Electron Cyclotron Heating

in a Non-Uniform Magnetic Field

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ABSTRACT

The power absorbed by a plasma from an rf electric field is calculated from a real conductivity resulting from collisions or other randomizing processes. For small collision frequencies ($\nu \ll \omega$), the power absorbed in a magnetic flux shell $d\psi$ is independent of collision frequency and is proportional to $d^2V/dBd\psi|_{B_0}$, i.e. to the volume in which the magnetic field is within dB of the resonance value. The model assumes low density and low temperature, although the results are in agreement with a more general single particle treatment by Kuckes.[1] For a Maxwellian velocity distribution and finite ionization time τ_i, the electron temperature initially increases linearly with time, but after $\sim\tau_i$ it approaches a constant, and the density then increases like e^{t/τ_i}. Experimental observations in the Wisconsin toroidal octupole confirm most of the above predictions and indicate that microwave power absorption requires either high background gas pressure or preionization. The first case produces a predominately cold electron (< 10 eV) plasma, while the second case produces a short-lived, hot electron (> 1 keV) component as evidenced by a burst of x-rays during the heating pulse.

INTRODUCTION

Most previous theories of electron cyclotron resonance heating have dealt primarily with uniform or mirror magnetic fields.[2-4] Microwave heating in multipoles and other nonuniform magnetic fields has been successfully employed,[5-10] but theoretical treatments are rare. This paper will outline a simple theoretical model which can be used to estimate the electron cyclotron heating rate in an arbitrary nonuniform magnetic field, and the results will be applied to the Wisconsin toroidal octupole in an attempt to explain some of the experimental observations.

THEORY

The power absorbed in a magnetic flux shell $d\psi$ by a plasma in the presence of an rf electric field is given by

$$\frac{dP}{d\psi} = \int \sigma_{\perp} \overline{E_{\perp}^{2}} \frac{d\ell}{B} \qquad (1)$$

where $\overline{E_{\perp}^{2}}$ is the mean square perpendicular electric field, and $\sigma_{\perp}$ is the real perpendicular conductivity. In the low temperature limit, the steady state conductivity can be written as

$$\sigma_{\perp} = \varepsilon_{o}\omega_{p}^{2} \frac{\omega^{2} + \omega_{c}^{2}}{(\omega^{2} - \omega_{c}^{2})^{2} + 4\omega^{2}\nu^{2}}$$

where ν is a collision frequency which may result from particle collisions or other phase randomizing effects. For $\nu << \omega$, most of the power is absorbed very near resonance, and the magnetic field can be expanded in a Taylor series about its resonance value

$$B(\ell) = B_{o} + \ell \left.\frac{\partial B}{\partial \ell}\right|_{B_{o}} .$$

The integral in Eq. (1) can be evaluated explicitly giving the result

$$\frac{dP}{d\psi} = \frac{\pi}{2} \frac{ne\overline{E_{\perp}^{2}}}{B_{o}} \sum \left.\frac{1}{\partial B/\partial \ell}\right|_{B_{o}}$$

$$= \frac{\pi}{2} ne\overline{E_{\perp}^{2}} \left.\frac{d^{2}V}{dBd\psi}\right|_{B_{o}} . \qquad (2)$$

Note that the power absorbed is independent of collision frequency and is proportional to the volume in which the magnetic field is within dB of the resonance value. An equivalent result has been derived by Kuckes[1] by solving the equation of motion of a single particle during one transit through the resonance region. A stochastic treatment by Lichtenberg[4] also gives similar results. The electric field is assumed to be unperturbed by the plasma near resonance. This assumption places an upper limit on the density for which the model applies by requiring that the width of the resonance be much less than the skin depth for wave penetration

$$\int (\mathrm{Im}k)\, dx \ll 1$$

or

$$\omega_p^{\,2} \ll \omega c \frac{|\nabla B|}{B} .$$

Also, the use of a steady state conductivity is justified only if the particle undergoes many gyrorevolutions during a transit through the resonance region

$$v_{\parallel} \ll \omega B_o \left.\frac{\partial B}{\partial \ell}\right|_{B_o} .$$

These assumptions are well satisfied in many heating experiments.

