Experimental Determination of Microwave Electron Distribution Functions Using Particle Energy Analysis

by

G. W. Kuswa

J. C. Sprott

October, 1969

PLP 314

Plasma Studies
University of Wisconsin

EXPERIMENTAL DETERMINATION OF MICROWAVE ELECTRON DISTRIBUTION FUNCTIONS USING PARTICLE ENERGY ANALYSIS

PLP 280[3] discusses techniques for measuring electron energy distribution functions in the toroidal octupole and gives some preliminary examples of measurements. Here the previous work is expanded, and it is shown that the energetic portions of the distribution functions which result from electron cyclotron resonance heating at several frequencies all resemble $f \sim e^{a\sqrt{E}}\, e^{bE}$, where b is a small positive number. The lifetime of electrons as a function of energy is examined under various conditions, and the attainment of critical density in the resonance zones is examined and shown to be an impediment to the production of very hot electrons some tens of microseconds after the heating pulse starts.

I. LIFETIME MEASUREMENTS

In a microwave heated plasma many electrons above 100 eV are present, and ionization of background gas may be nearly perfect. Under such conditions the predominant loss of electrons should be to obstacles. The production of particles over 100 eV after the heating pulse will be negligible so that a loss rate can be determined with some confidence. At low energies electron e-folding times are not to be interpreted as particle loss rates; both loss and repartitioning of energy as the plasma cools are involved. A plot of electron lifetime against energy is useful as an aid in understanding the distributions to be presented. Figure 1 is a plot of electron lifetime against energy under various conditions for gun produced plasmas and μ-wave heated plasmas after the μ-waves turn off.

Measurements of hot electron lifetime were made using a 32 microsecond burst of X-band microwaves to heat gun-injected plasma. The iron extractor[2] extending to within 3cm of the center of the horizontal midplane guided electrons from the octupole to a 127^o electrostatic analyzer, and the detector was a photo-multiplier and a scintillator covered with a thin aluminum film held at high positive potential[3]. All objects except the extractor pipe were removed from the machine.

The area of the hoop supports was 152 cm^2, and the extractor area was 47 cm^2. An obstacle consisting of the miniature electrostatic energy analyzer[5] with area of 115 cm^2 could be inserted. The flow of particles with velocity v to an object of area A from an isotropic velocity distribution of number density n is given by

$dN/dt = nvA/4$ for energies well above the plasma potential. The factor of 4 arises from a factor of 2 which comes from the average over velocity space and another factor of 2 which arises because the distribution has only one-half density at the surface of an obstacle. The obstacle-limited e-folding lifetime, neglecting electrostatic and convective processes, is

$$\tau = 4 \text{ volume}/vA ;$$

for an effective confinement volume of 250 liters and 199 cm^2

of obstacles,

$$= 8.45 \times 10^{-5} E^{-\frac{1}{2}}$$ seconds, where E is in eV.

Examination of Figure 1 shows that the measured decay of energetic electrons is about 2.5 times less than the prediction but has a 1/v dependence. Since density is not constant over support area and heated electrons have excess $W_\perp$ which keeps them away from the supports, the discrepency between prediction and experiment is not surprising. At energies below 100 eV the lifetimes have more complex dependencies; indeed, in the vicinity 10 eV growth is observed as the plasma cools.

II. ISOTROPY OF HOT ION DISTRIBUTION

The iron extractor pipe could be used only to examine particles incident from the radial direction. The miniature analyzer could readily be pointed in various directions but was limited to examination of energies below 1 keV. Figure 2 shows a plot of detector signal as a function of direction of the miniature analyzer for several energies below 1 keV using forward and reverse B fields. For these measurements gun plasma was injected at the usual 1750 microseconds after multipole excitation, and a 144 microsecond pulse of S-band microwaves was initiated 500 microseconds after the gun fired. Measurements were made during the wave pulse at 2.4 milliseconds; at this time the induced electric field, i.e. A, is very small. Changing the position of the resonance zone by varying the magnetic field had no strong effect on the features of the anisotropy. Reversing the direction of B reversed the sense of the anisotropy. No similar anisotropy was noted for gun plasma electrons. It is possible that the high quasi-D.C. potential fields present during microwave heating[7] cause E x B drifts leading to anisotropy, but more work is needed in this area to understand the details.

