On the Possibility of Ion Cyclotron Heating in the Toroidal Octupole

by

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INTRODUCTION

It has been observed previously (PLP 129, 153, 165, 176) that it is possible to impart energy to electrons at the electron cyclotron resonance frequency in the toroidal octupole by filling the cavity with electromagnetic radiation at microwave frequencies. It would be most desirable to extend the technique to lower frequencies in an attempt to heat ions at their cyclotron frequency.

The usual argument against ion cyclotron resonance heating (ICRH) asserts that since electromagnetic radiation does not penetrate a plasma if its frequency is less than the electron plasma frequency, ion cyclotron heating is possible only when $\omega_{pe} < \omega_{ci}$. In the octupole, with a typical magnetic field of 1 kG, this condition would limit the plasma density to ~ 1.7×10^5 cm^{-3}. In fact, it is often possible for low frequency waves to penetrate a considerable distance into a plasma having a much higher density. For example, in the case of zero magnetic field, the penetration depth of electromagnetic waves of frequency ω is given by

$$\delta = \frac{c}{\sqrt{\omega_{pe} - \omega^2}} \cong \frac{c}{\omega_{pe}} \qquad (1)$$

where the latter approximation is valid for $\omega \ll \omega_{pe}$. If we require that the waves propagate a distance of 10 cm, we conclude that ICRH should be possible at densities up to ~ 2.8×10^9 cm^{-3}. Since both the gun and microwave-produced plasmas, in the toroidal octupole, have densities of this order, it may well be possible to heat ions by a fairly simple means.

The frequency required for ICRH would be less than that required for ECRH by the electron/ion mass ratio m/M, or about 1 MHz. Since the lowest resonant mode of the present toroidal cavity is ~ 300 MHz, the usual method of resonating the cavity would be ineffective. At 1 MHz, perhaps the simplestway to generate electric fields within the plasma is by applying a voltage between the hoops and the wall. This case is also easy to treat theoretically since there would be no azimuthal electric field ($E_\theta = 0$) and no electric field parallel to the magnetic field ($E_\ell = 0$). The elctric field can then be written as the gradient of a potential in ψ (magnetic flux) space:

$$\vec{E} = -\hat{s}\,\frac{\partial\Phi}{\partial s} = -\,\nabla\psi\,\frac{\partial\Phi}{\partial\psi}. \qquad (2)$$

MKS units are used throughout. A useful conversion factor for the octupole case is

$$1 \text{ dory} \cong 3.1 \times 10^{-3} \text{webers} = 3.1 \times 10^{5} \text{ gauss-cm}^2$$

where 10 dorys are the total flux in the toroid at the normal operating field (2 kV on the capacitor bank, 800 guass at the outside wall midplane).

CALCULATION OF POTENTIAL DISTRIBUTION IN PLASMA

The potential distribution inside an infinite parallel plate capacitor with a uniform transverse magnetic field and a nonuniform plasma density can be easily derived in the low frequency limit ($\omega \ll \omega_{ci}$). If the plasma does not touch the capacitor plates, the plasma polarizes in such a way as to satisfy the boundary condition εE = const. The zero temperature low frequency perpendicular dielectric constant is given by

$$\varepsilon = \varepsilon_o + \frac{M + m}{B^2}\,n. \qquad (3)$$

For B = 1 kG and densities well above 5×10^7 cm^{-3}, Eq. (3) reduces to

$$\varepsilon \cong \frac{Mn}{B^2}.$$

The potential distribution is found by integration:

$$\Phi(s) = \int \vec{E}\cdot d\vec{s} = \int \frac{\text{const.}}{\varepsilon}\,ds$$

$$= \text{const.} \int \frac{B^2}{n} ds$$

In ψ-space, $B \propto \frac{\partial\psi}{\partial s}$, and

$$\Phi(\psi) = \text{const.} \int \frac{B}{n} d\psi.$$

The constant can be determined from the boundary condition that the potential difference between the capacitor plates is Φ_o:

$$\Phi(s) = \Phi_o \frac{\int_0^s \frac{ds}{n(s)}}{\int_0^d \frac{ds}{n(s)}} \qquad (4)$$

where d is separation of the capacitor plates. The prediction of Eq. (4) is that the potential tends to be constant in regions where the density is highest.