The effect of ionizing collisions can be included by introducing an ionization time τ_i such that

$$\frac{dn}{dt} = \frac{n}{\tau_i} .$$

The plasma energy density can be written as

$$\frac{d\mu}{d\psi} = \frac{dV}{d\psi}\, n(kT_e + U_i)$$

where U_i is the ionization energy. If we make the somewhat unrealistic assumption that the velocity distribution is Maxwellian with density and temperature constant along the magnetic field, and if we neglect all other loss mechanisms such as excitation, radiation, and diffusion, we can combine the last two equations with Eq. (2) to get a differential equation for the electron temperature

$$\frac{d}{dt}(kT_e) + \frac{kT_e + U_i}{\tau_i} = \frac{\pi}{2} e\overline{E_\perp^{\,2}} \frac{1}{V'} \frac{d^2V}{dBd\psi}\bigg|_{B_o} \tag{3}$$

where $V' = \frac{dV}{d\psi}$ is the volume of a unit tube of magnetic flux. In a multipole, V' is infinite on the separatrix, and so the model predicts negligible heating there. For $t \ll \tau_i$, the second term in Eq. (3) can be neglected, and the temperature increases linearly with time

$$k\Delta T_e(\psi,t) = \frac{\pi}{2} e\overline{E_\perp^{\,2}}\, t \frac{1}{V'} \frac{d^2V}{dBd\psi}\bigg|_{B_o}$$

while the density remains constant

$$n(\psi,t) = n_o(\psi) .$$

For $t \gg \tau_i$, the first term in Eq. (3) can be neglected, and the temperature reaches an equilibrium value

$$kT_e(\psi,t) = \frac{\pi}{2} e\overline{E_\perp^{\,2}}\, \tau_i \frac{1}{V'} \frac{d^2V}{dBd\psi}\bigg|_{B_o} - U_i$$

while the density increases exponentially

$$n(\psi,t) = n_o(\psi)e^{t/\tau_i} .$$

Since τ_i is inversely proportional to the background gas pressure, low pressures would favor the production of hot tenuous plasmas, while high pressures would favor the production of cold dense plasmas.

EXPERIMENT

Figure 1 shows a computer calculated magnetic flux plot in a cross sectional plane of the Wisconsin toroidal octupole. The light lines are magnetic field lines (or ψ lines), and the heavy lines are contours of constant magnetic field strength. Heating is expected to be most favorable wherever the resonance B contour is tangent to a ψ-line. Heating is least efficient near the separatrix where the volume per unit flux is infinite.

Figure 2 shows the result of a heating experiment in the Wisconsin toroidal octupole. A 10 kW, 144 μsec pulse of 3250 MHz microwaves was applied to a background gas at a pressure of 10^{-4} torr with no preionization. The ion saturation current to a special type floating double probe[11] was measured as a function of ψ between the hoops and the wall 60 μsec after the beginning of the heating pulse. The curve has a pronounced dip on the separatrix, where the model predicts negligible heating, and peaks which approximately coincide with infinite values of $d^2V/dBd\psi$. The energy content of the plasma is in quantitative agreement with the theoretical prediction if we assume an average electron energy of the order of 100 eV. By varying the voltage on the capacitor bank used to excite the magnetic field, the location of the poles of $d^2V/dBd\psi$ can be moved to produce a wide variety of ion saturation current distributions. Many of these distributions are MHD unstable, and a considerable readjustment or the density occurs after the microwaves are switched-off.[12]

