Comment on PLP 325

by

J. C. Sprott

February, 1970

PLP 332

Plasma Studies
University of Wisconsin

Rudmin, et al., in PLP 325 have proposed a heating mechanism in which a periodic sawtooth modulation of the confining magnetic field in a plasma containment device is supposed to adiabatically and stochastically heat the ions. The purpose of this note is to calculate the heating rate by a different method and to show that the proposed technique has little to offer over ordinary ion cyclotron resonance heating.

The sawtooth waveform can be Fourier analyzed approximately as

$$B(t) \simeq B_o + \frac{1}{2}\delta B - \frac{\delta B}{\pi}\sum_{k=1}^{\infty}\frac{1}{k}\sin\frac{k\omega_{ci}t}{10} \quad ,$$

to produce an rf electric field of

$$E(t) = R\dot{B}(t) = \frac{R\omega_{ci}\,\delta B}{10\pi}\sum_{k=1}^{\infty}\cos\frac{k\omega_{ci}t}{10} \; .$$

The heating produced by a sinusoidal electric field of frequency ω can be calculated from

$$\frac{dW}{dt} = \frac{\sigma\overline{E^2}}{n} = \frac{e^2\overline{E^2}\,\nu(\omega^2+\omega_{ci}^{\;2})}{m_i[(\omega^2-\omega_{ci}^{\;2})^2+4\omega^2\nu^2]}$$

$$= \frac{e^2R^2(\delta B)^2\,\nu\omega_{ci}^{\;2}}{2\pi^2 m_i}\sum_{k=1}^{\infty}\frac{k^2+100}{(k^2-100)^2\omega_{ci}^{\;2}+400k^2\nu^2} \; .$$

Note that in the absence of collisions (ν=0) there is no heating. The stochastic heating calculated by Rudmin arises from the implicit assumption that the phase of the particle gyration with respect to the electric field is random every time the field jumps back to B_o. The actual collision time may be either longer or shorter than this time, but for the sake of comparison we will take $\nu = \omega_{ci}/10$ to obtain

$$\frac{dW}{dt} = \frac{e^2R^2(\delta B)^2\omega_{ci}}{20\pi^2 m_i} \sum_{k=1}^{\infty} \frac{k^2 + 100}{(k^2 - 100)^2 + 4k^2}$$

The resonant Fourier component (k = 10) gives a heating rate of

$$\frac{dW}{dt} = \frac{e^2R^2(\delta B)^2\omega_{ci}}{40\pi^2 m_i}. \tag{1}$$

The other components give smaller but nonnegligible contributions. Equation (1) is identical to the stochastic part of Rudmin's result.

For a non-uniform magnetic field, the effective collision frequency ν is determined by the transit time of a particle through resonance, and the heating rate is given (as calculated in PLP 282) by

$$\frac{dW}{dt} = \frac{\pi e\overline{E^2}}{2B_o} G \tag{2}$$

where G is a geometrical factor of order unity given by

$$G = B_o \int n_o \frac{d^2V}{dBd\psi}\bigg|_{B_o} d\psi \Big/ \int \oint \frac{n d\ell}{B} d\psi,$$

and $\overline{E^2}$ is the mean square electric field at the ion cyclotron frequency. For G = 1, equation (2) predicts a heating rate of

$$\frac{dW}{dt} = \frac{e^2R^2(\delta B)^2\omega_{ci}}{400\pi m_i}$$

for Rudmin's sawtooth wave. For a non-uniform field, resonance may occur for several Fourier components, in which case the above rate would be correspondingly increased.

The main drawback of the sawtooth waveform is that large voltages are required to produce it. For the parameters considered by Rudmin, we require

$$V = 2\pi RE = 2\pi R^2\dot{B} = 10R^2\omega_{ci}\delta B \simeq 1000 \text{ volts.}$$

If the same voltage is used to sinusoidally modulate the field at ω_{ci}, equation (2) predicts a heating rate of

$$\frac{dW}{dt} \simeq \frac{\pi e E^2}{2B_o} = \frac{eV^2}{8\pi B_o R^2} \simeq 4 \times 10^7 \text{eV/sec}$$

which is considerably larger than the (optimistic) calculation of Rudmin that gave 7.28×10^4eV/sec.

Reflection of the low frequency waves from the boundary of a dense plasma should not affect the comparison because, as previously shown, it is the Fourier component of the sawtooth at the ion cyclotron frequency that produces most of the heating. Furthermore, it is technically easier to produce sine waves than sawtooth waves. The sawtooth wave would produce a more uniform heating, although in a non-uniform magnetic field, the heating is fairly uniform even for a sinusoidal electric field.