A Comparison of Electron Cooling Mechanisms in the Toroidal Octupole

by

J.C. Sprott

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INTRODUCTION

A theoretical comparison of electron cooling mechanisms in the toroidal octupole is important for at least three reasons: 1) it is desirable to understand the causes of the observed temperature decay during the confinement period; 2) when the plasma is heated with rf electric fields, the equilibrium temperature is determined by a balance between the rf heating rate and the sum of all cooling mechanisms; 3) since electron temperature is a parameter in many theories, it would be desirable to predictably vary the electron temperature.

In this paper, we will list all possible cooling mechanisms and then treat each one in turn to determine quantitatively its importance for typical parameters in the toroidal octupole. A Maxwellian electron velocity distribution will be assumed. For comparison, the cooling rates will be expressed in terms of the parameter γ defined as follows:

$$\frac{d\, T_e}{dt} = -\gamma\, T_e$$

ELECTRON COOLING MECHANISMS

1. Ionization

It was shown in PLP 213 that in the presence of ionizing collisions the electron temperature obeys the differential equation

$$\frac{d}{dt}\ (kT_e) + \frac{kT_e + U_i}{\tau_i} = 0$$

where U_i is the ionization energy and τ_i is the ionization time. For molecular hydrogen, U_i is taken to be 15.8 eV (ionization energy of hydrogen atom + 1/2 dissociation energy of hydrogen molecule). The ionization time can be expressed in terms of the neutral density n_n, the ionization cross section $\sigma_i(v)$, and the velocity distribution function f(v) as

$$\frac{1}{\tau_i} = n_n \int_0^\infty \sigma_i(v)\ v\ f(v)\ dv.$$

By approximating $\sigma_i(v)$ with an analytic expression[1] and assuming a Maxwellian f(v), the integral can be evaluated by computer. The result has been tabulated.[2] The cooling rate is then given by

$$\gamma = \left[1 + \frac{U_i}{kT_e}\right] \quad n_n \int_0^\infty \sigma_i(v)vf(v)dv. \tag{1}$$

The first term in the bracket results from the fact that in every ionizing collision the energy is shared between two electrons. The second term comes from the fact that an amount of energy equal to the ionization energy is lost in every ionizing collision. Equation (1) has been plotted in Fig. 1 for a neutral hydrogen pressure of 10^{-6} Torr. The cooling rate is very large above a few eV and scales linearly with pressure.

2. Excitation

Inelastic electron-neutral collisions which do not result in ionization are classed as excitation. The cooling rate for this process is given by

$$\gamma = \frac{n_n}{kT_e} \sum_j U_j \int_0^\infty \sigma_j(v)vf(v)dv \tag{2}$$

where U_j is the energy above the ground state of the jth excited state and $\sigma_j(v)$ is the cross section for excitation to that state. If impurities or excited states are present, the calculation is much more complicated. Since the dominant excitation is from the ground state to the first excited state, only the first term in the sum in Eq. (2)

will be considered. Using published formulas[2] and oscillator strengths[7] Eq. (2) is evaluated for a pressure of 10^{-6} Torr and plotted in Fig. 1. Excitation is an important process below about 10 eV, but is dominated by ionization at high temperatures. The excited state decays in about 2×10^{-9} sec with the emission of a Lyman α photon with a wavelength of 1216 Å.

3. Cyclotron Radiation

Electrons in a magnetic field experience a centrifugal acceleration causing them to radiate at harmonics of the electron cyclotron resonance frequency. The power radiated per unit volume can be derived by integrating the radiation of a single particle over the relativistic distribution[3] with the result

$$\frac{dP}{dV} = \frac{e^4 B^2}{3\pi\varepsilon_o m^2 c} \left(\frac{n_e k T_e}{mc^2}\right) \frac{K_3(mc^2/kT_e)}{K_2(mc^2/kT_e)}$$

where K_2 and K_3 are modified Bessel functions. In the non-relativistic limit, the above equation reduces to the simple form[4]

$$\frac{dP}{dV} = \frac{e^2 \omega_c^2}{3\pi\varepsilon_o c} \left(\frac{n_e k T_e}{mc^2}\right).$$