Figure 3 shows the result of an experiment in which a gun plasma with a density of 10^9 cm^{-3} was used for preionization. The microwaves were applied with a background gas pressure of 10^{-6} torr. The x-rays emitted during the heating pulse were detected by a plastic scintillator embedded in the tank wall and connected to a photomultiplier tube. As the capacitor bank voltage was varied, the x-ray signal reached a sharp peak at a value which corresponds to $d^2V/dBd\psi$ infinite on a field line which is about one 10 keV electron gyroradius from the surface of the inner hoops. To verify that these x-rays are in fact produced by electrons striking the hoops, a collimator was inserted which obstructed the view of the hoops. The peak was completely removed, but some x-rays were still observed, presumably formed by electrons striking the vacuum tank walls or by Bremsstrahlung from the body of the plasma. If the x-rays emitted by the hoops are from the copper K lines, we conclude that electrons with energies greater than 9 keV are present in the plasma. In all cases the x-ray signal disappeared within a few microseconds after the end of the heating pulse.

Figure 4 shows how the density increases during the heating pulse. The average density in the cavity 500 μsec after heating was measured as a function of pulse length using Langmuir probes and 12 mm microwaves. This was necessary because the large fluctuating potentials during the heating period made probes difficult to use, and the high power microwaves interfered with the microwave diagnostics. It was established that the decay rate is independent of density, so that the density at 500 μsec is proportional to the density at the end of the microwave pulse. The initial growth rate is exponential as predicted, and the growth time agrees within a factor of two with the ionization time. Other measurements showed that the growth rate is approximately proportional to pressure. The density levels off at a value such that the microwave frequency (9000 MHz) is about equal to the electron plasma frequency during the heating pulse. By lowering the microwave frequency 3250 MHz, the density levels off an order of magnitude lower, as expected, provided the magnetic field strength is readjusted to maintain the same resonance regions.

Figure 5 summarizes most of the experimental observations. All traces in a row have the same scale, and the columns show the effect of preionization and pressure. With low background pressure (10^{-6} torr) and no preionization, very little microwave power is absorbed by the gas as evidenced by the lack of ion saturation current, light output, and x-rays. When a gun plasma with a density of 10^9 cm^{-3} is used for preionization, the ion saturation current increases only slightly after the microwave pulse is over, and a burst of x-rays is observed during the heating pulse indicating the presence of a hot tenuous plasma component. Preionization from a filament produces similar results. The floating potential is positive during the heating pulse as would be expected if hot electrons were being rapidly lost to the walls and hoops. At higher pressures (10^{-4} torr), a colder denser plasma is produced as evidenced by the large ion saturation current, greater light output, and almost total absence of x-rays. At high pressure, the results are essentially the same with and without preionization.

