# CYCLOTRON HEATING NEAR THE MULTIPOLE B = 0 AXIS

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Ion and Electron Cyclotron heating rates in multipoles are generally difficult to calculate except numerically because of the complicated shape of the magnetic field. Near the B = 0 axis, however, the magnetic field has a simple asymptotic form,

$$B = |\vec{B}| = \alpha r^{\ell-1}$$

where $2\ell$ is the multipole number and $\alpha$ is a constant. The cyclotron resonance zone then is in the shape of a large aspect ratio toroid (or cylinder in the case of a linear multipole) of circular cross section. This note is a collection of formulas and graphs useful for both electron and ion cyclotron resonance heating near the B = 0 axis in a multipole.

The average cyclotron heating rate is given by

$$\frac{d\langle W\rangle}{dt} = \frac{\pi \int n e E_\perp^2 \delta(B-B_o)\, dV}{2\int n\, dV} \cong \frac{n_o e\langle E^2\rangle}{\bar{n} B_o} G$$

where

$$G = \frac{\pi B_o \int \delta(B-B_o)\, dV}{\int dV} = \frac{\pi B_o}{3V} \left.\frac{dV}{dB}\right|_{B_o} .$$

For a multipole (near the B = 0 axis),

$$\frac{dV}{dB} = \frac{4\pi^2 r_o R_o\, dr}{dB} = \frac{4\pi^2 R_o}{(\ell-1)\alpha} r_o^{3-\ell} ,$$

where $r_o$ and $R_o$ are the minor and major axes of the resonance toroid.

Hence

$$G = \frac{4\pi^3 R_o}{3(\ell-1)V} \left(\frac{\alpha}{B_o}\right)^{\frac{\ell-1}{2}} .$$

For an octupole ($\ell = 4$),

$$G = \frac{4\pi^3 R_o}{9V}\left(\frac{B_o}{\alpha}\right)^{\frac{2}{3}}$$

For the small Wisconsin octupole, $V = 3 \times 10^5\ cm^{-3}$, $R_o = 42$ cm, and $\alpha = 5.86 \times 10^{-4}\ kG/cm^3$. The computed value of G for the small octupole is plotted in Fig. 1 vs the resonance field ($B_o$) normalized against B at the outer wall midplane. Also shown is the corresponding plot for the levitated octupole.

Note that the cyclotron heating rate is the same for electrons and for ions if the appropriate electric field and resonance magnetic field are used. Furthermore, the existence of a B = 0 axis implies that whenever a frequency is chosen for ion cyclotron resonance, there will also be a region of electron cyclotron resonance nearer to the axis. It is then reasonable to ask what fraction of the energy goes into the electrons and ions. For a given perpendicular electric field, the heating rate is

$$\frac{d\langle W\rangle}{dt} = \frac{2\pi^3 n_o e\langle E_\perp^2\rangle R_o}{\bar{n}V(\ell-1)\alpha}\left(\frac{B_o}{\alpha}\right)^{\frac{3-\ell}{\ell-1}} ,$$

and so the ratio of electron to ion heating is

$$\frac{d\langle W_e\rangle}{dt}\Big/\frac{d\langle W_i\rangle}{dt} = \left(\frac{m_e}{m_i}\right)^{\frac{3-\ell}{\ell-1}} .$$

The ratio of heating rates for electrons and protons in various multipoles is given below:

| | $\ell$ | $\frac{d\langle W_e\rangle}{dt} \Big/ \frac{d\langle W_i\rangle}{dt}$ | electrons | protons |
|---|---|---|---|---|
| quadrupole | 2 | $m_e/m_i$ | 0.05% | 99.95% |
| sextupole | 3 | 1 | 50% | 50% |
| octupole | 4 | $(m_i/m_e)^{\frac{1}{3}}$ | 92.5% | 7.5% |

This is a curious result because it says that quadrupoles and octupoles behave very differently in the presence of ion cyclotron resonance heating. A weak toroidal field of order $(m_e/m_i)B_o$ should suppress the electron heating in an octupole and allow the rf to couple entirely to the ions, however.