The decay rate is given by

$$\gamma = \frac{1}{n_e k T_e} \frac{dP}{dV} = \frac{e^4 B^2}{3\pi m^2 \varepsilon_o c^3}$$

or

$$\gamma = 3.9 \times 10^{-3}\ B^2 \qquad (3)$$

where γ is in sec^{-1} and B is in kG. The cooling due to cyclotron radiation is shown in Fig. 1 for a typical magnetic field of 1 kG. A comparison with ionization losses shows that for non-relativistic energies cyclotron radiation is an important cooling mechanism only in ultra high vacua with strong magnetic fields. Note that the cooling rate scales as the square of the magnetic field strength and is independent of temperature.

4. Bremsstrahlung

An electron can radiate as a result of the acceleration it experiences when colliding with 1) another electron, 2) an ion, 3) a neutral molecule, or 4) an obstacle in the plasma. In the non-relativistic limit, electron-electron collisions do not radiate since the acceleration of the electrons are equal and opposite. Electron-neutral collisions are not too important since the orbital electrons shield the electron from the electric field of the nucleus. Collisions with an obstacle generally result in the electron giving up all its energy and being lost from the plasma. This effect will be treated later. The electron-ion Bremsstrahlung can be calculated in a straightforward way[5] with the result that the radiated power density is given by

$$\frac{dP}{dV} = \frac{e^4 n_i n_e}{12\pi\varepsilon_o{}^3 c^3 mh} \sqrt{\frac{3kT_e}{\pi m}} \quad .$$

The cooling rate is then

$$\gamma = 9.5 \times 10^{-14} \quad n_i / \sqrt{kT_e} \tag{4}$$

where n_i is in cm^{-3} and kT_e is in eV. For $n_i = 10^9$ cm^{-3}, Bremsstrahlung losses are so small that they cannot be plotted in Fig. 1. Nevertheless, Bremsstrahlung is important for plasma diagnostics since the radiation has a frequency given approximately by $h\nu \sim kT_e$.

5. Electron-ion scattering

In a plasma with electron temperature T_e and an ion temperature T_i ($<T_e$), energy is transferred from the electrons to the ions by elastic collisions. The rate of electron temperature decay has been calculated by Spitzer[6] as

$$\frac{dT_e}{dt} = (T_i - T_e) \frac{\sqrt{2\pi}\, n_i\, e^4 \;\ell n\, \Lambda}{6\pi^2 \varepsilon_o^2 m M k^{3/2}} \left(\frac{T_e}{m} + \frac{T_i}{M}\right)^{-\frac{3}{2}}$$

If we assume $T_e >> T_i$, the result can be written as

$$\frac{dT_e}{dt} = - \frac{\sqrt{2\pi}\, e^4\, m^{1/2} \;\ell n\, \Lambda}{6\pi^2 \varepsilon_o{}^2 M k^{3/2}} \; \frac{n_i}{T_e{}^{1/2}}$$

For $\ell n\, \Lambda = 20$, the cooling rate is

$$\gamma = 6.35 \times 10^{-8}\, n_i \,/\, (KT_e)^{3/2} \qquad (5)$$

where n_i is in cm^{-3} and kT_e is in eV. Note that cooling due to elastic scattering dominates Bremsstrahlung losses over the entire range of non-relativistic velocities. For an ion density of $10^9\ cm^{-3}$, fig. 1 shows that this process is important in the octupole only at very low temperatures. If the ions have a finite temperature, the cooling is reduced at low temperatures, and $\gamma \to 0$ at $T_e = T_i$.

