Theory of Off Resonance Heating

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I. Introduction

Experiments by Dandl, et al.[1] at Oak Ridge have shown that microwave power applied to a plasma in a magnetic mirror above or below the electron cyclotron frequency can produce some rather striking effects. Upper off resonance heating is found to be very efficient, while lower off resonance heating results in expelling the plasma from the field. These experiments have been repeated with identical results in a magnetic mirror at the University of Wisconsin,[2] and Upper off resonance heating has been observed in the Wisconsin levitated toroidal octupole.[3] Lower off resonance heating will be attempted in the small Wisconsin octupole with B_θ. The purpose of this note is to interpret the experimental results in terms of a heating model[4] that has previously proved successful for calculating resonance heating rates.

II. Upper off Resonance Heating

The heating rate for a cold, tenuous, uniform plasma in a uniform magnetic field can be written as

$$\frac{d\overline{W}}{dt} = \frac{e^2 E_\perp^{\,2} \nu \, (\omega^2+\omega_c^{\,2})}{m\left[(\omega^2-\omega_c^{\,2})^2+4\omega^2\nu^2\right]} + \frac{e^2 E_\parallel^{\,2} \nu}{m\omega^2} \qquad (1)$$

where $E_\perp (E_\parallel)$ is the perpendicular (parallel) component of the electric field, ω is the applied rf frequency, ω_c is the electron cyclotron frequency, and ν is the collision frequency ($\nu \ll \omega$). Equation (1) has been used[4] to calculate the stochastic heating of a collection of electrons that traverse a resonance in a non-uniform magnetic field. This is done by integrating Eq. (1) over the volume of the plasma, assuming $\nu << \omega$ so that the dominant contribution to the integral comes from the immediate vicinity of the resonance. The magnetic field is then expanded in a Taylor series about the resonance value, and the resulting heating rate is independent of the collision frequency ν.

It is often said that there can be no heating of a collisionless plasma, and Eq. (1) suggests this since $d\overline{W}/dt \to 0$ for $\nu \to 0$. This statement is true only if one broadens the usual concept of a collision. For example, the singularity in the denominator of Eq. (1) at $\omega = \omega_c$ allows heating for $\nu = 0$, but in a non-uniform field this can be thought of as a collision of the particle with the resonance. Other phase randomizing mechanisms can also simulate collisions, as will be shown later.

Consider now the case where $\omega^2 >> \omega_c^2$ (upper off resonance heating). Equation (1) becomes

$$\frac{d\overline{W}}{dt} = \frac{e^2 E^2 \nu}{m\omega^2} , \qquad (2)$$

which is the usual result for collisional heating in the absence of a magnetic field. The more general case of ω only slightly greater than ω_c is straightforward but more complicated because the heating rate depends on the geometry of the field and on the particle trajectories (or density distribution). The heating rate (averaged over a bounce period) is calculated from

$$\left<\frac{d\overline{W}}{dt}\right> = \int n(\ell)\, \frac{d\overline{W}}{dt}(\ell)\, \frac{d\ell}{B(\ell)} \Big/ \int n(\ell)\, \frac{d\ell}{B(\ell)} \; . \qquad (3)$$

Calculations of the heating rate of a hot, relativistic plasma should take into account the following effects:

1) Variation of ω_c with energy: $\omega_c = eB/\gamma m$

2) Doppler shift of ω due to v : $\omega_D = \omega \pm kv_{\parallel}$

3) Variation of E over the gyrodiameter of the particle

4) Effect of $E_{\parallel}$ on the turning points of the bounce motion

5) Non-conservation of the particle's magnetic moment

Existing heating models[5,6,7] neglect one or more of the above effects. By restricting the present discussion to the case $\omega^2 >> \omega_c{}^2$, the problem is reduced to that of determining an appropriate effective collision frequency ν, valid for arbitrary energy, which when substituted into Eq. (2) gives a heating rate in agreement with experiment.

