^2}{\omega^2}\right)^2} + \frac{\omega_c^2 \cos^2\theta}{\omega^2}\right)^{\frac{1}{2}}\right]}$$

where ω_c is the electron cyclotron frequency:

$$\omega_c = \frac{eB}{mc}$$

and θ is the angle of the wave vector $\vec{k}$ with respect to the magnetic field $\vec{B}$. The procedure is to find the average absorption/unit length of a wave by integrating $K = \text{Im } k$ over all volume and all solid angle:

$$\bar{K} = \frac{1}{4\pi V} \iint K(\vec{r},\theta)\, dV d\Omega .$$

The problem can be simplified somewhat by assuming that all elements of solid angle are equally probable in which case the angle integration gives

$$\bar{K} = \frac{\pi\omega_p^2 \nu}{\mu c[(\omega \pm \omega_c)^2 + \nu^2]}$$

where $\mu = \frac{c}{\omega}$ Re k. The volume integral can be performed by converting to an integral over ω_c:

$$\bar{K} = \frac{mc}{4eV}\int_{-\infty}^{\infty} \frac{\omega_p^2 \nu}{\mu c[(\omega - \omega_c)^2 + \nu^2]} \frac{dV}{dB}\, d\omega_c \ .$$

For $\nu << \omega$, the main contribution to the integral comes from $\omega \approx \omega_c$, and dV/dB, ω_p, and ν can be evaluated at $B = \frac{mc}{e}\omega$ and brought out of the integral. For $\omega_p^2 << \omega\nu$, $\mu \rightarrow 1$ and the average absorption is

$$\bar{K} = \frac{m\omega_p^2 \nu}{4\ eV}\frac{dV}{dB}\int_{-\infty}^{\infty} \frac{d\omega_c}{\omega_c^2 - 2\omega\omega_c + (\omega^2 + \nu^2)}$$

$$= \frac{\pi m\omega_p^2}{4\ eV}\frac{dV}{dB} = \frac{\pi^2 e}{V}\frac{dV}{dB}\, n. \qquad (16)$$

For $\omega_p^2 >> \omega\nu$, the above absorption is increased by a factor of $\sim\omega_p/\sqrt{2\omega\nu}$. The independence of $\bar{K}$ on ν for low densities can be understood physically since the absorption at resonance is proportional to $1/\nu$ but the volume over which resonance occurs is proportional to ν/ω_c.

The result of equation (16) can be compared with the resistive heating case of equation (9) by determining an effective collision frequency ν^* which will allow resonance heating to be treated in the same way as resistive heating. The result is

$$\nu^* = \frac{\pi}{2}\left(\frac{B}{V}\frac{dV}{dB}\right)\omega \ . \qquad (17)$$

The quantities dV/dB and (B/V)(dV/dB) are plotted in Fig. 5. The quantity (B/V)(dV/dB) is especially useful since it is independent of B provided the abscissa of Fig. 6 is interpreted as the constant B surface on the standard B plots (normalized to B = 1.0 at outside wall midplane) rather than kilogauss. Note that since (B/V)(dV/dB) is of order unity, $\nu^* \sim \omega$ and resonance heating is more efficient than resistive heating by order ω/ν. Normally the magnetic field in the octupole is reduced in order to give resonance at B $\cong$ 5.0 in order to produce a density distribution which is peaked on the separatrix. For this case, $\nu^* \approx 0.1\,\omega$, and the power absorbed by the plasma is

$$P \cong 0.1\, P_o Q \frac{\omega_p^2}{\omega^2} .$$

Efficient absorption occurs when $\omega_p^2 > 10\,\omega^2/Q$ or $n > 2.8\text{x}10^8 \text{cm}^{-3}$ for 3 GHz or $n > 8.3 \times 10^8$ cm^{-3} for 9 GHz. Hence, the gun injected plasma ($n \sim 10^9$ cm^{-3}) should provide adequate preionization for microwave heating. If this level of preionization is not available, then a high background pressure is required to build the density up rapidly.