A much more general expression can be derived taking into account non-uniform magnetic fields and arbitrary frequency. Assume that charges can flow freely parallel to $\vec{B}$ so that Poisson's equation can be written for each field line as

$$\oint \left(\nabla\cdot\vec{E} - \frac{\rho}{\varepsilon_o}\right) \frac{d\ell}{B} = 0.$$

As is frequently done, the term $\nabla\cdot\varepsilon_o\vec{E} - \rho$ can be written in terms of a general permittivity ε which includes the effect of the polarization currents:

$$\nabla\cdot\varepsilon_o\vec{E} - \rho = \nabla\cdot\varepsilon\,\vec{E}.$$

From this we obtain the result:

$$\oint (\nabla\cdot\varepsilon\,\vec{E}\,) \frac{d\ell}{B} = 0$$

Equation (5) is a general result valid for an arbitrary magnetic field provided the density and potential are constant on a magnetic field line.

A more useful form of Eq. (5) can be obtained by expanding and evaluating the divergence term:

$$\nabla\cdot\varepsilon\vec{E} = \varepsilon\ \nabla\cdot\vec{E} + \vec{E}\cdot\nabla\psi$$

$$= \varepsilon\frac{\partial E}{\partial s} + E\frac{\partial \varepsilon}{\partial s}\ .$$

$$= \varepsilon\frac{\partial E}{\partial \psi}\nabla\psi + E\frac{\partial \varepsilon}{\partial \psi}$$

For an azimuthally symmetric poloidal magnetic field, $\vec{B}$ can be written as

$$\vec{B} = \frac{1}{2\pi}\nabla\theta x\nabla\psi = \frac{1}{2\pi R}\hat{\theta}\ x\ \nabla\psi,$$

and Eq. (5) reduces to

$$\oint\left|\varepsilon\frac{\partial E}{\partial \psi} + E\frac{\partial \varepsilon}{\partial \psi}\right| Rd\ell = 0 \qquad (6)$$

where R is the distance to the major axis of the toroid. From Eq. (6) one can calculate $E(\psi)$ and hence $\Phi(\psi)$ by applying the appropriate permittivity and boundary conditions.

As a fairly general example, we will consider a collisionless plasma in an arbitrary magnetic field. The permittivity for an oscillating electric field linearly polarized perpendicular to $\vec{B}$ can be derived from the equation of motion for ions and electrons. The result is

$$\varepsilon = \varepsilon_o\left[1 - \frac{\omega_{pe}^{\ 2}}{\omega^2 - \omega_{ce}^{\ 2}} - \frac{\omega_{pi}^{\ 2}}{\omega^2 - \omega_{ci}^{\ 2}}\right]. \qquad (7)$$

Note that in the limit of $\omega >> \omega_{ce}$, Eq. (7) reduces to the familiar result:

$$\varepsilon = \varepsilon_o\left[1 - \frac{\omega_{pe}^{\ 2}}{\omega^2}\right].$$

Also for $\omega \ll \omega_{ci}$, Eq. (7) reduces to

$$\varepsilon = \varepsilon_o \left[1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} + \frac{\omega_{pi}^2}{\omega_{ci}^2}\right]$$

$$= \varepsilon_o \left[1 - \frac{M + m}{\varepsilon_o B^2} n\right]$$

which is identical to Eq. (3).

To simplify things somewhat, we will consider the wave frequency to be much smaller than the electron cyclotron frequency ($\omega \ll \omega_{ce}$) but comparable to the ion cyclotron frequency ($\omega \lesssim \omega_{ci}$). For this case

$$\varepsilon \cong \varepsilon_o \left[1 + \frac{\omega_{pi}^2}{\omega_{ci}^2 - \omega^2}\right]$$

and

$$\frac{\partial \varepsilon}{\partial \Psi} \cong \varepsilon_o \left[\frac{\omega_{pi}^2}{\omega_{ci}^2 - \omega^2} \frac{1}{n} \frac{\partial n}{\partial \psi} - \frac{2\omega_{pi}^2 \omega_{ci}^2}{(\omega_{ci}^2 - \omega^2)} \frac{1}{B} \frac{\partial B}{\partial \psi}\right] .$$