III. DISTRIBUTION FUNCTIONS

Since it has been demonstrated that anisotropy of at least 5:1 exists at the center of the horizontal midplane in one location, one must not rely on density measurements made with the iron pipe. One should also note that scintillator probe results are also subject to anisotropy errors since the direction of x-rays produced by electrons impinging on a surface is sensitive to the direction of the electrons. A suggested design for an improved scintillator probe is included in the appendix. This design would permit rapid testing for anisotropy at high energies, which is not possible with present equipment.

If one pretends that the distribution is isotropic, the number of particles and the average energy can be calculated numerically using the relations below:

$$n = \int_0^\infty 4\pi v^2 f(v)\,dv$$

$$E = \frac{1}{e}\int_0^\infty 4\pi v^2 \frac{mv^2}{2} f(v)\,dv$$

$$\langle E \rangle = E/n$$

A transformation from velocity to energy in electron volts in the integrals gives the following relationships with n in units of $(m)^{-3}$.

$$n = 1.31 \times 10^{18} \int_0^\infty E^{1/2} f(E)\, dE$$

$$n \approx 1.31 \times 10^{18} \sum_{m=0}^{\infty} \left[\frac{E_m + E_{m+1}}{2}\right]^{1/2} f\left(\frac{E_m + E_{m+1}}{2}\right) \Delta E_m$$

$$E = 1.31 \times 10^{18} \int_0^\infty E^{3/2} f(E)\, dE$$

$$E \approx 1.31 \times 10^{18} \sum_{m=0}^{\infty} \left[\frac{E_m + E_{m+1}}{2}\right]^{3/2} f\left(\frac{E_m + E_{m+1}}{2}\right) \Delta E_m$$

In the last four equations f has the same units as in the preceeding equations; $(\text{velocity})^2$ is merely expressed as 2Ee/m in the argument of f.

A. S-BAND MICROWAVE HEATING

The microwave parameters[6] were:
3250 MHz, 9.2cm, $B_{resonant}$= 1160 Gauss, peak power = 10kW.

Figure 3 shows the variation of signal from the energy analyzer as a function of time for various electron energies. In this case gun plasma was injected 500 microseconds before the 144 microsecond s-band pulse was switched on. The fast rise of the very hot electrons and their decay even while the microwave pulse is still on is typical behavior for all frequencies tested. The most negative floating potentials exist in the heating zones early in time.[7] Figure 4 shows some distribution functions which result from s-band resonant heating of gun plasma with different background pressures. The number of hot electrons is greatly reduced by background gas. Application of the formulas for numerical integration were used to determine the density and average energy of particles above 100eV for the case of $1x10^{-6}$ torr; $n \approx 1.3x10^{8}$ cm^{-3} and $\langle E \rangle \approx 800$ eV at time 40 μseconds after the 10 kW pulse of microwaves started. Assuming a volume of 250 liters and uniform microwave power, about 1% of the power available has been absorbed and preserved in trapped plasma at 40 μsec. Much higher efficiencies (i.e. > 10%) exist at early times.

B. X-BAND MICROWAVE HEATING

The microwave parameters were:
9000 MHz, 3.3cm, $B_{resonant}$=3200 Gauss, peak power = 10 kW.

Figure 5 shows the distribution function measured 50 μ sec after the start of an x-band microwave burst turned on 500 μsec after gun injection. Application of the numerical integration formulas gave $n = 1.1x10^{8}$ cm^{-3} and average energy 1.6 keV for the group of electrons above 800 eV. Another experiment was carried out with x-band waves to determine how fast heating took place and the distribution relaxed. Figure 6 shows the time-resolved results of heating gun plasma with a 32 microsecond burst of x-band radiation. It is evident that the very hot electrons are produced during the first few microseconds. For this run the miniature analyzer was present in the machine.