Physically, ν can be thought of a the reciprocal of the time during which the particle gyrofrequency or phase remains fixed relative to that of the wave. A simple Coulomb collision with an ion or neutral atom is the simplest example of such a phase destroying mechanism. A collision can be either a single, large angle event in which the phase is abruptly altered to some new random value or a series of small angle collisions which slowly change the phase. In the weakly ionized plasmas generally produced by microwave heating, electron-neutral collisions dominate, and the collision frequency for electron energies greater than a few eV is

$$\left.\begin{aligned}\nu &= 2.3 \times 10^{9}\ p\ (\text{torr})\ \text{for}\ H_e \\ \nu &= 4.5 \times 10^{9}\ p\ (\text{torr})\ \text{for}\ H_2\end{aligned}\right\} . \qquad \text{(electron-neutral)}$$

For comparison, the electron-ion collision frequency is

$$\nu \simeq \frac{20n(\text{cm}^{-3})}{T_e^{3/2}(^{o}\text{K})}\ , \qquad \text{(electron-ion)}$$

and is nearly independent of ion mass. These collision rates are much too small to explain the off resonance heatin observed in the experiments.

In the absence of real particle collisions, the electron energy oscillates with frequency ω and amplitude

$$W_o = \frac{2e^2E^2}{m\omega^2}\ ,$$

and so there is no net heating since the energy averaged over a period of the oscillation is constant. But if the particle initially had no energy, and if the rf is turned off at some random phase, the particle will on the average be left with an energy

$$\Delta W = \frac{1}{2} W_o = \frac{e^2E^2}{m\omega^2}\ ,$$

and we can say the particle was heated at a rate

$$\frac{d\overline{W}}{dt} = \frac{\Delta W}{T} = \frac{e^2E^2}{m\omega^2 T}$$

where T is the duration of the rf pulse. Comparing this result with

Eq. (2) shows that the reciprocal of the time during which the rf is on plays the role of a collision rate.

If the rf is turned on and off repeatedly, but in a random manner, the particle executes a random walk in velocity space, so that the magnitude of the velocity after n steps is $v = \sqrt{n}\,\Delta v$, and the energy is $W = n\Delta W$. The energy then increases linearly with time, and the effective collision frequency is the inverse of the correlation time of the rf. Noise modulated rf sources thus provide a useful means of plasma heating.[8]

Electrons trapped in a mirror field in a multimode cavity move through regions where the electric field reverses on the average every half wavelength. If the cavity mode number is large, these reversals occur randomly as shown by rf field measurements in the Wisconsin octupole.[9] If the particle's bounce motion is absolutely periodic and if the rf mode structure is time independent, then the acceleration is a periodic, although non-sinusoidal, function of time, and there is no net heating. In real situations, the bounce motion is probably non-periodic for a variety of reasons, and this fact has led some workers[7,10] to assume that phase randomization occurs in one bounce period, so that

$$\nu = \omega_\beta = \sqrt{\frac{\mu}{m}\frac{d^2B}{d\ell^2}} \quad , \tag{4}$$

where μ is the particle's magnetic moment $(W_\perp/B)$. Grawe[7] assumes phase randomization after a bounce period but strangely arrives at a heating rate <u>less</u> than would be obtained by substituting Eq. (4) into (2).

We argue here, that in a multimode cavity in which the rf wavelength is short compared with the amplitude of the bounce motion, if one assumes phase randomization after a bounce period, it is inconsistent not to assume that randomization occurs in the shorter time represented by the transit of the particle through a cavity mode. This assumption gives an effective collision frequency of

$$\nu = v_{\parallel}/\lambda\!\!\!^{-} = \omega v_{\parallel}/c. \tag{5}$$

For highly relativistic plasmas, $\nu \to \omega$, and the heating rate is given by

$$\frac{d\overline{W}}{dt} = \frac{e^2E^2}{m\omega} . \tag{6}$$

This result corresponds closely to the off resonance heating rates observed experimentally for relativistic plasmas at Oak Ridge.[1] Grawe's[7] calculation also correctly predicts the Oak Ridge results, and so this experiment does not provide a conclusive test of the theories. Grawe's theory predicts a strong dependence of the heating rate on particle energy in contrast with the present theory that predicts a heating rate that varies linearly with velocity:

$$\frac{d\overline{W}}{dt} = \frac{e^2E^2v_{\parallel}}{mc\omega} \tag{7}$$

Thus a quantitative measurement of the off resonance heating rates for the non-relativistic plasmas in the Wisconsin experiments should provide a conclusive comparison of the theoretical predictions.