Assuming that the density is high enough that

$$\left[\frac{\omega_{pi}^2}{\omega_{ci}^2 - \omega^2}\right] \gg 1$$

(i.e., $n \gg 5 \times 10^7$ cm^{-3} at B = 1 kG), the above quantities can be substituted into Eq. (6) to obtain the result:

$$\oint \left[\frac{1}{B^2 - B_o^2} \frac{\partial E}{\partial \psi} + \frac{1}{B^2 - B_o^2} \frac{E}{n} \frac{\partial n}{\partial \psi} - \frac{2B^2}{(B^2 - B_o^2)^2} \frac{E}{B} \frac{\partial B}{\partial \psi}\right] R d\ell = 0$$

where $B_o = \frac{M}{e} \omega$ is the value of B which gives ion cyclotron resonance at the wave frequency. The quantities n, $\frac{\partial n}{\partial \psi}$, and $\frac{\partial \Phi}{\partial \psi} = \frac{E}{2\pi RB}$ are assumed to be independent of ℓ and can be brought out of the integral leaving

$$\frac{\partial^2 \Phi}{\partial \psi^2} \oint \frac{R^2 B}{B^2 - B_o{}^2} \, d\ell + \frac{\partial \Phi}{\partial \psi} \left[\oint \frac{RB}{B^2 - B_o{}^2} \frac{\partial R}{\partial \psi} \, d\ell \right.$$

$$+ \oint \frac{R^2}{B^2 - B_o{}^2} \frac{\partial B}{\partial \psi} \, d\ell + \frac{1}{n} \frac{\partial n}{\partial \psi} \oint \frac{R^2 B}{B^2 - B_o{}^2} \, d\ell$$

$$\left. - 2 \oint \frac{R^2 B^2}{(B^2 - B_o{}^2)^2} \frac{\partial B}{\partial \psi} \, d\ell \right] = 0 \tag{8}$$

Equation (8) is the first major result of this paper. From it one can calculate $\Phi(\psi)$ from a knowledge of n (ψ) and the geometry of the field. It is useful to stop at this point and list the assumptions that were made in its derivation:

1) Density constant on a magnetic flux surface: $n = n(\psi)$
2) Potential constant on a magnetic flux surface: $\Phi = \Phi(\psi)$
3) Plasma not in contact with the electrodes producing the electric field
4) Gyroradii small compared with characteristic distances such as $B/|\nabla B|$, $\Phi/|\nabla\Phi|$, etc.
5) Azimuthally symmetric poloidal magnetic field
6) Collisionless plasma
7) Low frequency electric field: $\omega \ll \omega_{ce}$, $\omega \lesssim \omega_{ci}$
8) High density:

$$\left| \frac{\omega_{pi}{}^2}{\omega_{ci}{}^2 - \omega^2} \right| \gg 1$$

The above assumptions may seem rather restrictive but they are reasonably well satisfied in the plasmas normally studied in the toroidal octupole.

Equation (8) will not be evaluated in this paper, but one limiting case, one in which $\omega \ll \omega_{ci}$, will be considered. In this low frequency limit the resonances at $B = B_o$ do not appear and Eq. (8) reduces to

$$\frac{\partial^2 \Phi}{\partial \psi^2} \oint \frac{R^2 d\ell}{B} + \frac{\partial \Phi}{\partial \psi} \left[\oint \frac{R}{B} \frac{\partial R}{\partial \psi} d\ell \right.$$

$$\left. - \oint \frac{R^2}{B^2} \frac{\partial B}{\partial \psi} d\ell + \frac{1}{n} \frac{\partial n}{\partial \psi} \oint \frac{R^2 d\ell}{B} \right] = 0. \qquad (9)$$

The last three integrals are of order $\frac{1}{R} \frac{\partial R}{\partial \psi}$, $\frac{1}{B} \frac{\partial B}{\partial \psi}$, and $\frac{1}{n} \frac{\partial n}{\partial \psi}$ respectively. Unfortunately these terms are all of the same order except perhaps the first which is slightly smaller than the others in the octupole case. For a straight, uniform field, $R \to \infty$ and $\frac{\partial B}{\partial \psi} \to 0$ so only the third integral remains. In this case Eq. (9) reduces to

$$\frac{\partial^2 \Phi}{\partial \psi^2} + \frac{1}{n} \frac{\partial n}{\partial \psi} \frac{\partial \Phi}{\partial \psi} = 0.$$

This equation can be solved for $\frac{\partial \Phi}{\partial \psi}$ by integration:

$$\int \frac{d\left(\frac{\partial \Phi}{\partial \psi}\right)}{\left(\frac{\partial \Phi}{\partial \psi}\right)} + \int \frac{dn}{n} = 0$$

or

$$\frac{\partial \Phi}{\partial \psi} = - \frac{c}{n(\psi)}$$

where c is a constant to be determined from the boundary conditions. One further integration gives $\Phi(\psi)$:

$$\Phi(\psi) = -c \int \frac{d\psi}{n(\psi)}$$

For the octupole, take $\Phi(\psi = +5) = 0$ and $\Phi(\psi = -5) = \Phi_o$, then $\Phi(\psi)$ can be written as

$$\Phi(\psi) = \Phi_o \frac{\int_\psi^5 \frac{d\psi}{n(\psi)}}{\int_{-5}^5 \frac{d\psi}{n(\psi)}} \tag{10}$$

Note that Eq. (10) is of the same form as Eq. (4) indicating that Eq. (8) is correct in the limit of low frequencies and uniform fields.