Two special cases which are of some experimental importance will be considered in detail: 1) High background gas pressure ($\sim 10^{-4}$ torr) and low preionization, and 2) Low background gas pressure ($\sim 10^{-6}$ torr) and considerable preionization ($\sim 10^9$ cm^{-3}):

Case I (High pressure, low preionization):

Following the same procedure used to obtain equation (13) leads to the result

$$(E_i + k\,T_e)\sqrt{kT_e} = \alpha_3 P_o/\sigma_i \qquad (18)$$

where $\alpha_3 = .725\,\frac{Q}{V}\,\frac{e^2}{\sqrt{m}\;\omega^2 n_n} = 6.7 \times 10^{-43} erg^{1/2} cm^2 sec$

for 9 GHz radiation in the octupole. Equation (18) allows a prediction of the equilibrium temperature of a resonantly heated weakly ionized plasma. The result is shown in Fig. 6. Note the vastly improved efficiency over the resistive heating case of Fig. 3. For presently available power levels, temperatures of 100 eV - 1 KeV would be expected on the basis of this model.

There are at least 3 effects which would tend to reduce this theoretically predicted temperature:

1) Excitation losses, diffusion to the wall and hoops and hanger losses have been neglected.

2) An equilibrium temperature may not be reached unless the microwave pulse length is many ionization times.

3) If plasma does not diffuse out of the resonance region sufficiently fast, the local density may reach the point where the radiation can no longer penetrate into the resonance region and the power absorbed will be reduced.

An experimental measurement of the temperature near the end of the microwave pulse would be useful for determining the accuracy of the preceding prediction. This is experimentally difficult, however, because of the fluctuating plasma potential and the presence of high microwave electric fields. Measurements several hundred μsec after the end of the pulse always show temperatures < 10 eV, but this is consistent with what would be expected from ionization losses.

As in the case of resistive heating, after thermal equilibrium has been reached, the density increases exponentially with a time constant given by

$$\tau_i = 1/n_n \sigma_i v$$

For a typical case of 10 kw at 3 GHz, τ_i = 5 μsec. This result is almost identical to that obtained in PLP 142 by a somewhat different method. In 100 μsec, the density should increase by ~ 9 orders of magnitude. This is in reasonable agreement with experimentally obtained results.

Case II (Low pressure, high preionization)

In order to achieve the maximum possible electron temperature by microwave heating, a high level of preionization ($\sim 10^9$ cm^{-3}) and low background pressure is desired. These conditions can be approximated in the present machine by injecting a gun plasma with a background gas pressure of $\approx 10^{-6}$ torr. Under these conditions, a microwave pulse should produce essentially a fully ionized plasma that absorbs all of the rf input power. The final density would be $\sim 6.5 \times 10^{10}$ cm^{-3} and the temperature would be determined solely by the input energy:

$$nV(E_i + kT_e) = U$$

or

$$E_i + kT_e = 300\ U$$

where E_i and kT_e are in eV and U is in joules. Hence, with presently available energies (~10 joules) we expect to be able to produce a 3 keV fully ionized plasma with $n \sim 6.5 \times 10^{10}$ cm^{-3}. Furthermore, as the background pressure is reduced, the temperature increases. Studies are presently under-way to test this intriguing prediction.

In PLP 142, an objection was raised to the possibility of achieving high electron temperatures by electron cyclotron resonance heating in the toroidal octupole because the nonuniformity of the magnetic field would allow particles to get out of resonance as their gyroradius increased. The maximum energy which a particle can acquire is given by

$$kT_e = 0.29\, m[a\omega\left(\frac{B}{\nabla B}\right)^2]^{2/3} \tag{20}$$

where a is the average acceleration of the particle,

$$a = \frac{e}{m}\bar{E}.$$

The average electric field was calculated assuming a Q of unity. The correct expression for $\bar{E}$ should be

$$\bar{E} = \sqrt{\frac{4\pi P_o Q}{V}} \tag{21}$$

The average electric field and corresponding electron acceleration are plotted vs. P_o in Fig. 7. For P_o = 100 kW, $a \simeq 10^{18}$ cm/sec^2. For resonance on the separatrix behind the hoops, $B/\nabla B \approx 7$ cm, and equation (20) predicts a possible energy of ~10 keV for 9 GHz. This revised estimate is consistent with the possibility of obtaining a 3 keV plasma and presents a considerably more optimistic view of plasma heating in the octupole than does PLP 142.

