CLASSICAL LOSSES, SPACE POTENTIAL AND STABILITY OF MIRROR PLASMAS

by

J. C. Sprott

June 1973

PLP 518

Plasma Studies

University of Wisconsin

Certain features of hot-electron, steady state, mirror-confined plasmas are not yet well understood. The purpose of this note is to derive for such plasmas expressions for 1) the classical, ambipolar loss rate including collisions with neutrals; 2) the space potential; and 3) the neutral density required to insure MHD stability. The derivations parallel those of an earlier report (Univ. of Wisc. PLP 484) in which collisions with neutrals were neglected.

If electron-ion pairs are produced isotropically at the mirror midplane, and if the mean free path is greater than the length of the mirror, the electron current into the loss cone if given by

$$I_{oe} = Ne(\nu_{ioniz} \frac{\Omega_{LC}}{4\pi} + \nu_{scatt}) ,$$

where N is the number of electrons in the mirror, ν_{ioniz} is the ionization rate (= ν_{loss} in the steady state), ν_{scatt} is the scattering rate into the loss cone, and Ω_{LC} is the loss cone solid angle. It is simple to show that

$$\nu_{scatt} = \nu_e/\log R$$

and

$$\frac{\Omega_{LC}}{4\pi} = 1 - \sqrt{1 - \frac{1}{R}} ,$$

where $\nu_e = \nu_{ee} + \nu_{ei} + \nu_{en}$ is the 90° scattering time for electrons and R is the mirror ratio. Substitution gives

$$I_{oe} = Ne[\nu_{loss} (1 - \sqrt{1 - 1/R}) + \nu_e/\log R].$$

Similarly, for ions,

$$I_{oi} = Ne[\nu_{loss}\,(1 - \sqrt{1 - 1/R}\,) + \nu_i/\log R].$$

If the particles flow along the field to conducting ends, a sheath will develop at the wall, and the plasma will charge up to a potential such that the electron and ion currents are equal. For maxwellian electrons and ions, these currents are given by

$$I_e = I_{oe}\min[1, \exp(-e\Phi/kT_e)]$$

and

$$I_i = I_{oi}\min[1, \exp(e\Phi/kT_i)]$$

where Φ is the potential of the plasma relative to the end walls. In steady state,

$$I_e = I_i = Ne\nu_{loss}\ .$$

Eliminating Φ and solving for ν_{loss} gives

$$\nu_{loss} = \frac{\min(\nu_i,\ \nu_e)}{\sqrt{1 - 1/R}\ \log R}\quad . \tag{1}$$

Numerically, for a hydrogen plasma (density n) in a neutral hydrogen background (density n_o),

$$\left.\begin{aligned} \nu_e &= 1.14 \times 10^{-4} nT_e^{-3/2} + 1.77 \times 10^{-7} n_o \\ \text{and}\qquad \nu_i &= 1.33 \times 10^{-6} nT_i^{-3/2} + 8.23 \times 10^{-9} n_o \end{aligned}\right\}$$

where n is in cm^{-3}, T is in eV, and ν is in sec^{-1}.

The space potential can be calculated in a similar manner, and the result is

$$e\Phi = \begin{cases} -kT_i \ln[1 - \sqrt{1 - 1/R} + \frac{\nu_i}{\nu_e}\sqrt{1 - 1/R}\,] & \Phi < 0 \\ kT_e \ln[1 - \sqrt{1 - 1/R} + \frac{\nu_e}{\nu_i}\sqrt{1 - 1/R}\,] & \Phi > 0. \end{cases}$$

In the large mirror ratio limit, the potential can be written as

$$e\Phi = kT_e \max(0, \ln\nu_e/\nu_i) + kT_i \min(0, \ln\nu_e/\nu_i) \qquad (2)$$

Note that at low neutral pressure,

$$\frac{\nu_e}{\nu_i} = 86\left(\frac{T_i}{T_e}\right)^{3/2} < 1 \quad \text{for } T_e >> T_i,$$

and the space potential is negative and given by

$$e\Phi = -\frac{3}{2} kT_i \ln(T_e/19.5T_i),$$

whereas at high neutral pressure,

$$\frac{\nu_e}{\nu_i} = 21.5,$$

and the space potential is positive and given by

$$e\Phi = 3.07kT_e.$$

Guest and Harris (Phys. Rev. Letters 27, 1499(1971)) have suggested that the stability of these hot electron mirror plasmas results from their positive space potential. If such is the case, we can estimate the critical neutral density required for stability by determining the value of n_o for which $\Phi = 0$. This occurs when $\nu_e = \nu_i$ or,

$$\frac{n_o}{n} = \frac{7.88}{T_i^{3/2}} - \frac{675}{T_e^{3/2}} \quad . \qquad (3)$$

In the ELMO experiment, for example, the critical pressure is $\sim 2 \times 10^{-5}$ torr (gauge) for $n \sim 10^{12}$ cm^{-3}, which would be consistent with equation (3) if $T_i = 2.9$eV. The space potential in ELMO is observed to be positive in the high pressure, stable regime and negative in the low pressure, unstable regime, lending credence to the theory.