Finally, for $\omega_{ci} << \omega << \omega_{ce}$, Eq. (8) reduces to

$$\frac{\partial^2\Phi}{\partial\psi^2} \oint R^2 B d\ell + \frac{\partial\Phi}{\partial\psi} \left[\oint RB \frac{\partial R}{\partial\psi} d\ell + \oint R^2 \frac{\partial B}{\partial\psi} d\ell + \frac{1}{n}\frac{\partial n}{\partial\psi} \oint R^2 B d\ell\right] = 0$$

For a straight uniform magnetic field, this equation can be solved for $\Phi(\psi)$ in a manner similar to that of Eq. (9), giving a result identical to Eq. (10). This fact suggests that Eq. (10) is more general than is implied by the assumptions required for its derivation.

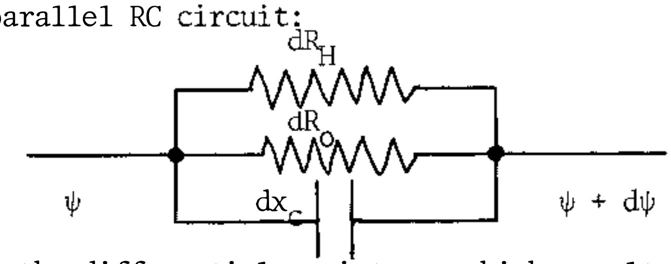
CALCULATION OF REQUIRED POWER

If the rf power could be coupled to the plasma with 100% efficiency, the power required to heat the ions from zero temperature to T_i would be given by

$$P = \frac{NkT_i}{t}$$

where N is the number of particles in the cavity and t is the rf pulse duration. For the s band microwave plasma, $N \simeq 10^{15}$. Taking a pulse length of t = 1 msec, we conclude that only .16 W are required to raise the temperature by 1 eV.

In reality, there are other losses which would almost certainly lower the efficiency. It is possible to represent the electrical coupling between two ψ lines as a parallel RC circuit:



dR_H represents the differential resistance which results in ion heating and dR_O represents other loss mechanisms such as diffusion, particle loss to the walls and hoops, and perhaps others. dx_C is the differential capacitive reactance which distributes the potential according to the derivation in the preceeding section.

The power which is useful for ion heating is given in terms of dR_H by

$$dP = Id\Phi = \frac{d\Phi}{dR_H}\,d\Phi = \left(\frac{d\Phi}{d\psi}\right)^2 \frac{d\psi}{dR_H}\,d\psi \qquad (11)$$

where $\frac{d\Phi}{d\psi}$ can be found from Eq. (8) or, since dR_O may be smaller than dx_C, $\frac{d\Phi}{d\psi}$ should probably be measured experimentally.

The resistance dR_H can be calculated by expressing it as a function of a real conductivity, :

$$dR_H = \frac{d\Phi}{I} = d\Phi \;/\; \oint\!\!\int \sigma\, ERd\theta d\ell .$$

Or by writing E as $2\pi \frac{d\Phi}{d\psi} RB$, we obtain

$$\frac{d\psi}{dR_H} = 4\pi^2 \oint \; BR^2 d\ell . \qquad (12)$$

Note that the R in the integral is the distance to the major axis of the toroid and not a resistance.

In order to proceed further, it is necessary to express σ as a function of ψ and ℓ. The conductivity can be written as

$$\sigma = -\omega \ \mathrm{Im} \ \varepsilon. \qquad (13)$$

Note that if a permittivity such as that of Eq. (7) is used, the perpendicular conductivity is zero. This fact raises an interesting paradox. It means that in a collisionless plasma, no power can be absorbed from a wave even if its frequency is the same as the electron or ion cyclotron frequency. The explanation lies in the fact that Eq. (7) is derived from the steady state equation of motion of a charged particle. Since an oscillating electric field at the cyclotron frequency will continuously accelerate particles, a steady state can never be reached. In reality there is always some dissipative effect such as collisions which limits the particle energy and causes power to be absorbed.

