ION CYCLOTRON HEATING RATE IN THE SMALL OCTUPOLE

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The spatially averaged cyclotron heating rate in a arbitrary non-uniform magnetic field is given by

$$\frac{d\bar{W}}{dt} = \frac{\pi e \int n E_\perp^2 \, \delta \, (B - B_o) \, dV}{2 \int n dV} , \tag{1}$$

where n is the density, $E_\perp$ is the component of the RF electric field perpendicular to the magnetic field, B, and B_o is the magnetic field at cyclotron resonance. Near the axis of a linear multipole, the B = const. contours are nearly circular and the volume integration can be written

$$dV = r \, dr \, d \, \phi \, dL ,$$

The heating rate becomes

$$\frac{d\bar{W}}{dt} = \frac{\pi e r_o \int n \, (r_o, \phi) \, E_\perp^2 \, (r_o, \phi) \, d\phi}{2 \left. \frac{dB}{dr} \right|_{r_o} \iint n \, (r, \phi) \, r \, dr \, d \, \phi} , \tag{2}$$

where the subscript o refers to the value of a quantity on the resonance surface. For an octupole, the density along a mod B contour can be approximated by

$$n \, (r, \phi) \cong \frac{n_A + n \, (r,0)}{2} - \frac{n_A - n \, (r,0)}{2} \cos 4 \, \phi ,$$

where the subscript A refers to the value of a quantity on the axis. For a gun-injected plasma in the small octupole, the density is known to be approximately

$$n \, (\psi) \cong n_A e^{-|\psi - \psi_A| / \psi_1} ,$$

where ψ is the magnetic flux function. Near the axis,

$$B \propto \frac{d\psi}{dr} \propto r^3 \Rightarrow \psi \propto r^4$$

so that

$$n \, (r,o) = n_A e^{-(r/r_1)^4} .$$

Also,

$$r_o / \left.\frac{dB}{dr}\right|_{r_o} = \frac{r_o^2}{3B_o} .$$

Finally, it has been shown experimentally that for an electric field produced by a hoop at a distance r_H from the axis, the field varies approximately as

$$E_\perp^2 (r) \cong E_A^2 \left[1 + \frac{r^2}{r_H^2} + \frac{2r}{r_H} \cos \phi\right]^{-1.6}$$

Substituting into equation (2) gives

$$\frac{d\overline{W}}{dt} = \frac{\pi e r_o^2 E_A^2 \int_o^\pi \left[1 + e^{-(r_o/r_1)^4} - (1 - e^{-(r_o/r_1)^4})\cos 4\phi\right]\left[1 + \frac{r_o^2}{r_H^2} + \frac{2r_o}{r_H} \cos \phi\right]^{-1.6} d\phi}{6B_o \int_o^\pi \int_o^{r_H} \left[1 + e^{-(r/r_1)^4} - (1 - e^{-(r/r_1)^4}) \cos 4\phi\right] r \, dr \, d\phi} .$$

This equation can be written in the form

$$\frac{d\overline{W}}{dt} = \frac{\pi e E_A^2}{6 B_o} F (r_o, r_H, r_1) .$$

For typical conditions (r_H = 6" and r_1 = 4.5"), the function F has been calculated numerically as a function of the resonance zone position ($0 < r_o < r_H$), and the result is plotted in the attached figure.

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