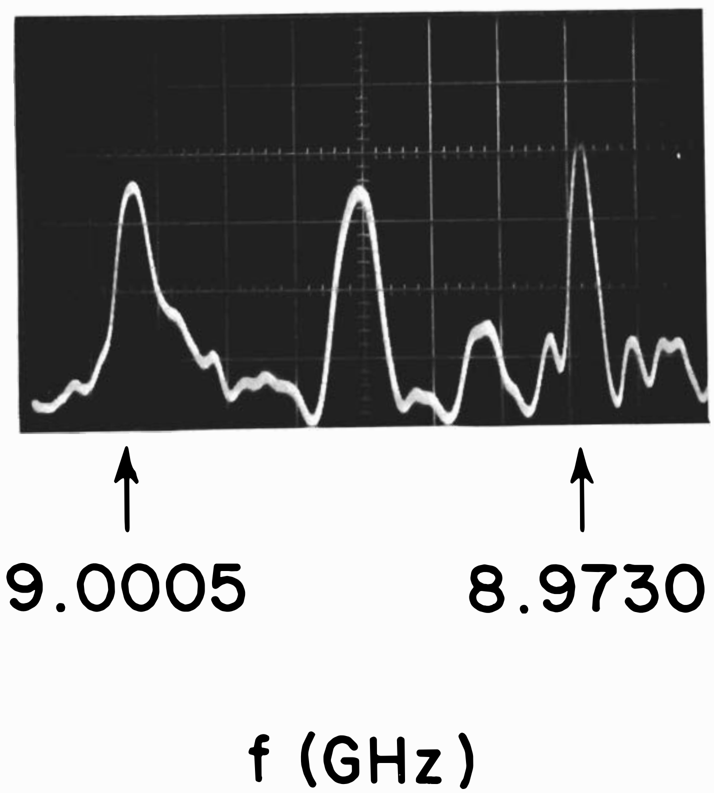
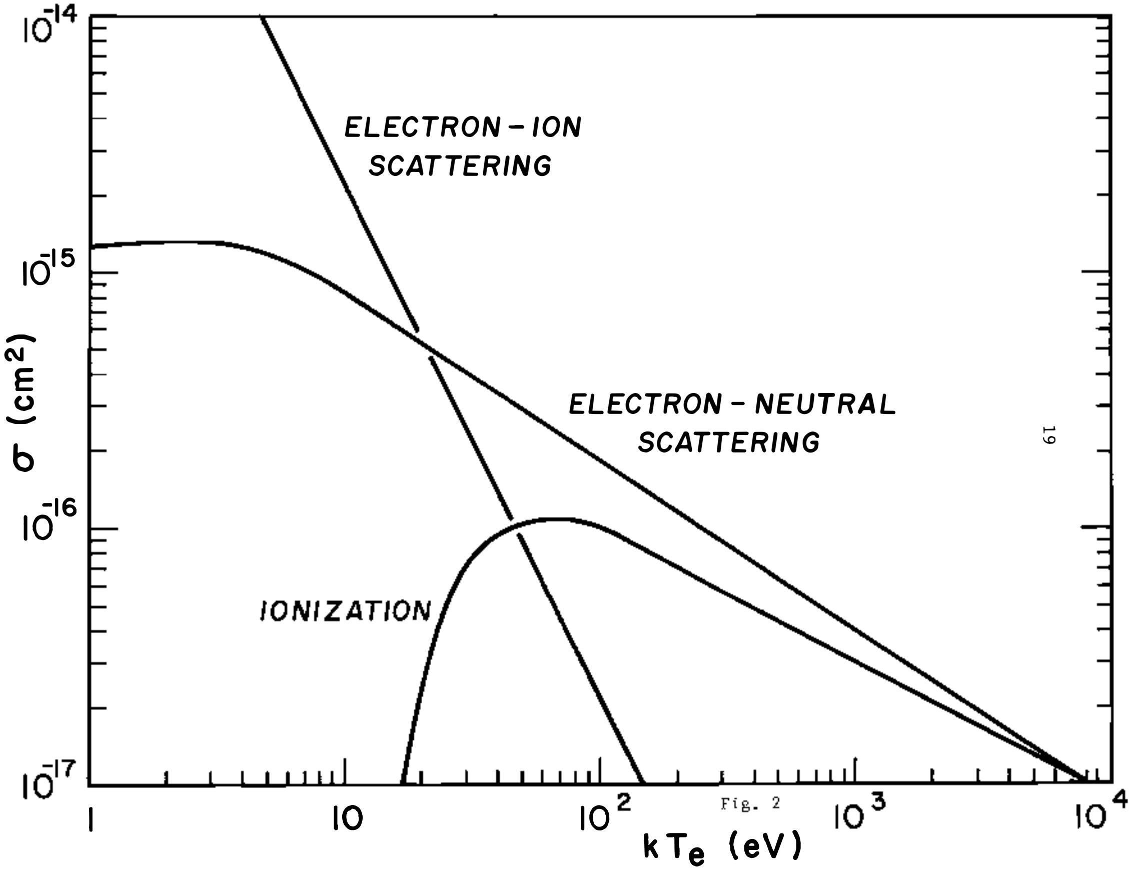