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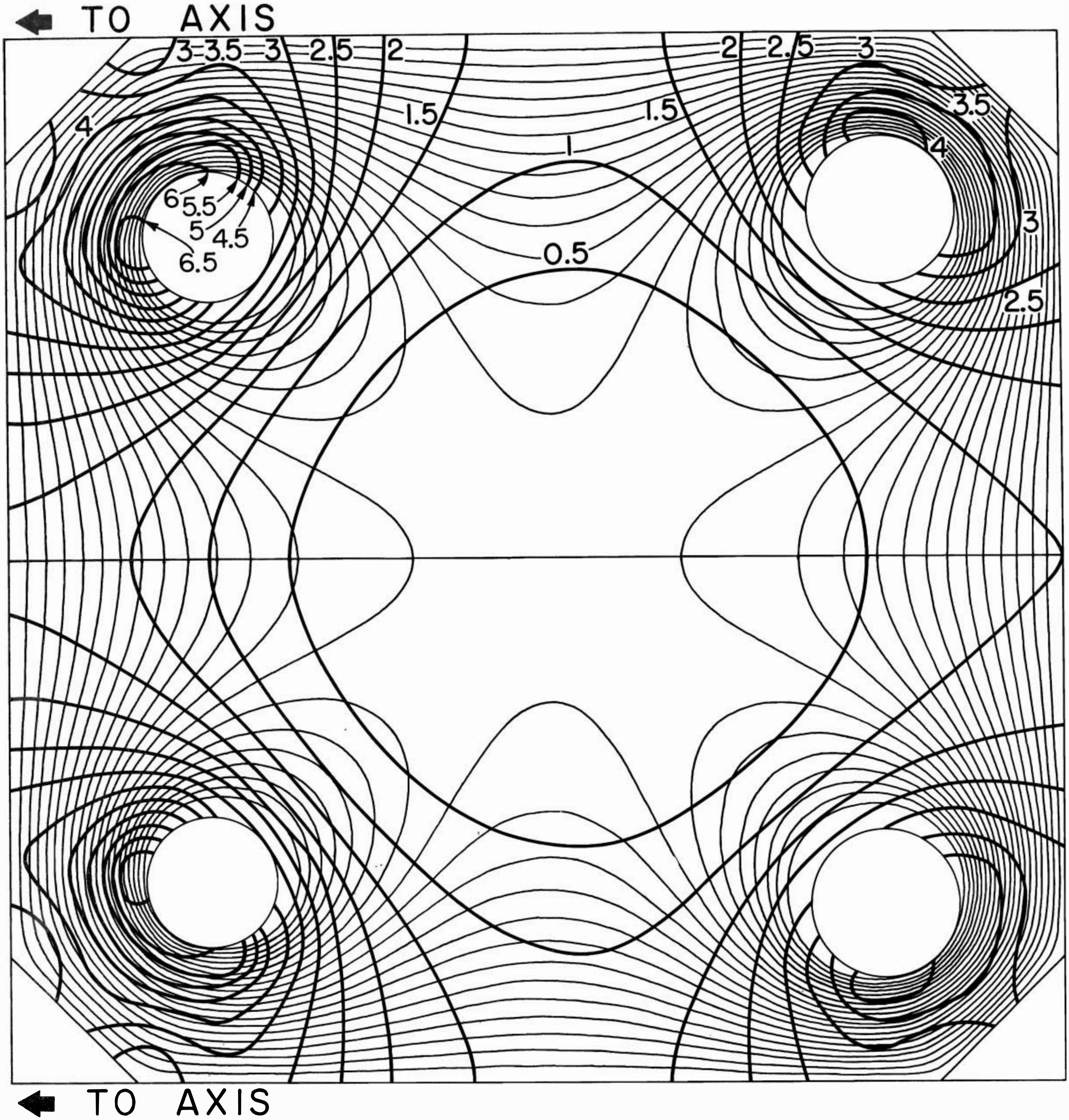
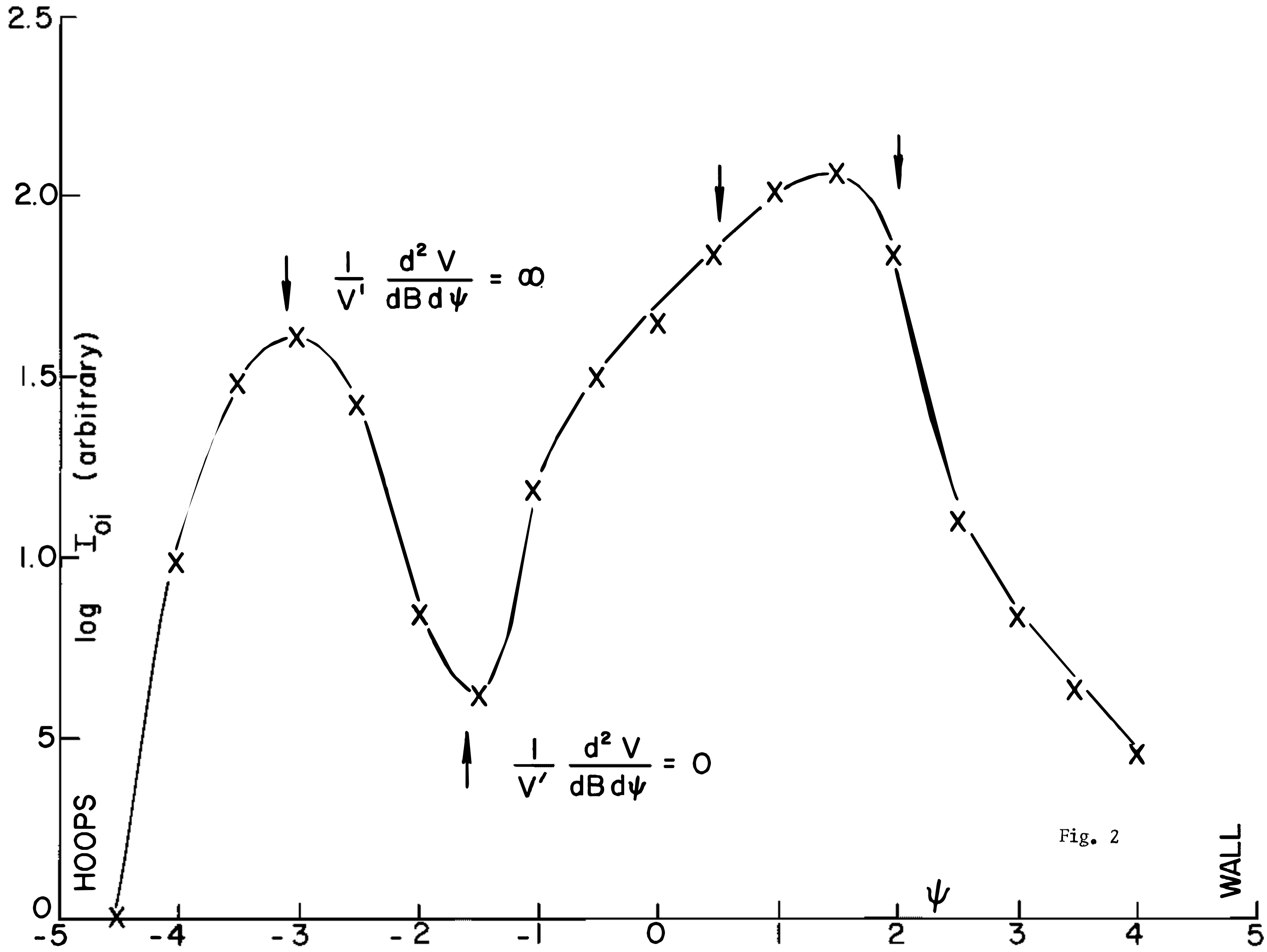