For resonance very near the B = 0 axis, we may expect the theory to fail when the gyroradius of the particles becomes large compared with the scale length of the magnetic field. Near the axis,

$$\frac{B}{|\nabla B|} = \frac{r}{\ell-1} = \frac{1}{\ell-1}\left(\frac{\alpha}{B}\right)^{\ell-1} .$$

This restriction is more severe for ions than for electrons.

When a toroidal field ($B_\theta$) is added to a multipole, we expect good heating when the resonance occurs in the approximately uniform field near the minor axis. If the toroidal field has the usual 1/R variation, then we can write the total field near the multipole axis as

$$B = \frac{R_o}{R_o + r\cos\phi}\sqrt{\alpha^2 r^{2\ell-2} + B_\theta^2}$$

where $\phi$ is the poloidal angle. A computer code (GCAL) was written to calculate the normalized heating rate vs resonance field with this B variation. Using numbers appropriate to the small octupole, the results in Fig. 2 were obtained for several values of $B_\theta$. ($B_\theta$ is the toroidal field on axis normalized to the poloidal field at the outside wall midplane.) Note the peak for resonance near the axis and the sharp drop where resonance diappears from the machine. The $B_\theta = 0$ case is not quite identical to that shown in Fig. 1 because the toroidal curvature has been included to lowest order.

For ion cyclotron resonance heating, it is possible to get a quantitative measure of the heating rate only by considering the detailed shape of the coupling structure. One method of producing the necessary perpendicular electric fields is to modulate the poloidal magnetic field with a current $\delta I$ at frequency $\omega$ applied to the synchrotron gap. This produces an electric field on the separatrix given by

$$\oint \vec{E} \cdot d\vec{\ell} = - \frac{d\psi_s}{dt} = \omega\psi_s = \omega \frac{\psi_s}{\psi_T} L\delta I$$

or

$$E_\perp = \frac{\omega L \delta I \psi_s}{2\pi R_o \psi_T} ,$$

where L is the poloidal inductance. The corresponding heating rate is

$$\frac{d\langle W\rangle}{dt} = \frac{2\pi^3 e E_\perp^2 R_o}{(\ell-1)\alpha V} \left(\frac{B_o}{\alpha}\right)^{\frac{3-\ell}{\ell-1}} = \frac{\pi e \omega^2 L^2 (\delta I)^2 \psi_s^2}{2(\ell-1)\alpha V R_o \psi_T^2} \left(\frac{B_o}{\alpha}\right)^{\frac{3-\ell}{\ell-1}} .$$

A useful measure of the heating rate is the inverse Q of the coupling structure, which in this case is given by

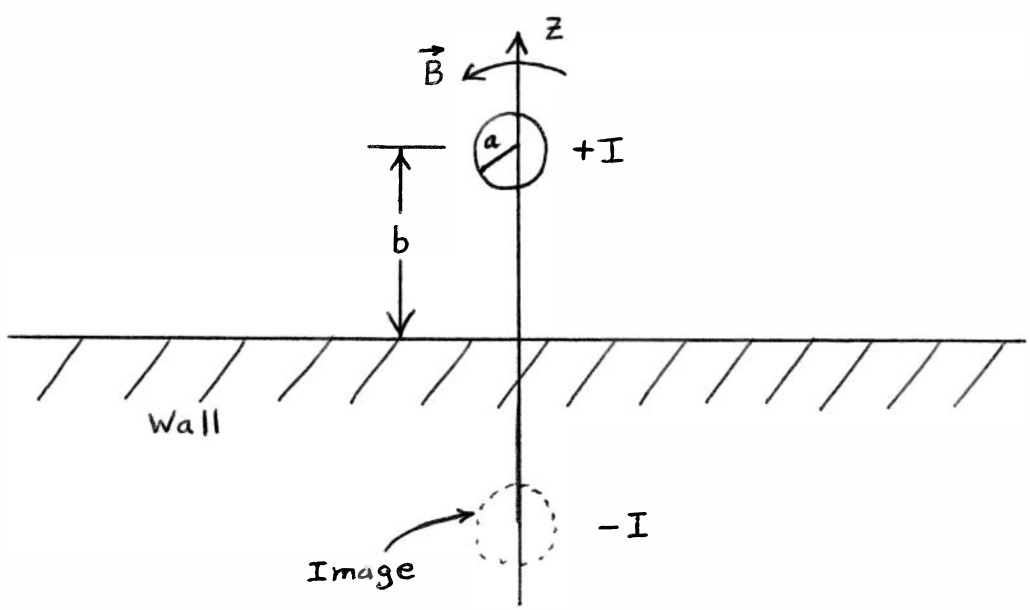
$$\frac{1}{Q} = \frac{n_o V}{\omega L (\delta I)^2} \frac{d\langle W\rangle}{dt} = \frac{\pi \varepsilon_o \omega_{pi}^2 L \psi_s^2}{2(\ell-1) R_o \psi_T^2} \left(\frac{B_o}{\alpha}\right)^{\frac{2}{\ell-1}} .$$