6. Electron-neutral elastic scattering

In a weakly ionized plasma, elastic scattering of hot electrons off cold neutrals leads to electron cooling. Simple kinematics shows that the energy equipartition time is longer than the 90° scattering time by the neutral/electron mass ratio:

$$\gamma = \frac{m}{M_n} \frac{1}{\tau_s} = \frac{m}{M_n} n_n \sigma_s v$$

where σ_s is the scattering cross section. A rigorous treatment would require that the rate coefficient $<\sigma_s v>$ be evaluated by integrating over the velocity distribution. However, since $\sigma_s (v)v$ is only very weakly velocity dependent, a sufficient approximation will result if we take

$$v = \sqrt{\frac{2kT_e}{m}} \quad .$$

The cooling rate then becomes

$$\gamma = \frac{m}{M_n} n_n <\sigma_s v> \tag{6}$$

Using published values[2] of σ_s for electrons on molecular hydrogen, the cooling rate for electron-neutral elastic scattering has been plotted in fig. 1 for a pressure of 10^{-6} torr. Since cross section data is available only for 1-100 eV, the higher energies were obtained by extrapolation as indicated by the dotted line in fig. 1. Electron-neutral scattering is apparently negligible compared with ionization and excitation for temperatures above about 1 eV.

7. Magnetic field expansion

A detailed prediction of T_e (ψ, t) in a time varying magnetic field in the toroidal octupole has been made by Lencioni[8] under the assumption that the magnetic field is frozen into the plasma. Since we are concerned here with comparing the importance of various cooling mechanisms, it will suffice to use a simplified model in order to estimate the magnitude of the effect. If we postulate some diffusion mechanism which keeps the density distribution fixed in space while the field changes,[9] and if we assume that particle motions are adiabatic so the

$$\frac{U_\perp}{B} = \frac{U_{\perp o}}{B_o} ,$$

the electron temperature for an isotropic velocity distribution is given by

$$T_e = T_{eo}\, B/B_o .$$

For the octupole, B can be approximated by

$$B(t) = B_M \sin \omega t \qquad (o < \omega t < \pi)$$

so that the cooling rate is

$$\gamma = - \frac{1}{T_e} \frac{dT_e}{dt} = - \omega \frac{\cos \omega t}{\sin \omega t}$$

or

$$\gamma = - 628 \cot \omega t \qquad (7)$$

where γ is in sec^{-1}. The plasma is heated for the first quarter period and cooled for the second quarter period. The cooling rate is zero at peak field and infinite when the field goes to zero. Magnetic field expansion can therefore be an important mechanism for electron cooling.

8. Obstacles

Loveberg[10] has shown that the heat flux to an insulator or floating conductor in a plasma is given by

$$Q = \frac{1}{4} n \bar{v}_i \; [\, (eV_s + 2\, kT_e) + 2kT_i]$$

where $\bar{v}_i$ is the average ion velocity and V_s is the sheath potential given by[11]

$$V_s = \frac{kT_e}{2e} \ln \left(\frac{2M}{\pi m} \right) = 3.6\; kT_e \; .$$

For $T_i << T_e$, the heat flux is

$$Q = \frac{1}{4} n \bar{v}_i \; (5.6\; kT_e).$$

By applying the continuity equation for energy

$$\nabla \cdot \vec{Q} + \frac{d}{dt} \; (nkT_e) = 0$$

and particles

$$\nabla \cdot n \vec{v}_i + \frac{dn}{dt} = 0 \; ,$$

it follows that

$$n \frac{d}{dt} \; (kT_e) = - \; 4.6\; kT_e \; \nabla \cdot n \; \vec{v}_i \; .$$

The average cooling rate is then

$$\gamma = 4.6 \times \frac{1}{4} \; \bar{v}_i \; \frac{\int ndA}{\int ndV} \; .$$

For $T_i < T_e$, $\overline{v_i}$ is given in terms of the electron temperature by[12]

$$\overline{v_i} = \sqrt{\frac{4kT_e}{M}} \quad .$$

For n constant in space, we obtain for the octupole ($V = 3 \times 10^5\ cm^3$):

$$\gamma = 7.5\ A\ \sqrt{kT_e} \qquad (8)$$

where A is in cm^2 and kT_e is in eV. Equation (8) is plotted in fig. 1 for $A = 1\ cm^2$. Obstacle cooling is apparently very important, particularly at high energies. Hoop supports can be treated as obstacles with an effective area of $\sim 100\ cm^2$. For $T_e \sim 5$ eV, supports would dominate the cooling for pressures below about 10^{-5} torr. Observations[13] of T_e (t) at various pressures for the gun plasma support this conclusion.