C. L-BAND MICROWAVE HEATING

The microwave parameters were:
1300 MHz, 23cm, $B_{resonant}$ = 465 Gauss, peak power = 40kW.

Figure 7 shows the distribution function measured 20 and 40 microseconds after the 144 microsecond nominal pulse of microwaves started. The pulse seemed to short out at $\sim 60\,\mu$sec. The multipole capacitor bank was charged to only 1.5kV to bring the resonance heating zone close to the center of the machine. Gun plasma was injected 500 μsec prior to turn-on of the microwaves for this run, as it was for other runs. Numerically integrating f above 800 eV showed a density of 2×10^{8} cm^{-3} and an average energy of 2 kV. The obstacles in the machine were the extractor pipe, a scintillator probe, and Molvik's hoop collectors.

IV. INTERPRETATION OF RESULTS

A. LOSS MECHANISM FOR HOT ELECTRONS AT LOW PRESSURES

All of the distributions measured have a form similar to $f(v) = Ce^{-a\sqrt{E} + bE}$, where b is a small positive constant. The constants for the three microwave bands examined are given below for E in kilo-electron volts and f in $m^{-6}sec^{3}$. This relation can be trusted only above several hundred electron volts. The background pressure was 1×10^{-6} torr.

	S-BAND	X-BAND	L-BAND
a	7.15	5.92	4.9
b	.37	.42	.311
C	$5.6x10^{-6}$	$8.6x10^{-7}$	$5x10^{-7}$
time after start of wave pulse	40 μsec	50 μsec	40 μsec

Since the loss rate of electrons as a function of electron energy has been found, one could, in principle, determine the heating rate as a function of energy (if the microwave input were a constant). Since the input power was not constant and some gas is liberated from the walls during the pulse, such an energy balance is subject to some error. One can conclude that the form of f is dominated by loss of particles which is velocity-dependent.

B. EFFECT OF BACKGROUND GAS

It can be seen from Figure 4 that a ten-fold rise in pressure of background hydrogen greatly reduces the production of s-band hot electrons. The relation between heating and critical density (at critical density $\omega_{microwaves} = \omega_{plasma}$) was qualitatively examined for X and S band waves; background gas density is greater than critical density for L band waves. A table of critical density and the corresponding pressure of monatomic molecules for the three frequencies used follows. (Note that to obtain actual Torr from ionization gauge readings for hydrogen, the gauge reading must be multiplied by a correction factor of approximately 2.5;

atomic density of hydrogen is about 5 x gauge reading x 3.5×10^{16} cm^{-3}.)

	L-BAND	S-BAND	X-BAND
$n_{critical}$ cm^{-3}	2.1×10^{10}	1.3×10^{11}	1×10^{12}
pressure (Torr) of a monatomic gas	$.6\times10^{-6}$	3.7×10^{-6}	2.85×10^{-5}

The heating of 10 KeV S-band electrons was reduced by 1/e when H_2 was admitted to give an ion gauge reading of 2×10^{-6} Torr. For X-band waves the ion gauge reading must be about 3×10^{-5} Torr to similarly attenuate the same energy electrons. The pressures at which attenuation first was noticeable for a given energy electron were found to differ by about one order of magnitude for s and x-band heating; the ratio of critical densities for the two cases is 7.6. The effect of this attenuation increases during the first tens of microseconds of the microwave pulse as the electron density increases. The effect of increasing the pressure is first noticed on the hot particles; for example, using S-band heating and increasing the pressure from 1×10^{-6} to 5×10^{-5} Torr reduces the flux of 6 keV electrons by a factor of 100, but the flux of 2 keV particles was reduced by only ten.