If a collision frequency ν is assumed, and if ω is taken to be well below the electron cyclotron frequency, the perpendicular permittivity can be written as

$$\varepsilon = \varepsilon_o \left[1 - \frac{\omega_{pi}^2}{\omega} \frac{\omega - i\nu}{(\omega - i\nu)^2 - \omega_{ci}^2} \right]$$

which for $\nu << \omega$ becomes

$$\varepsilon = \varepsilon_o \left[1 - \frac{\omega_{pi}^2}{\omega} \frac{\omega(\omega^2 - \omega_{ci}^2 + 2\nu^2) + i\nu(\omega^2 + \omega_{ci}^2)}{(\omega^2 - \omega_{ci}^2)^2 + 2\nu^2(\omega^2 + \omega_{ci}^2)} \right] .$$

The conductivity in Eq. (13) then becomes

$$\sigma = \varepsilon_o \omega_{pi}^2 \nu \frac{\omega^2 + \omega_{ci}^2}{(\omega^2 - \omega_{ci}^2)^2 + 2\nu^2(\omega^2 + \omega_{ci}^2)}.$$

The spatial dependence of σ is more easily recognized by making the substitutions:

$$\omega_{ci} = \frac{e}{M} B, \qquad \omega = \frac{e}{M} B_o, \qquad \nu = \frac{e}{M} \Delta B.$$

The conductivity is then written as

$$\sigma = ne \frac{(B_o{}^2 + B^2)\ \Delta B}{[(B_o{}^2 - B^2)^2 + 2(\Delta B)^2(B_o{}^2 + B^2)]}$$

and Eq. (12) becomes

$$\frac{d\psi}{dR_H} = 4\pi^2 ne \oint \frac{(B^2 + B_o{}^2)BR^2\Delta B}{(B^2 - B_o{}^2)^2 + 2(\Delta B)^2(B^2 + B_o{}^2)} d\ell \ .$$

For ΔB sufficiently small, most of the contribution to the integral comes from $B \cong B_o$, and B can be expanded in a Taylor series about B_o,

$$B(\ell) \cong B_o + \ell \frac{\partial B}{\partial \ell}\bigg|_{B_o} \ ,$$

where $\ell = 0$ is taken to be the position at which $B = B_o$. Then to first order, $B^2 - B_o^2$ becomes

$$B^2 - B_o^2 \cong 2B_o\ell \ \frac{\partial B}{\partial \ell}\bigg|_{B_o}$$

and the integral can be written as

$$\frac{d\psi}{dR_H} = 2\pi^2 ne \int_{-\infty}^{\infty} \frac{R^2 B_o \Delta B}{\ell^2 \left(\frac{\partial B}{\partial \ell}\right)^2 + (\Delta B)} d\ell$$

$$= 2\pi^3 ne\ B_o \sum_{res} R^2 / \frac{\partial B}{\partial \ell} \ . \tag{14}$$

The sum is taken over every intersection of the ψ line and the constant B surface $B = B_o$. Substituting Eq. (14) into Eq. (11) gives the power absorbed by the ions between ψ and $\psi + d\psi$:

$$dP = 2 \left(\frac{d\Phi}{d\psi}\right)^2 \pi^3 enB_o \sum_{res} R^2 / \frac{\partial B}{\partial \ell}\ d\psi. \tag{15}$$

Equation (15) is the second major result of this paper. From it one can calculate the amount of power absorbed by the ions on each ψ line provided the potential distribution $\Phi(\psi)$ is known. Equation (15) is valid under the same conditions as Eq. (8) except that no assumption about the magnitude of the density was made.

Equation (15) can be used to calculate the spatial dependence of the ion temperature:

$$kT_i(\psi) = \frac{tdP}{nV'd\psi}$$

where $V' = \oint \frac{d\ell}{B}$.

From Eq. (15) we obtain

$$kT_i(\psi) = \frac{2\pi^3 eB_o t}{V'} \left(\frac{d\Phi}{d\psi}\right)^2 \sum_{res} R^2 / \frac{\partial B}{\partial \ell}. \tag{16}$$

This prediction suggests that ion cyclotron heating should not be effective near the separatrix since $V' \to \infty$ there.