9. Other Mechanisms

There are a number of other mechanisms that are probably of negligible importance. For example, hot electrons striking the wall or hoops could cause secondary emission of colder electrons. However, in a strong magnetic field, these cold electrons would be deflected back to the wall. Recombination would preferentially remove the colder electrons causing a rise in temperature, but the recombination rates are exceedingly small at these low densities. Rotational and vibrational excitation of the hydrogen molecule and nuclear processes are of negligible importance.

Summary

In the temperature range of interest in rf heated plasmas (a few eV to a few hundred keV), the most important cooling processes are ionization and obstacles. Obstacle losses are complicated by the fact that in many heating experiments an isotropic velocity distribution is absent (since $v_\perp >> v_\parallel$) and a strong magnetic field is present. Ionization losses are fairly straightforward and scale linearly with pressure.

It is of interest to compare the cooling rates calculated here with the heating rates expected for typical microwave heating conditions. From PLP 213 we get

$$\frac{dP}{d\psi} = \frac{\pi}{3} n e \overline{E^2} \left. \frac{d^2V}{dBd\psi} \right|_{B_o}$$

or

$$\gamma = - \frac{1}{nV' kT_e} \frac{dP}{d\psi} = - \frac{\pi e \overline{E^2}}{3V' kT_e} \left. \frac{d^2V}{dBd\psi} \right|_{B_o}$$

For a density distribution that is constant in space we can average over ψ to get

$$\gamma = - \frac{\pi e \overline{E^2}}{3kT_e} \left. \frac{dV}{dB} \right|_{B_o}$$

For typical parameters we take E = 200 volts/cm, and dV/dB = 10^5 cm^3/kgauss, to obtain

$$\gamma \approx - 4 \times 10^8 / kT_e \tag{9}$$

where kT_e is in eV. This heating rate is quite large compared with the calculated cooling rates. In a levitated toroidal octupole, obstacle losses should be eliminated and the obtainable temperature would be limited only by the uality of the vacuum.

References

1. H. W. Drawin, Z. Physik, 164, 513 (1961), with correction in Z. Physik, 168, 238 (1962).
2. H. W. Drawin, Groupe de Recherches Sur La Fusion, EUR-CEA-FC-383, Fontenay-aux-Roses, Frances (1966).
3. B. A. Trubnilov and A. E. Bazhanova in Plasma Physics and the Problem of Controlled Thermonuclear Power, L. A. Leontovich, Editor, English Translation, Vol III, Pergamon Press, London (1959), pp 141-175.
4. D. J. Rose and Melville Clark, Plasmas and Controlled Fusion, MIT Press, Cambridge, Mass., (1961), pp 251.
5. Reference 4, page 232 ff.
6. Lyman Spitzer, Jr. Physics of Fully Ionized Gases, Interscience, New York (1962), pp 135.
7. W. L. Wiese, M. W. Smith, and B. M. Glennon, NSRDS-NB4 National Bureau of Standards, Washington (1966).
8. D. E. Lencioni, PLP 200.
9. Experimental observations indicate that this assumption is at least as reasonable as the frozen-in field approximation of Lencioni.
10. R. H. Loveberg in Plasma Diagnostic Techniques, R. H. Huddlestone and S. L. Leonard, editors, Academic Press, New York (1965), pp 95 ff.

11. F. F. Chen in Plasma Diagnostic Techniques, R. H. Huddlestone and S. L. Leonard, editors, Academic Press, New York (1965), pp 178.

12. Reference 11, page 150

13. J. C. Sprott, PLP 176.