Fig. 1



ELECTRON - ION SCATTERING
ELECTRON - NEUTRAL SCATTERING
IONIZATION
σ (cm²)
10⁻¹⁴
10⁻¹⁵
10⁻¹⁶
10⁻¹⁷
1
10
10²
10³
10⁴
kTₑ (eV)

Fig. 2

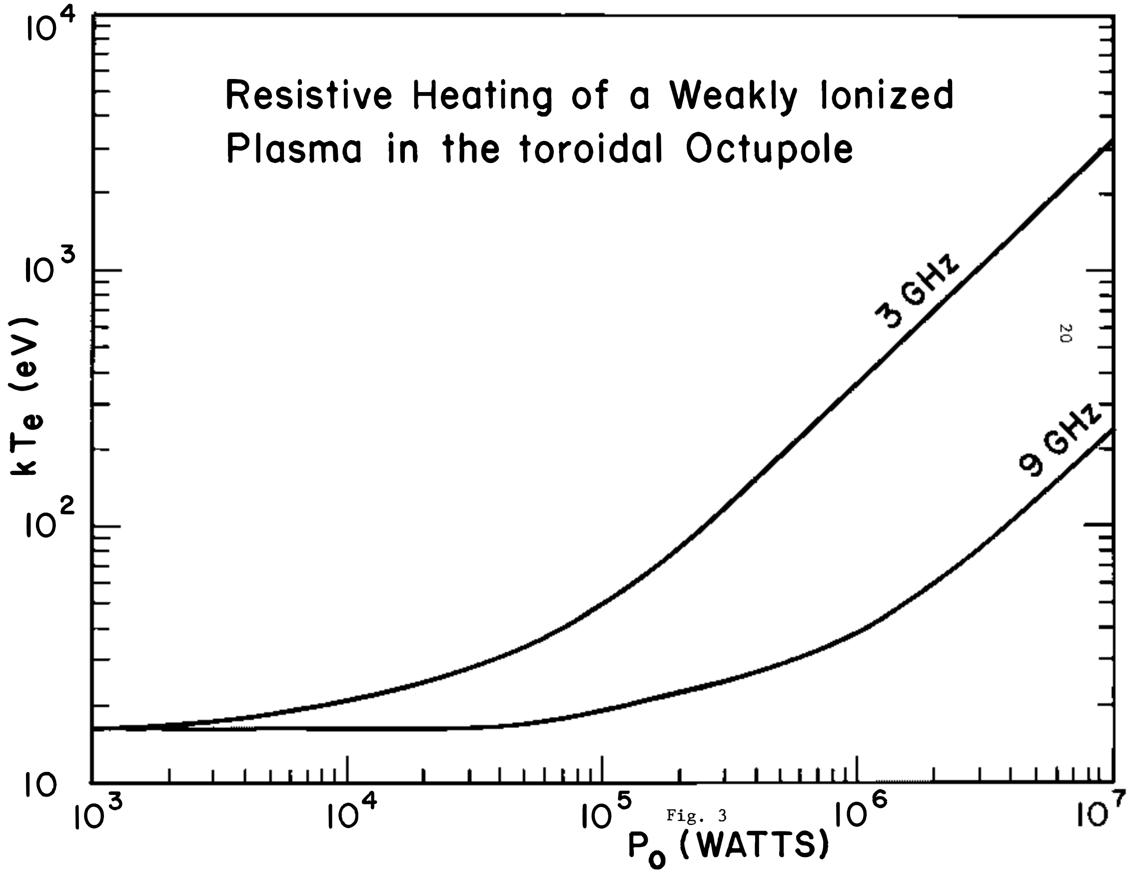

Resistive Heating of a Weakly Ionized
Plasma in the toroidal Octupole
3 GHz
9 GHz
kTe (eV)
P0 (WATTS)
10
10^2
10^3
10^4
10^3
10^4
10^5
10^6
10^7