One can conclude that the hottest electrons are produced in the most favorable parts of the resonance zones, and that the high concentration of energetic electrons in these zones can quickly ionize a large fraction of the background gas; if enough gas is present, the critical density is exceeded locally and efficient heating in the region is no longer possible. In the less-favorable resonance zones and/or the favorable zones operating at reduced efficiency, electrons are still heated but the maximum energy they can attain is lower.

Consideration is now given to ionization cross sections relevant to the creation of a critical density. If $\sigma(v)$ is the ionization crossection for $e + H_2 \rightarrow H + H^+ + 2e$ and $f_-(v)$ is the electron distribution with velocity v, the production of ions is governed by

$$\frac{dn_-}{dt} = \frac{dn_+}{dt} = \int_0^\infty f_-(v)\, n_o\, v\, \sigma(v)\, dv$$

where n_o is the neutral density. Since $v\sigma(v)$ is fairly constant above 100eV for H_2 (ref. 8), if most electrons have energies above 100eV, one can replace the integral by

$$\frac{dn_-}{dt} = \frac{dn_+}{dt} = n_-\, n_o\, v\, \sigma(v) = n_-(n_{o_{initial}} - n_-)\, v\, \sigma(v)$$

Solving,

$$\log \left[\frac{n_{crit}\ (n_o - n_{-initial})}{n_{-initial}\,(n_o - n_{-critical})}\right] \frac{1}{v\, n_o\, \sigma(v)} = T$$

gives the time needed for the build-up of critical density, not considering nascent hydrogen for which σ is $\sim\frac{1}{2}$ as large. Between 100 and 10,000 eV, $\sigma(v)v$ deviates no more than 20% from 5×10^{-8} $cm^3 sec^{-1}$ for H_2. Since $\sigma(v)v$ falls rapidly below 100 eV, the gun plasma will not ionize much background gas. As soon as the heating pulse starts, it is assumed that most of the gun plasma electrons in the efficient parts of the resonance zone are heated to at least 100 eV.

Some experimental parameters will be inserted in the above equation, and the calculated times for critical density build-up will be compared with experimentally determined times for the cessation of very hot electron production. At the usual heating time 500 μseconds after gun injection, the density in the machine is about 10^9 cm^{-3}. $n_{critical}$ for the x-band microwaves used is 1 x 10^{12} cm^{-3}.

T = time for build-up to critical density microseconds		Ion gauge reading Torr	H_2 density cm^{-3}
Experimental	Calculated		
~16	32	5x10^{-5}	4.5x10^{12}
~21	62	2.7	2.4
>32	~105	1.7	1.5
	not calculated	.82	.73
impossible to obtain $n_{crit.}$		.36	.32
"		.074	.066

A similar treatment for S-band heating indicates that starting from an initial pressure of 3.6x10^{-6} torr, a time of 330 microseconds would be required to reach the critical density. At such low pressures impurities could greatly influence the build-up to critical density.

Two criticisms can be levelled. Since thermal hydrogen can move several cm in ten microseconds, the neutral density in the resonance zone is probably closer to n_o than to $(n_o - n_-)$. A desirable consequence is that nascent hydrogen probably diffuses out of the resonance zone fast enough that it need not be considered. Electrostatic effects in the resonance zones have been neglected. As electrons gain $W_\perp$ they tend to congregate near the center of the resonance zones, and ions follow suit. The local density increase will hasten the attainment of $n_{critical}$.

Figure 8 shows the response of the production of 12 keV electrons to the admission of various pressures of H_2. The x-band pulse was 32 microseconds long. Since the analyzer did not directly view a resonance zone, but detected particles which diffused to the B = 0 axis, it was difficult to obtain exact numbers. The qualitative results show experiment and simple theory to agree within a factor of three. Considering the uncertainty in measuring pressure and the approximations made, agreement is deemed satisfactory.