III. Lower off Resonance Heating

For $\omega^2 << \omega_c^2$, Eq. (1) predicts a heating rate of

$$\frac{d\overline{W}}{dt} = \frac{e^2 E_{\perp}^2 \nu}{m\omega_c^2} + \frac{e^2 E_{\parallel}^2 \nu}{m\omega^2} . \tag{8}$$

Lower off resonance heating is anisotropic, in contrast to the upper off resonance case of Eq. (2). The first (second) term on the right of Eq. (8) represents the rate of change of perpendicular (parallel) energy. Physically this result demonstrates the effect of the magnetic field which prevents prolonged perpendicular particle acceleration by the rf electric field. In a multimode cavity, on the average, $E_{\perp}^2 = 2E_{\parallel}^2$, so that for $\omega << \omega_c$, the parallel heating dominates the perpendicular heating by a factor of the order ω^2/ω_c^2. The anisotropic heating would be expected to give rise to enhanced axial diffusion in agreement with experimental observations. In the Wisconsin mirror experiment, the mode number is rather small (~ 65), and so we expect small changes in frequency to change the ratio of $E_{\perp}$ to $E_{\parallel}$ (and thus the diffusion rate). This effect has been observed. The diffusion could also alter the axial density profile in such a way as to make the plasma flute unstable, giving rise to fluctuations and enhanced radial diffusion.

The theoretical model can be generalized to arbitrary ω by defining a heating anisotropy,

$$A = \frac{E_{\parallel}^2}{E_{\perp}^2} \frac{d\overline{W}_{\perp}}{dt} \Big/ \frac{d\overline{W}_{\parallel}}{dt} = \frac{(\omega^2+\omega_c^2)\omega^2}{[(\omega^2-\omega_c^2)^2+4\omega^2\nu^2]} , \tag{9}$$

such that A = 1 represents isotropic heating. Equation (9) is plotted in Fig. 1 as a function of ω/ω_c for $\nu = 0$ and for $\nu = 0.1\ \omega$. For $\omega = \omega_c$ (resonance heating), the heating is highly anisotropic with most of the energy in the perpendicular component. For $\omega > \omega_c$ (upper off resonance heating), the anisotropy is always greater than one, inhibiting axial diffusion. For $\omega < \omega_c/\sqrt{3}$, (lower off resonance heating), the anisotropy is less than one, enhancing axial diffusion. For ω only slightly below the cyclotron frequency ($\omega_c/\sqrt{3} < \omega < \omega_c$), we expect heating to occur without enhanced diffusion.

These results are changed if the rf electric field is strongly polarized, and we must also allow for the possibility of different effective collision frequencies in the perpendicular and parallel directions. The expulsion of the plasma in the Oak Ridge experiments probably results from the strong, parallel, off resonance heating that is comparable to the perpendicular, resonance heating for relativistic plasmas. In the Wisconsin experiments, the plasma is probably easily extinguished because of the small mirror ratio (1.02 or 1.19), and because the resonance heating occurs in short pulses after which the large anisotropy can disappear. For relativistic plasmas, the <u>hot</u> electron cyclotron frequency should be used in Fig. 1, so that particles of different energy are heated differently.

Finally, we attempt to estimate the lower off resonance power required to extinguish the plasma in a special case that should approximate the Oak Ridge experiment. Assume a multimode cavity with total absorption of both the resonance heating power (P_R) and off resonance heating power (P_{oR}). The plasma is expelled if the midplane anisotropy is less than $1/(R-1)$, where R is the mirror ratio:

$$\frac{W_{\perp}(0)}{W_{\parallel}(0)} < \frac{1}{R-1} .$$

Neglecting scattering or other processes that cause exchange of perpendicular and parallel energy, the respective energies are determined by the ratio

$$\frac{W_{\perp}(0)}{W_{\parallel}(0)} = \frac{P_R + P_{oR}(\perp)}{P_{oR}(\parallel)} ,$$

where $P_{oR}(\perp) + P_{oR}(\parallel) = P_{oR}$. Combining these equations with $A = P_{oR}(\perp)/P_{oR}(\parallel)$ from Eq. (9), the required off resonance power is

$$P_{oR} = \frac{(R-1)(A+1)}{1-A(R-1)} P_R. \qquad (10)$$

For $\omega^2 << \omega_c^2$, $A \ll 1$, and the required power is simply

$$P_{oR} = (R-1)P_R. \qquad (11)$$

In the Oak Ridge experiments, $R \simeq 2$, and so we expect the required off resonance power to be about equal to the resonance power in agreement with observations.

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