It is of interest to digress for a moment to consider how this treatment might apply to electron cyclotron resonance heating. Since Eq. (16) is independent of the particle mass, it applies equally well to the electrons provided of course that B_o is interpreted as the value of B for which electron cyclotron resonance occurs. For the high frequencies involved (> 1 GHz), ψ lines are not equipotentials and $\frac{d\Phi}{d\psi}$ must be written as $E/2\pi RB$. For short wavelengths and low densities ($\omega_{pe}^2 << \omega\nu$), it can be assumed that the electric field is random and can be averaged over a flux surface to give a mean square perpendicular electric field $\overline{E^2}$. The electron temperature is then

$$kT_e(\psi) = \frac{\pi e\overline{E^2}t}{2V'B_o} \sum_{res} 1 / \frac{\partial B}{\partial \ell} \tag{17}$$

Written in slightly different form, Eq. (17) becomes

$$kT_e(\psi) = \pi^2 e\overline{E^2}t \sum_{res} \frac{R}{V'|\nabla\psi \times \nabla B|} .$$

If we include the effect of ionizing collisions, we conclude that the electron thermal energy levels off at a value near the ionization energy U_i, and the density distribution can be written in terms of an initial distribution $n_o(\psi)$ as

$$n(\psi) \cong n_o(\psi) \frac{\pi^2 e\overline{E^2}t}{U_i} \sum_{res} \frac{R}{V'|\nabla\psi \times \nabla B|} \qquad (18)$$

Equation (18) is identical to Eq. (5) of PLP 142 except that the unknown constant. multiplying the summation has been evaluated. The result of PLP 142 came from purely geometrical considerations and lends confidence to the present treatment. The term $\sum_{res} R/V'|\nabla\psi \times \nabla B|$ is plotted vs. ψ for various values of B_o in Figs. 3-7 in PLP 142.

Furthermore, it is possible to calculate the total number of electrons produced by integrating Eq. (18) over-all ψ using the fact that

$$\frac{dV}{dB} = \int \frac{2\pi R d\Psi}{|\nabla\psi \times \nabla B|} .$$

The result is

$$N = n_o \frac{\pi}{2} \frac{e\overline{E^2}t}{U_i} \left.\frac{dV}{dB}\right|_{B_o} .$$

This same result was obtained by a similar method in PLP 186.

Another prediction of Eq. (15) is that the power absorbed by either electrons or ions and hence the temperature obtainable is proportional to $\overline{E^2}t$. For ECRH, electric fields of about 200 V/cm for times of about 100 μsec are required to ionize the gas (~ 15 eV) and raise the electron temperature to a few eV. By this reasoning we conclude that the order of 100 V/dory electric field for 1 msec would be required to effect appreciable ion cyclotron heating. By actual measurement of the hoop to wall impedance, R_o appears to be several hundred ohms for the s band microwave produced plasma and hence ~ 5 kW of rf power would be required to generate the necessary voltages. The efficiency of power conversion in this case would be about .01 %. By a very rough evaluation of Eq. (16), it appears that the theoretical prediction for ICRH gives a result that is the same order of magnitude as that deduced from the ECRH tests. More exact calculations of these quantities will be carried out by computer in the future.

One objection that one might raise to the preceeding theory is that it is based on the use of a zero temperature dielectric constant. In effect, we have assumed that the particle gyroperiod is short compared with the time required for the particle to drift into an appreciably different magnetic field so that a steady state exists at all times. If ℓ is the scale length of the magnetic field, $B/|\nabla B|$, this condition places an upper limit on the particle energy for which the above treatment is valid:

$$U_{max} = \frac{1}{2} m\ell^2 \; f^2 = \frac{\ell^2 e^2 B_o{}^2}{8\pi m}$$

If we take a typical value of, say, ℓ = 1 cm, we conclude that for electrons the theory is valid for energies up to 2.5 keV for s-band or 25 keV for x-band microwaves. For ions the situation is considerably worse as indicated by the mass in the denominator. For a frequency of 1 MHz we expect the model to break down above about 0.5 eV for protons. The problem is worse if one considers that near resonance even smaller changes in the particle position, of order $\frac{\nu}{\omega} B/|\nabla B|$ would be sufficient to destroy the steady state. However, if most of the particle energy is in perpendicular motion, the situation is improved by the factor $T_\perp/T_\parallel$.

Plasma was produced by a 5 kW, 144 μsec, pulse of 3250 MHz microwaves. The microwave pulse began 1 msec after the beginning of the magnetic field pulse, and the magnetic field was operated at .37 of its usual value. The background gas pressure was 5×10^{-5} torr (H_2).