BIBLIOGRAPHY

1. J.C. Sprott and G.W. Kuswa. PLP 286. University of Wisconsin Plasma Studies. 1969.

2. C.W. Erickson. Rev. Sci. Instr. 37:1388. 1966.

3. G.W. Kuswa. PLP 280. May, 1969.

4. G.W. Kuswa. PLP 305. August, 1969.

5. G.W. Kuswa. PLP 206. June, 1968.

6. J.C. Sprott. Ph.D. Thesis. University of Wisconsin. 1969.

7. G.W. Kuswa. PLP 307.

8. D.J. Rose and M. Clark. Plasma and Controlled Fusion. MIT Press. 1961. Page 39.

FIGURE 2

VARIATION OF ELECTRON SIGNAL AS MINIATURE ANALYZER IS ROTATED ABOUT ITS OWN AXIS IN PORT T-7

(A) location in center of horizontal midplane
B foreward and reverse 915eV.

(B) location 2" above and below center of horizontal midplane B foreward only 915eV.

(C) location in center of horizontal midplane
B foreward and reverse 450eV.

(D) location in center of horizontal midplane
B foreward and reverse 27eV.

The plastic probe carriage ring was in the machine for these measurements. Note that the entrance aperture is 1.2 centimeters out from the axis of rotation of the analyzer so that the point of measurement changes slightly as the analyzer is rotated.

PLP 314

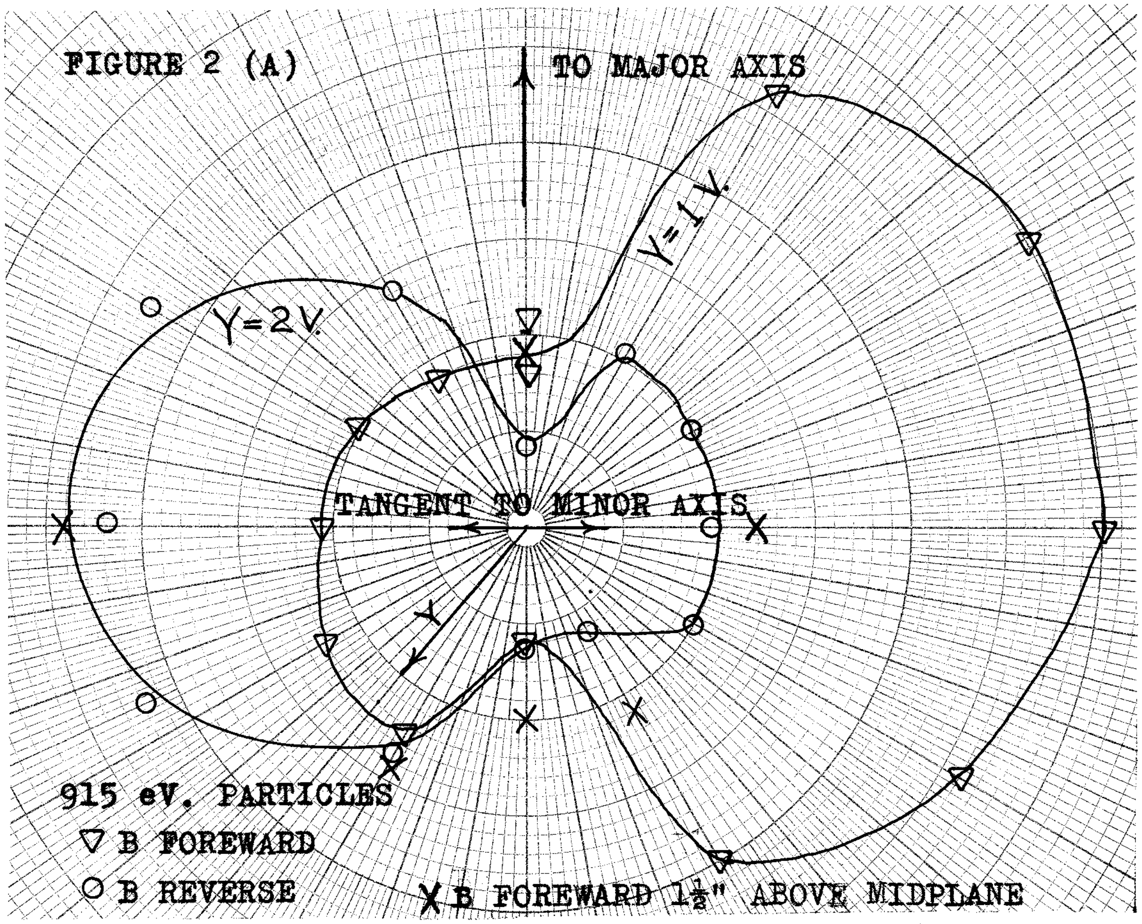
FIGURE 2 (A)
TO MAJOR AXIS
Y=1V.
Y=2V.
TANGENT TO MINOR AXIS
Y
915 eV. PARTICLES
▽ B FOREWARD
○ B REVERSE
X B FOREWARD 1½" ABOVE MIDPLANE