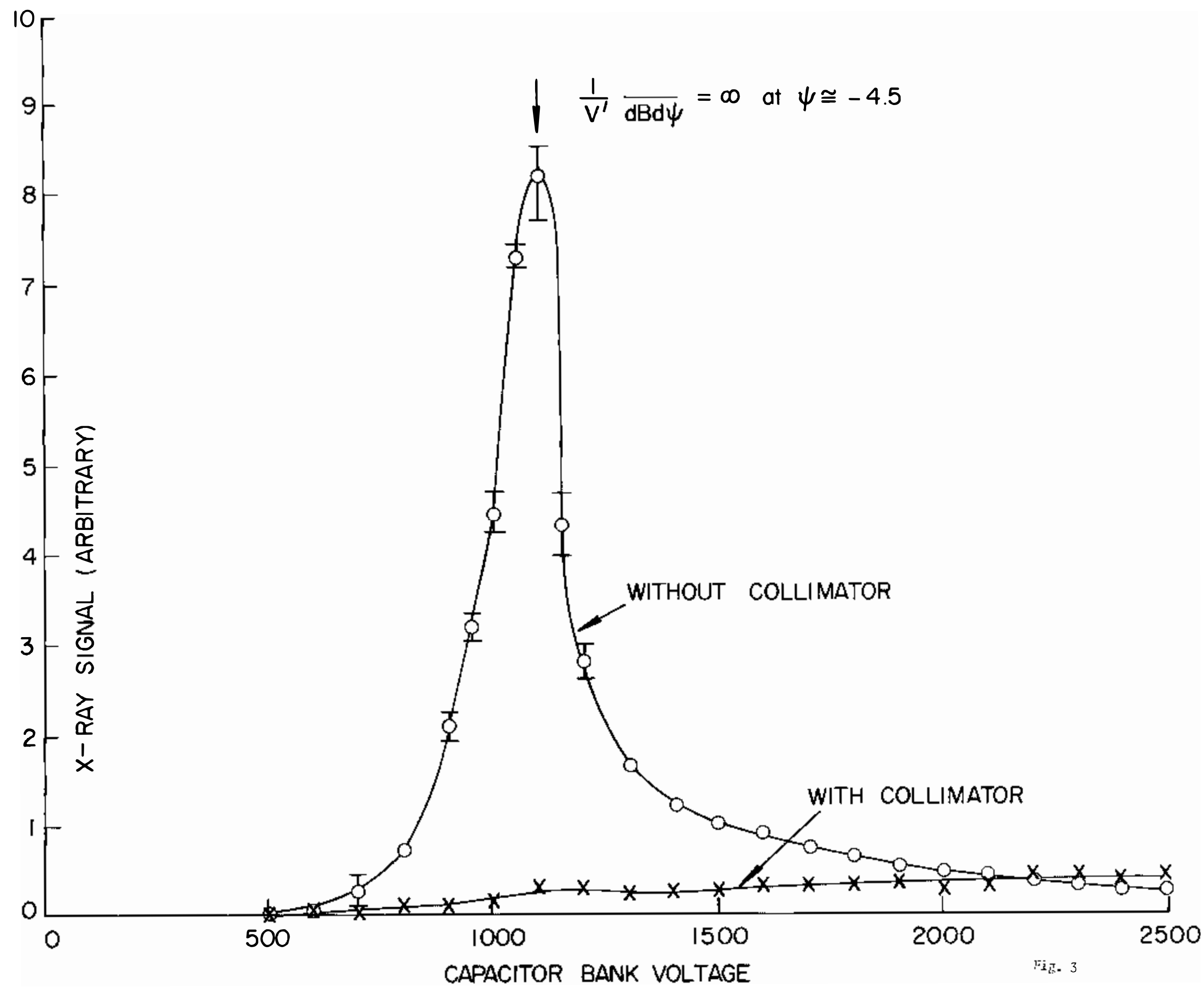
CONSTANT B SURFACES IN THE WISCONSIN TOROIDAL OCTUPOLE