Fig. 3

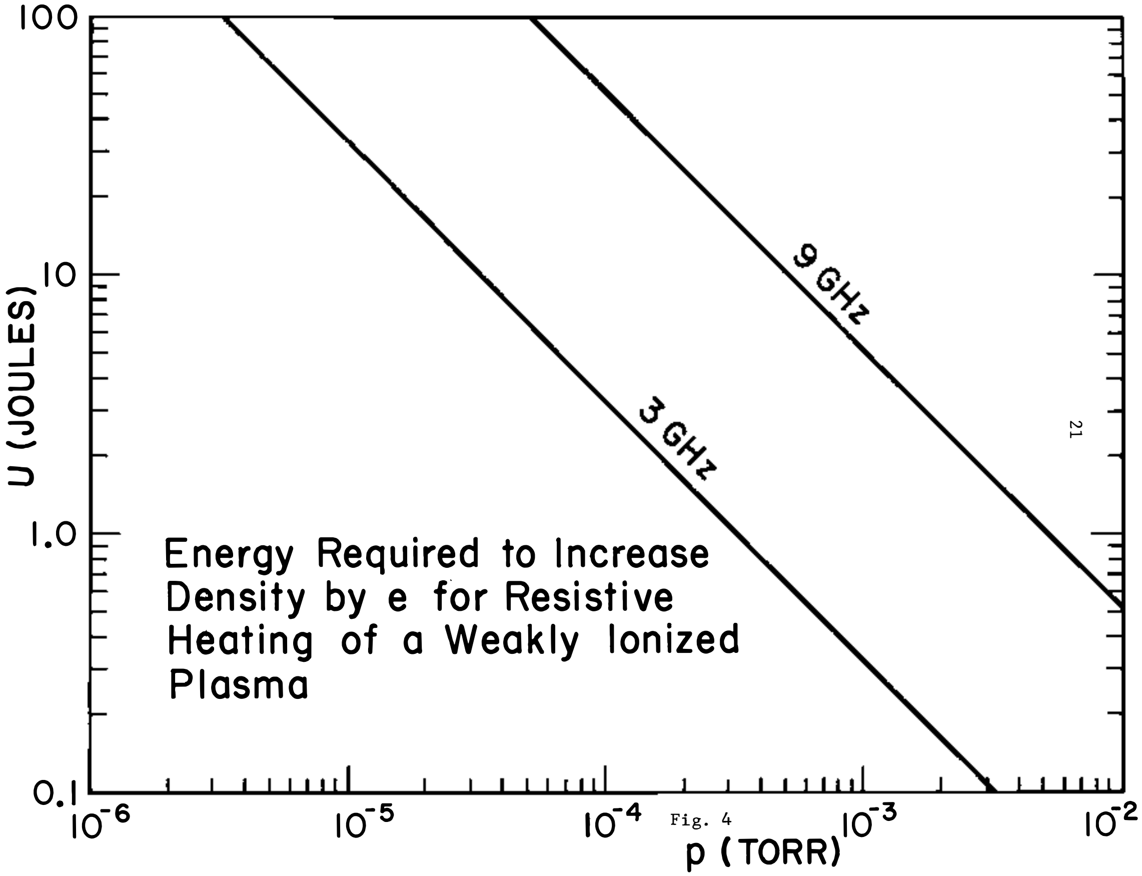

100
10
1.0
0.1
U (JOULES)
9 GHz
3 GHz
Energy Required to Increase Density by e for Resistive Heating of a Weakly Ionized Plasma
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
p (TORR)