For a toroidal octupole,

$$\frac{1}{Q} = \frac{\pi\varepsilon_o\omega_{pi}^2 L\psi_s^2}{6R_o\psi_T^2}\left(\frac{B_o}{\alpha}\right)^{\frac{2}{3}}.$$

For the small Wisconsin octupole, $\psi_s/\psi_T \simeq \frac{1}{3}$, and L = 0.16 μhy, and the calculated 1/Q for several densities is shown in Fig. 3, assuming hydrogen ions. Note that the Q is the order of $10^5$ at the densities typical of the octupole, indicating that it would be difficult to effectively couple the power in this way. For comparison, the unloaded Q of the synchrotron gap at 1 MHz is ~ $10^3$ implying a heating efficiency of ~ 1%. Furthermore, it is technically difficult to apply rf to the synchrotron gap because it is tightly coupled to a lossy iron core.

A more practical method of coupling ICRH power to a multipole plasma uses a fifth hoop coaxial to the main hoops and located near the wall so as to intercept as little plasma as possible. We can analyze this structure by considering an infinite straight rod of radius a located a distance b above an infinite conducting wall:



The magnetic field at a point on the z axis is given by

$$B = \frac{\mu_o I}{2\pi(z-b)} - \frac{\mu_o I}{2\pi(z+b)} = \frac{\mu_o Ib}{\pi z^2} ,$$

where the latter approximation is valid for z >> b. A small search coil that measures $\dot{B} = \mu_o \omega Ib/\pi z^2$ has been used to verify that this approximation is reasonably accurate near the B = 0 axis in the small octupole. The electric field is determined from

$$\nabla \times \vec{E} = \frac{\partial E_\perp}{\partial z} = -\dot{\vec{B}} = -\frac{\mu_o \omega Ib}{\pi z^2} ,$$

or

$$E_\perp = -\int \omega B dz = \frac{\mu_o \omega Ib}{\pi z} .$$

If the coil is located in the toroidal vertical mid-cylinder, its inductance is given by

$$L = 2\mu_o R_o \ln \frac{2b}{a}$$

for b >> a.

The heating rate is given by

$$\frac{d\langle W\rangle}{dt} = \frac{2\pi e\omega^2 I^2 \mu_o^2 b^2 R_o}{(\ell-1)\alpha V z_o^2} \left(\frac{B_o}{\alpha}\right)^{\frac{3-\ell}{\ell-1}} ,$$

and the inverse Q is

$$\frac{1}{Q} = \frac{2\pi\varepsilon_o \mu_o^2 \omega_{pi}^2 b^2}{(\ell-1) z_o^2 L} \left(\frac{B_o}{\alpha}\right)^{\frac{2}{\ell-1}} .$$

For a toroidal octupole,

$$\frac{1}{Q} = \frac{2\pi\varepsilon_o\mu_o^2\omega_{pi}^2 b^2}{3z_o^2 L}\left(\frac{B_o}{\alpha}\right)^{\frac{2}{3}} .$$

For the small octupole, we have typically $z_o = 18$ cm, a = 0.5 cm, b = 3 cm, L = 2.6 μhy, and these values are used to plot 1/Q for several densities in Fig 4, assuming hydrogen ions. The results are very similar to the case in which the synchrotron gap is driven. The incremental series resistance produced by the plasma is easily found from

$$\Delta R_s = \omega L/Q.$$

Both of these calculations assume that the electric field freely penetrates the plasma and that the inductance of the coupling structure is not significantly altered by the plasma. These assumptions have been verified experimentally for densities as high as $10^{11}$ $cm^{-3}$ in the small octupole.