Figure 1



2.5
2.0
1.5
1.0
5
0
log I_{oi} (arbitrary)
HOOPS
WALL
$\frac{1}{V'} \frac{d^2 V}{dB\, d\psi} = \infty$
$\frac{1}{V'} \frac{d^2 V}{dB\, d\psi} = 0$
-5
-4
-3
-2
-1
0
1
2
3
4
5
ψ

Fig. 2



$\frac{1}{V'}\ \overline{dBd\psi} = \infty$ at $\psi \cong -4.5$
WITHOUT COLLIMATOR
WITH COLLIMATOR
X-RAY SIGNAL (ARBITRARY)
CAPACITOR BANK VOLTAGE
0
500
1000
1500
2000
2500
10
9
8
7
6
5
4
3
2
1
0

Fig. 3

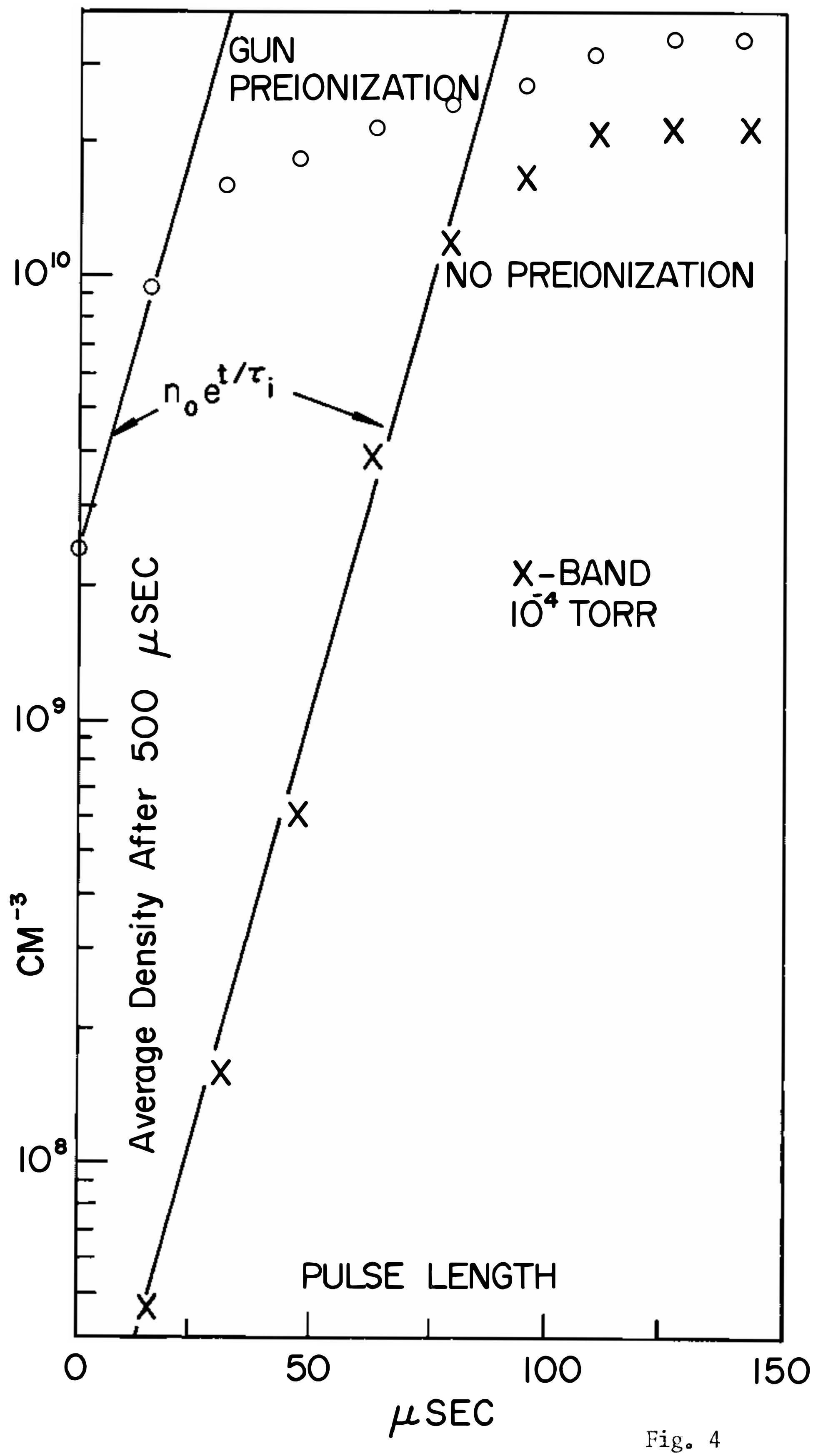