Figure 1 shows the magnitude of the impedance measured as a function of frequency at 2 msec after the beginning of the magnetic field pulse. From 10 kHz to 1 MHz the plasma can be approximated as a parallel RC circuit with R = 350 Ω and C = 3000 pF. Measurements of the phase of the voltage and current confirm that the load is indeed resistive at low frequencies and capacitive at high frequencies. The impedance does not vary appreciably for RF signal levels in the range 1 to 30 V peak-to-peak. Of the 3000 pF, approximately 300 is due to external wiring. An effective dielectric constant of the plasma can be estimated by comparing this capacitance with the measured vacuum capacitance:

$$\frac{\varepsilon}{\varepsilon_o} = \frac{3000 - 300}{850} = 3.2.$$

The increase in impedance above 1 MHz is unexplained, but it may be due to stray inductance in the wiring.

The next step consisted of actually measuring $\Phi(\psi)$ with an RF voltage applied between the upper outside hoop and the wall. The other three hoops were allowed to float. The potential was measured with a 10 MΩ floating probe. The results are shown in Fig. 2. Note that, as predicted, most of the voltage drop occurs near the boundaries of the plasma. In the region of highest density ($-4 < \psi < 0$), the electric field is essentially zero. The probe measures floating potential rather than plasma potential, but this does not matter since only the peak-to-peak value of the sine wave was measured.

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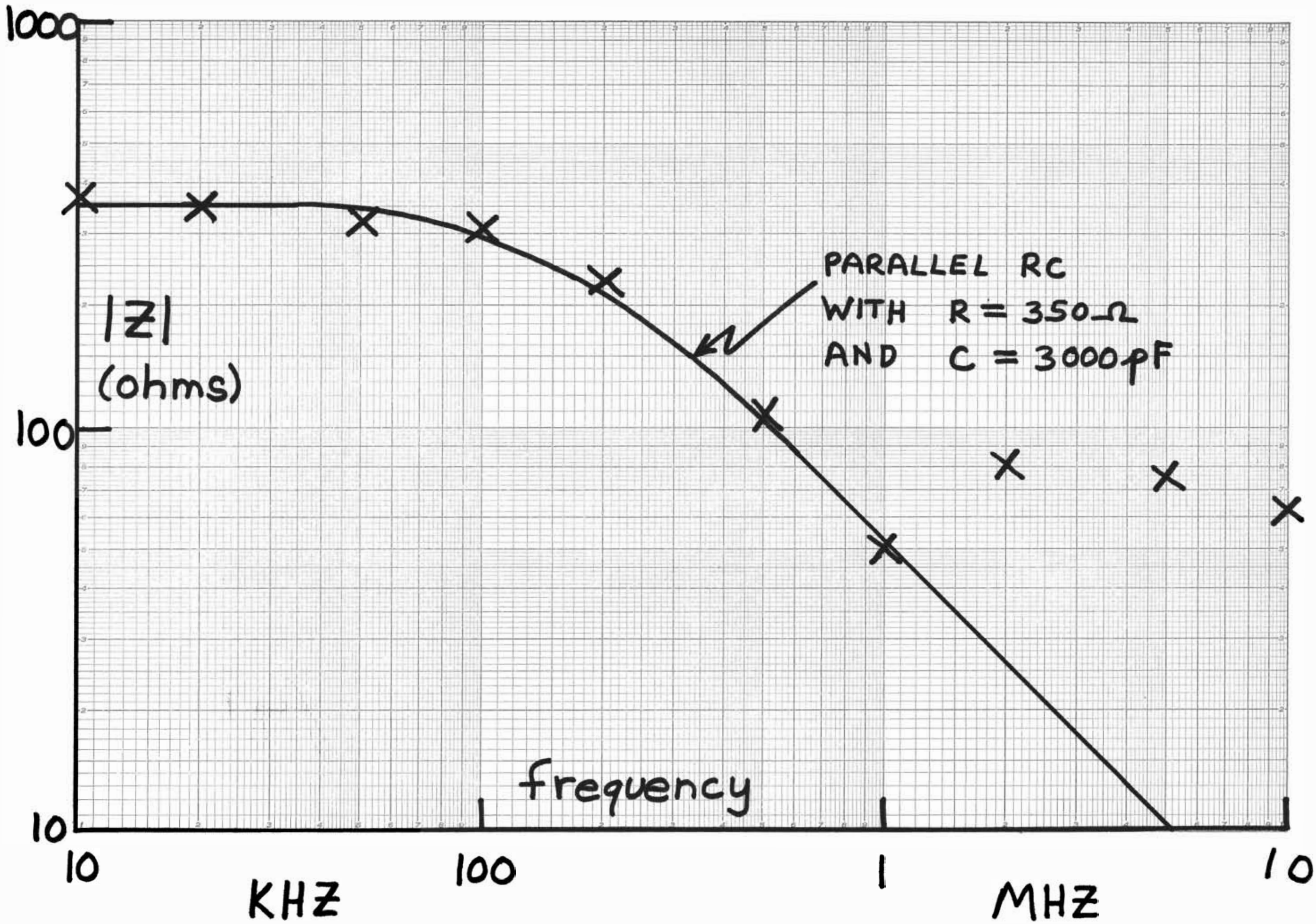