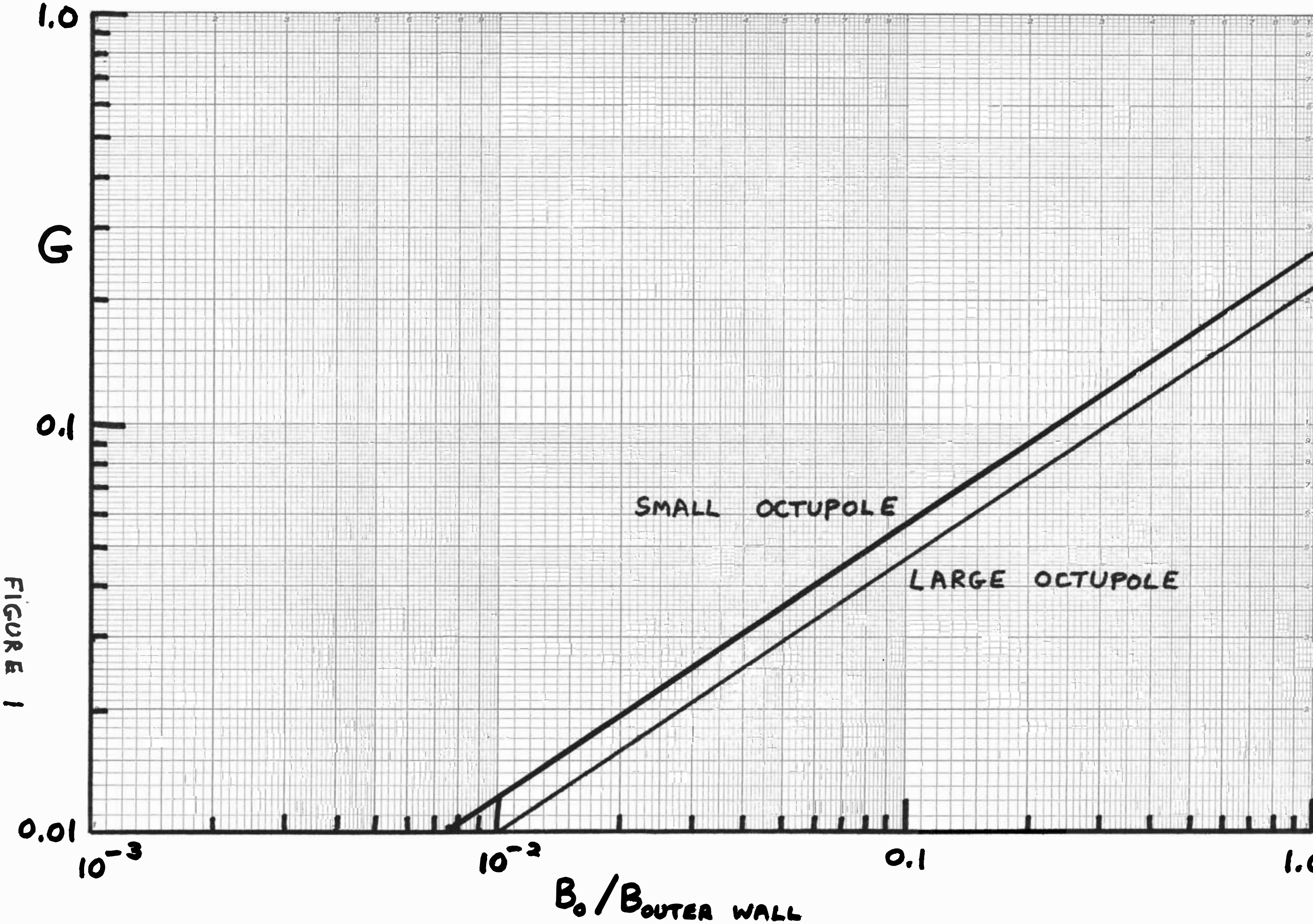

1.0
G
0.1
0.01
SMALL OCTUPOLE
LARGE OCTUPOLE
10^-3
10^-2
0.1
1.0
B_0 / B_OUTER WALL

FIGURE 1

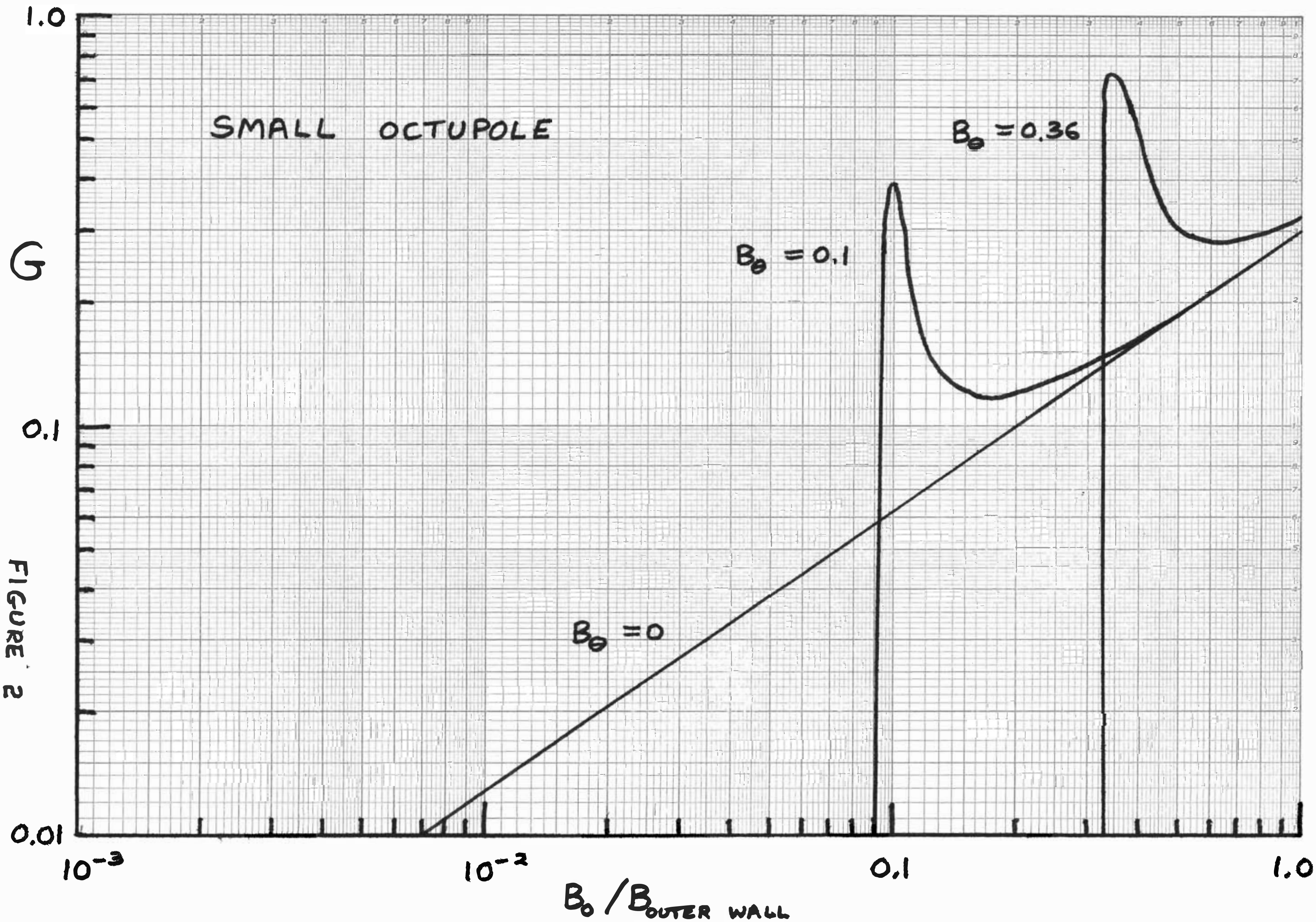

SMALL OCTUPOLE
$B_\theta = 0.36$
$B_\theta = 0.1$
$B_\theta = 0$
G
1.0
0.1
0.01
$10^{-3}$
$10^{-2}$
0.1
1.0
$B_0 / B_{\text{OUTER WALL}}$