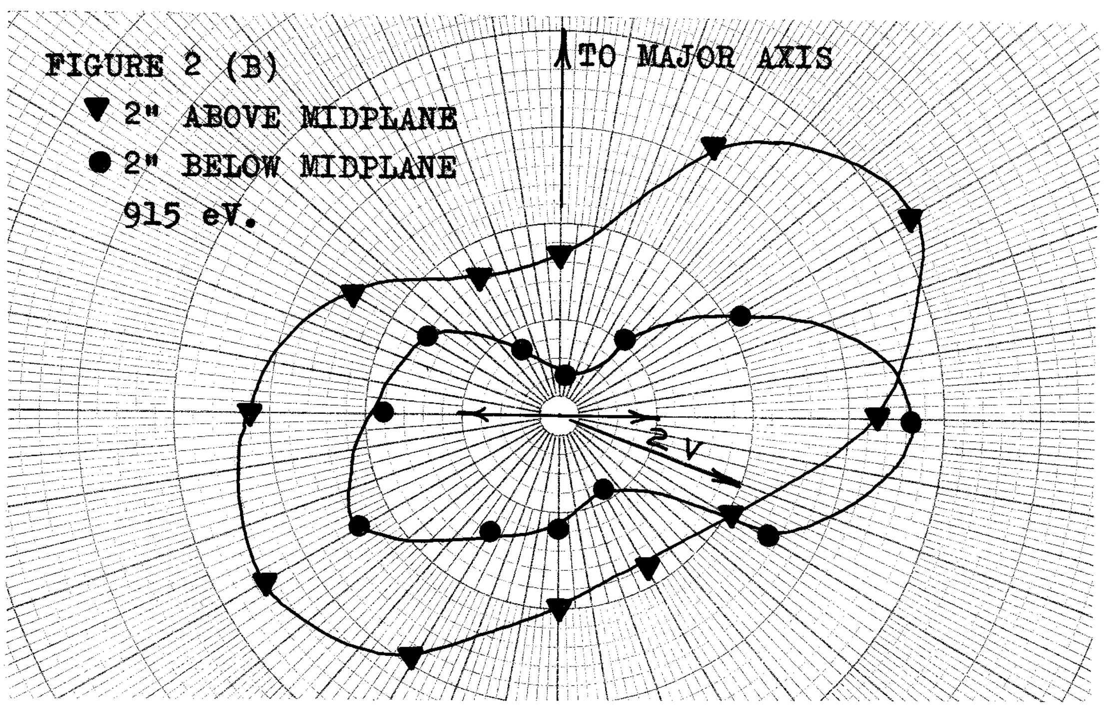
FIGURE 2 (B)
TO MAJOR AXIS
▼ 2" ABOVE MIDPLANE
● 2" BELOW MIDPLANE
915 eV.
2V