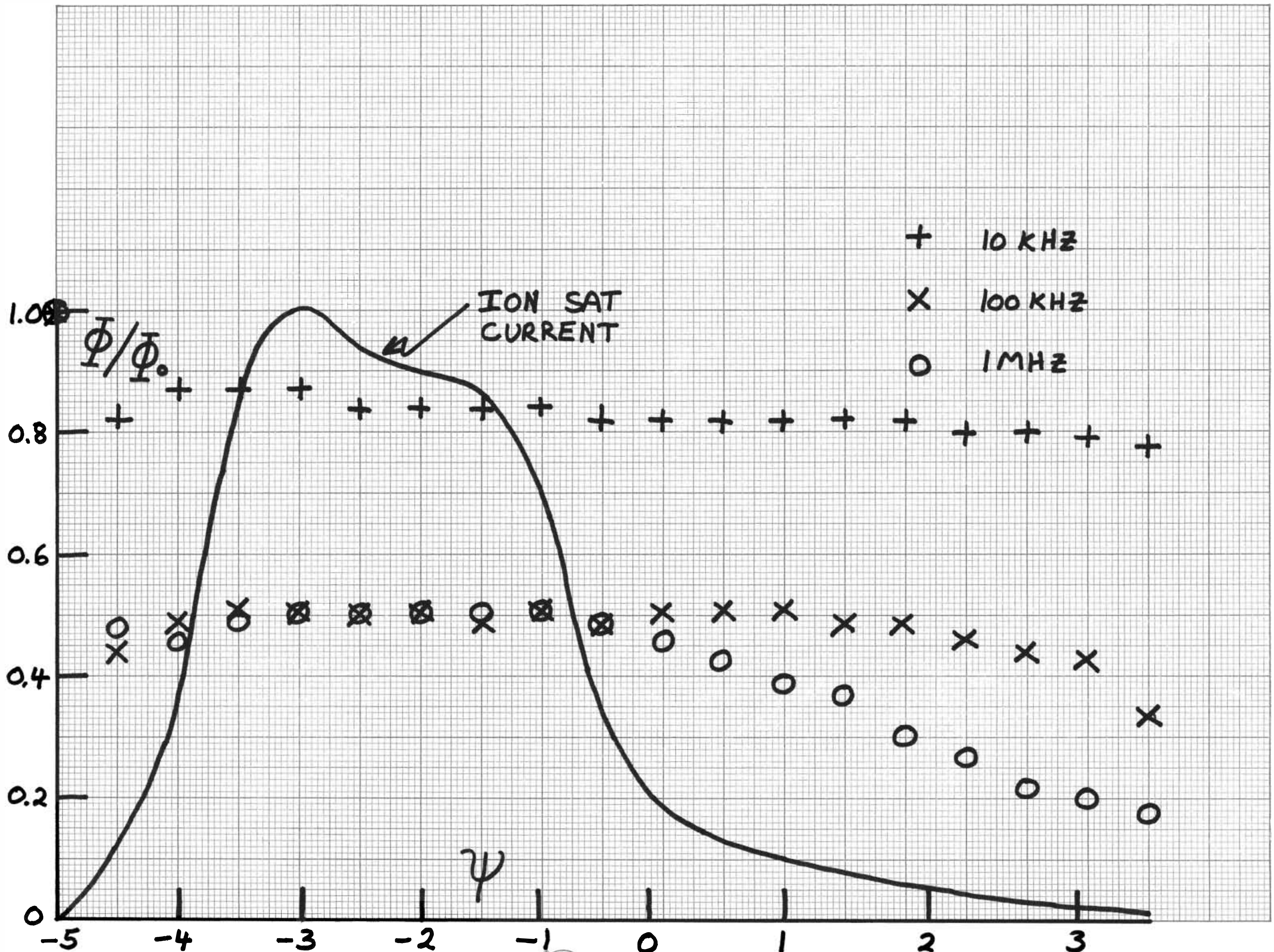

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