Fig. 4

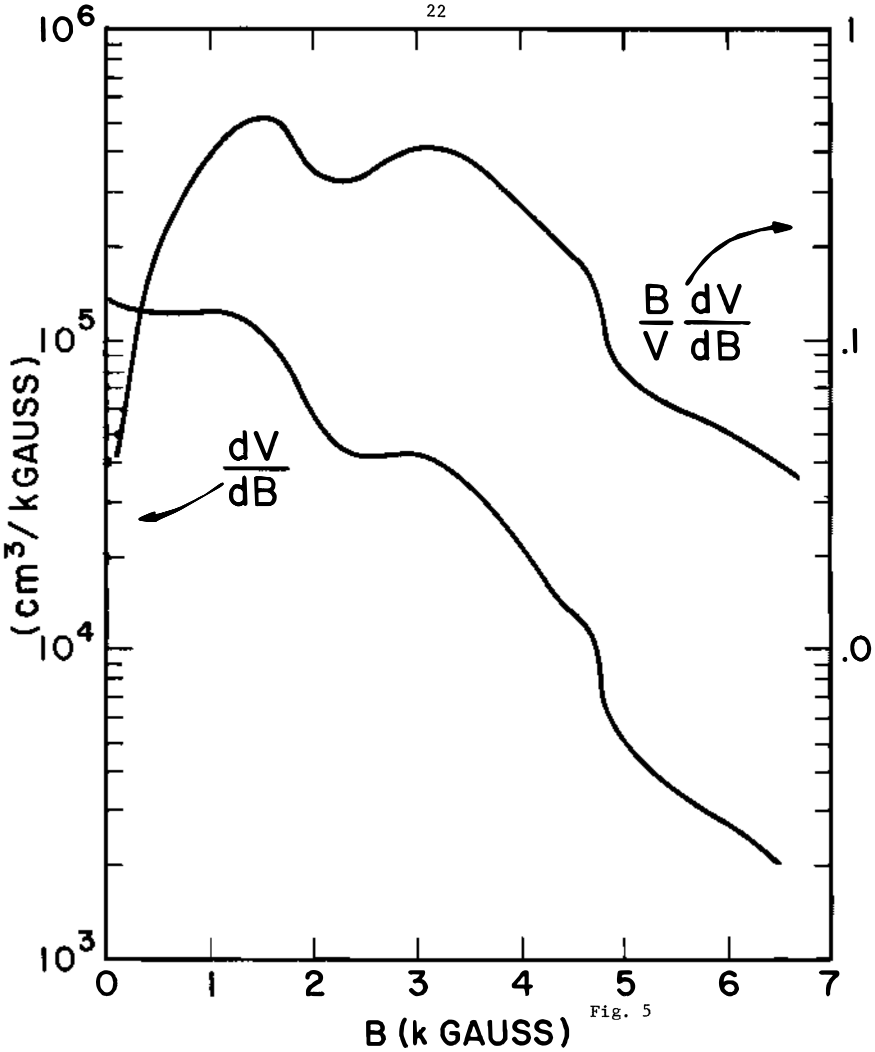


Fig. 5

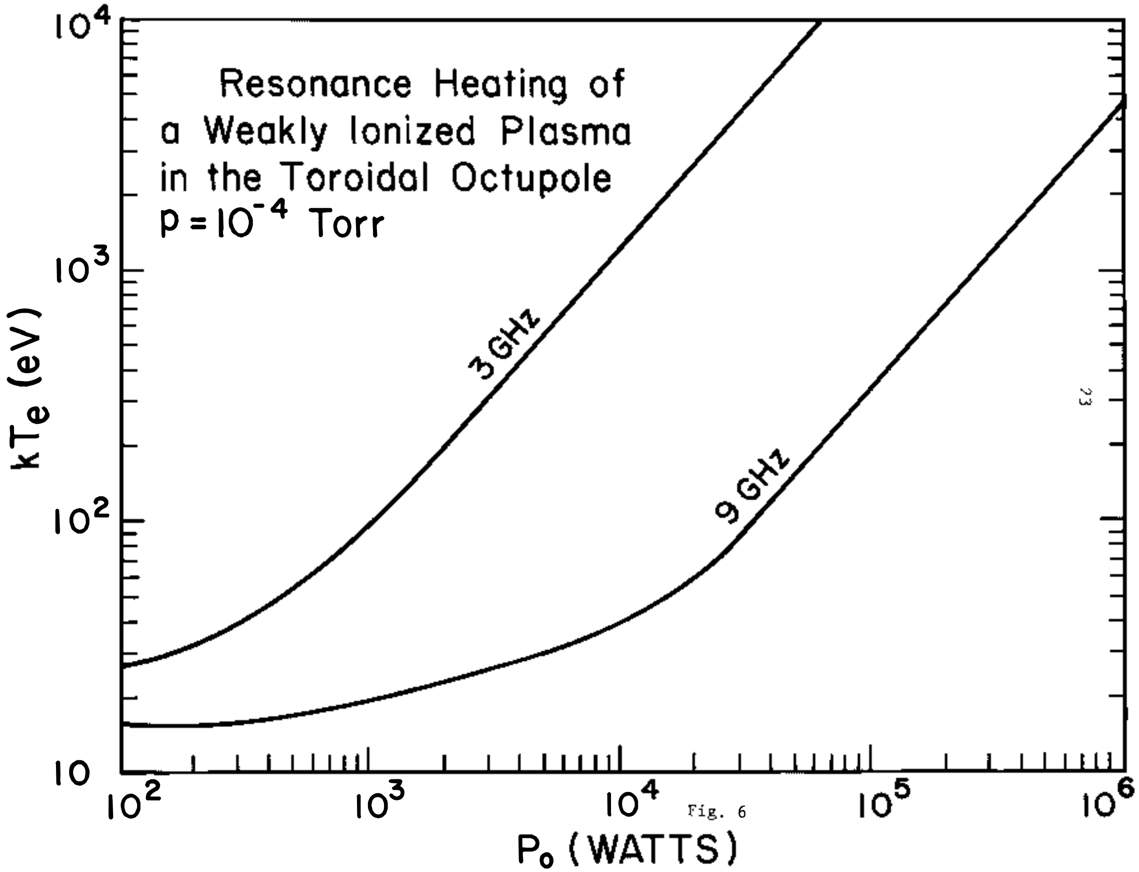

Resonance Heating of
a Weakly Ionized Plasma
in the Toroidal Octupole
p = 10^-4 Torr
3 GHz
9 GHz
kTe (eV)
Po (WATTS)
10
10^2
10^3
10^4
10^5
10^6