PLP 314

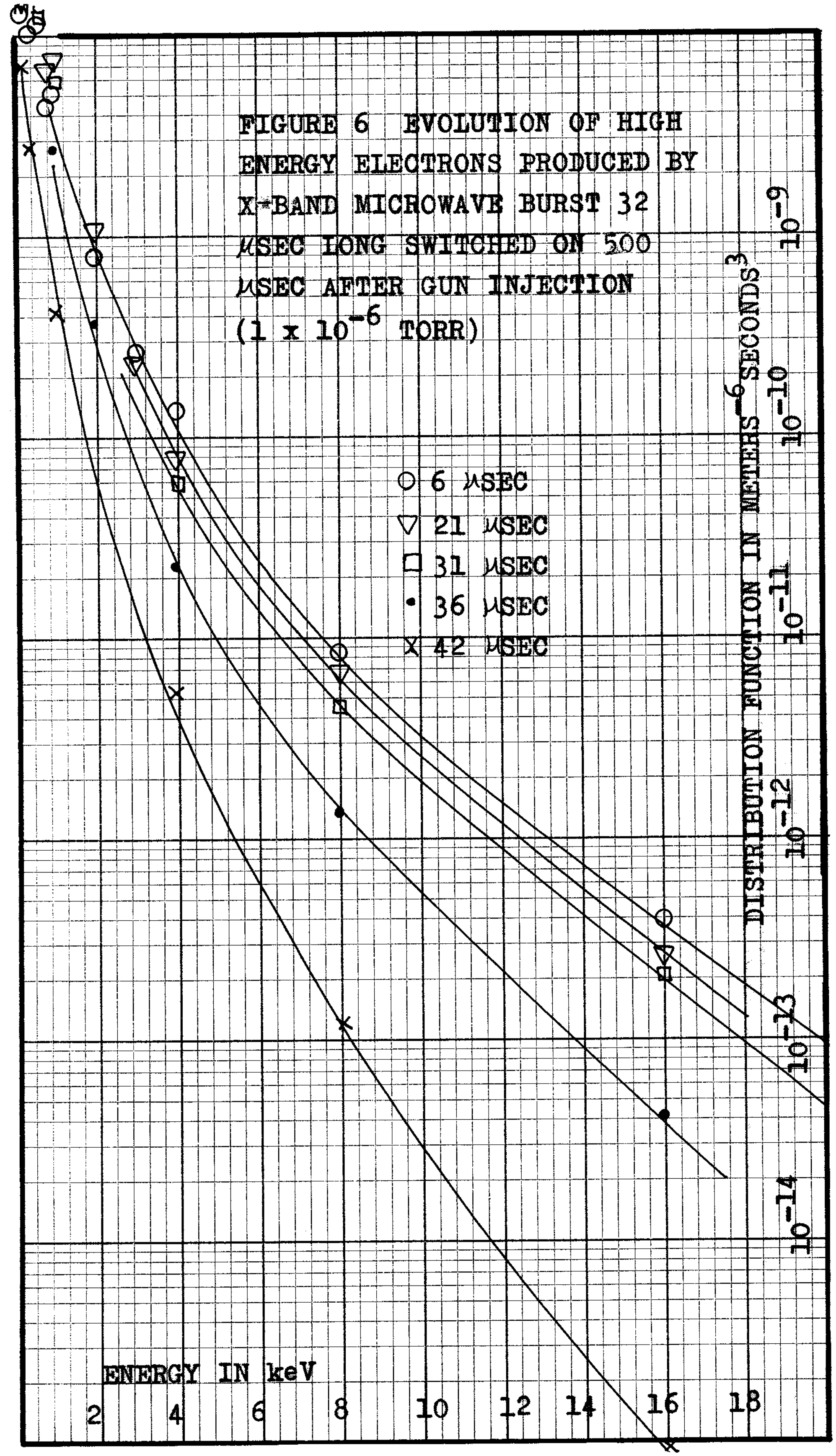

FIGURE 6 EVOLUTION OF HIGH ENERGY ELECTRONS PRODUCED BY X-BAND MICROWAVE BURST 32 μSEC LONG SWITCHED ON 500 μSEC AFTER GUN INJECTION (1×10^{-6} TORR)