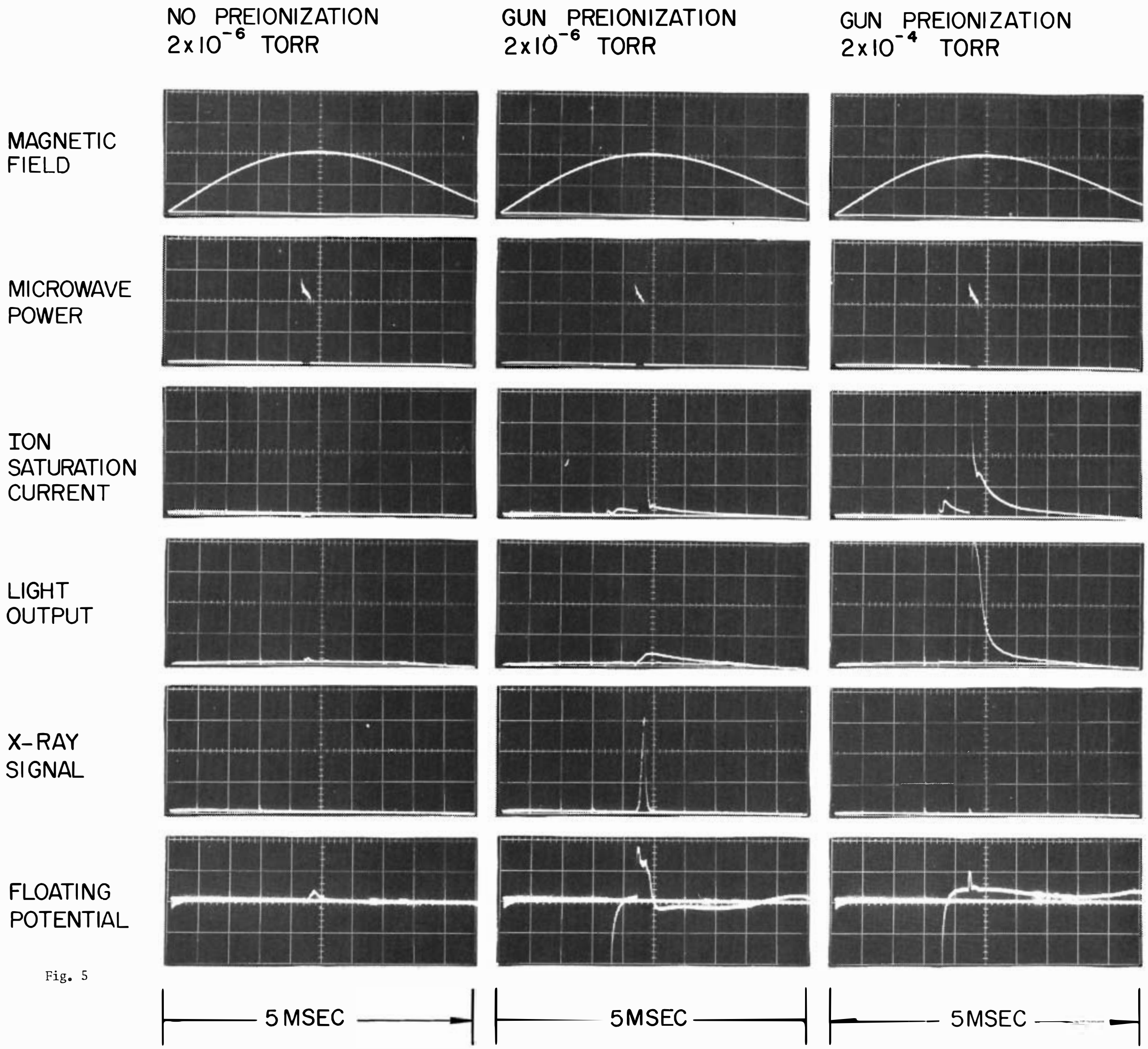

GUN
PREIONIZATION
NO PREIONIZATION
$n_0 e^{t/\tau_i}$
X-BAND
10^{-4} TORR
PULSE LENGTH
10^{10}
10^{9}
10^{8}
CM^{-3}
Average Density After 500 μSEC
0
50
100
150
μSEC

Fig. 4



NO PREIONIZATION
2x10⁻⁶ TORR
GUN PREIONIZATION
2x10⁻⁶ TORR
GUN PREIONIZATION
2x10⁻⁴ TORR
MAGNETIC FIELD
MICROWAVE POWER
ION SATURATION CURRENT
LIGHT OUTPUT
X-RAY SIGNAL
FLOATING POTENTIAL
5 MSEC
5 MSEC
5 MSEC

Fig. 5