Fig. 6

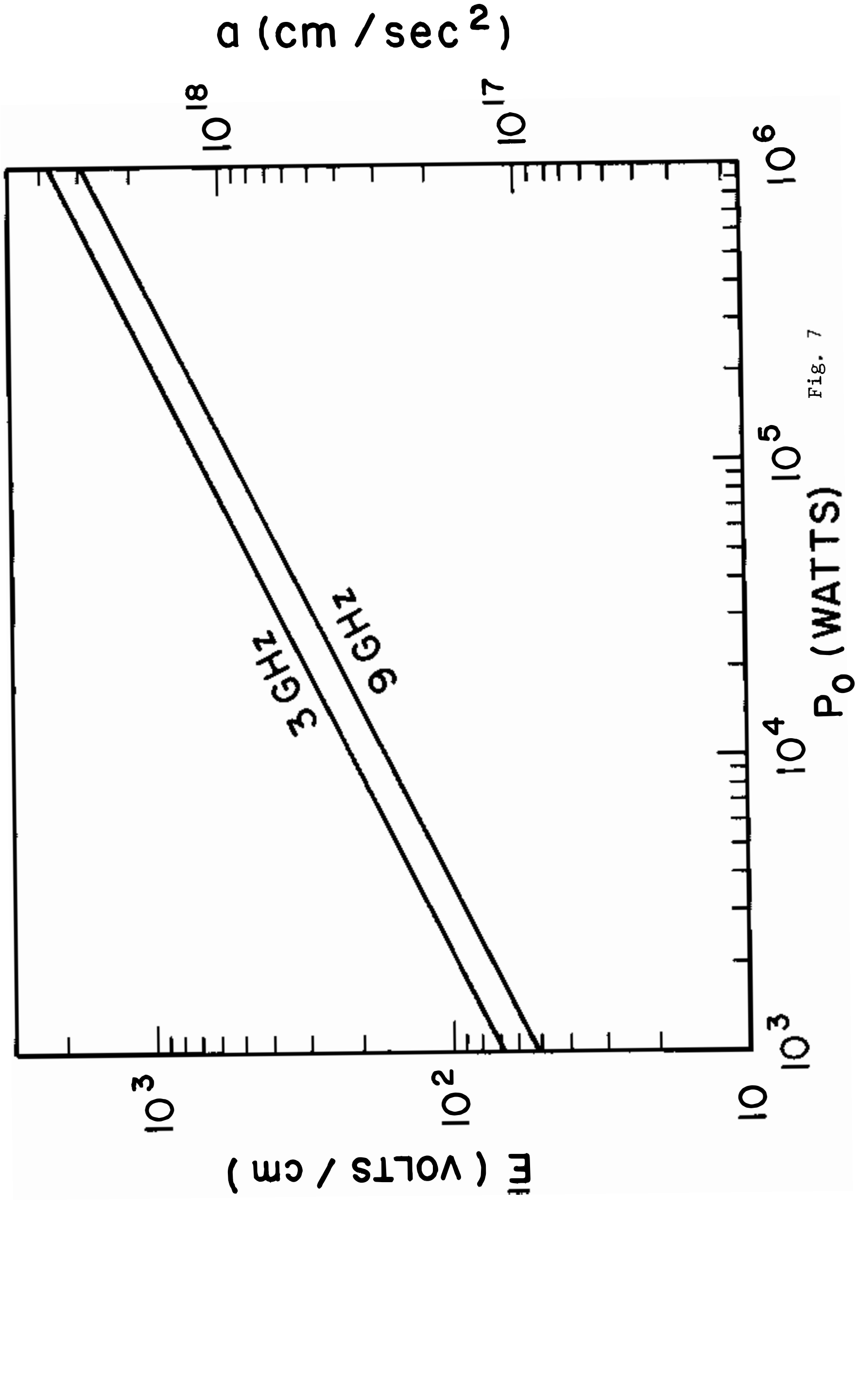

a (cm / sec²)
10¹⁸
10¹⁷
E (VOLTS / cm)
10³
10²
10
3 GHz
9 GHz
10³
10⁴
10⁵
10⁶
P₀ (WATTS)

Fig. 7