PLP 314

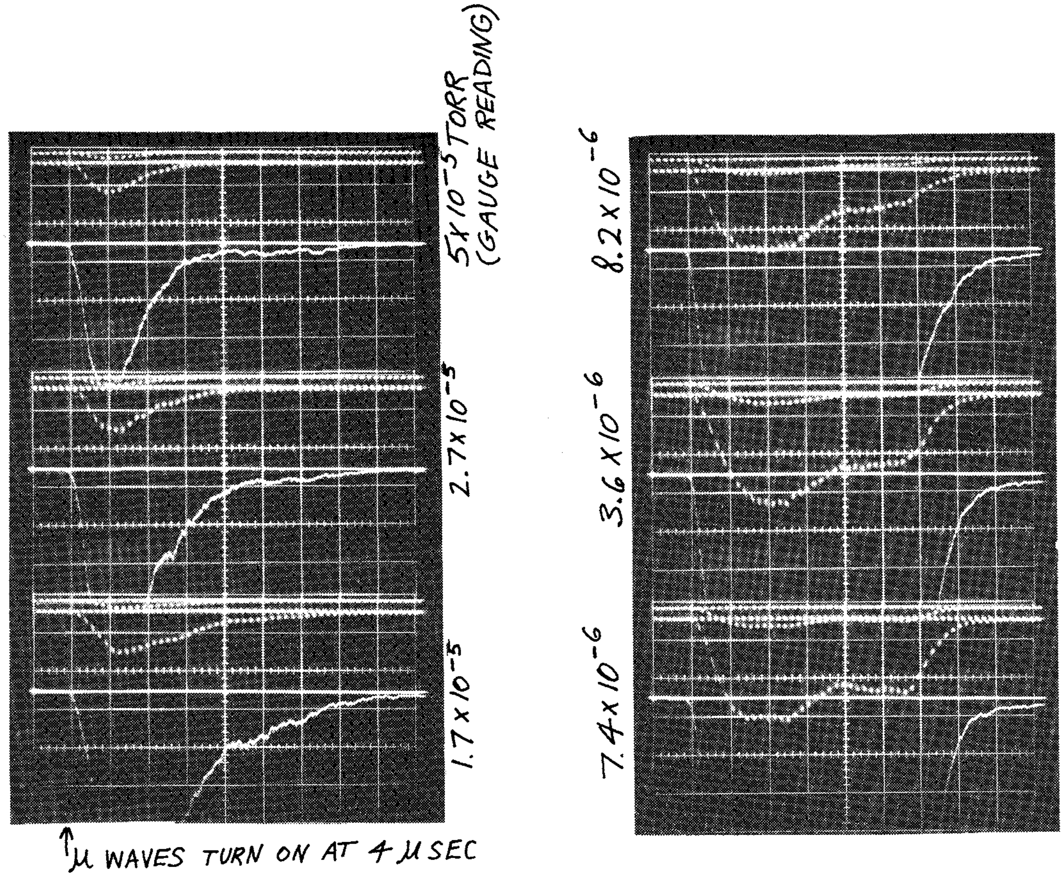

Figure 8 RESPONSE OF 12 keV ELECTRON PRODUCTION AS A FUNCTION OF H_2 PRESSURE IN THE OCTUPOLE

X-band microwaves were used to heat gun-injected plasma for 32 microseconds starting 500 microseconds after injection. The extractor pipe was located 3 cm from the center of the horizontal midplane in port S-6.

multipole excitation	=	2.5 kV
sweep speed		5 μsec/div.

The three traces on each picture are at 5, 1, and .2 volts per division and all display the detector output.