FIGURE 2

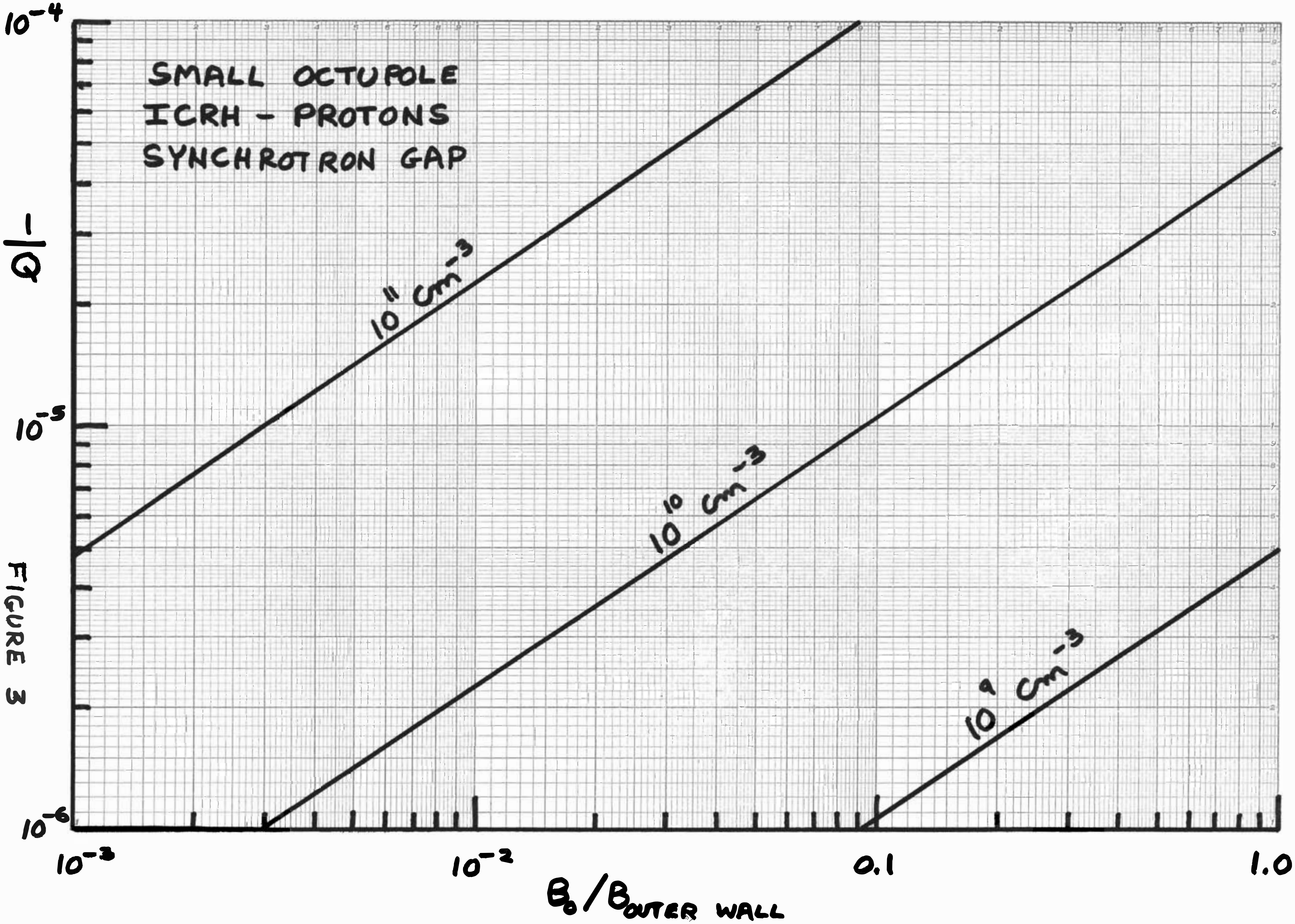

SMALL OCTUPOLE
ICRH - PROTONS
SYNCHROTRON GAP
$10^{-4}$
$10^{-5}$
$10^{-6}$
$\frac{1}{Q}$
$10^{11}$ cm$^{-3}$
$10^{10}$ cm$^{-3}$
$10^{9}$ cm$^{-3}$
$10^{-3}$
$10^{-2}$
0.1
1.0
$B_0/B_{\text{OUTER WALL}}$

FIGURE 3

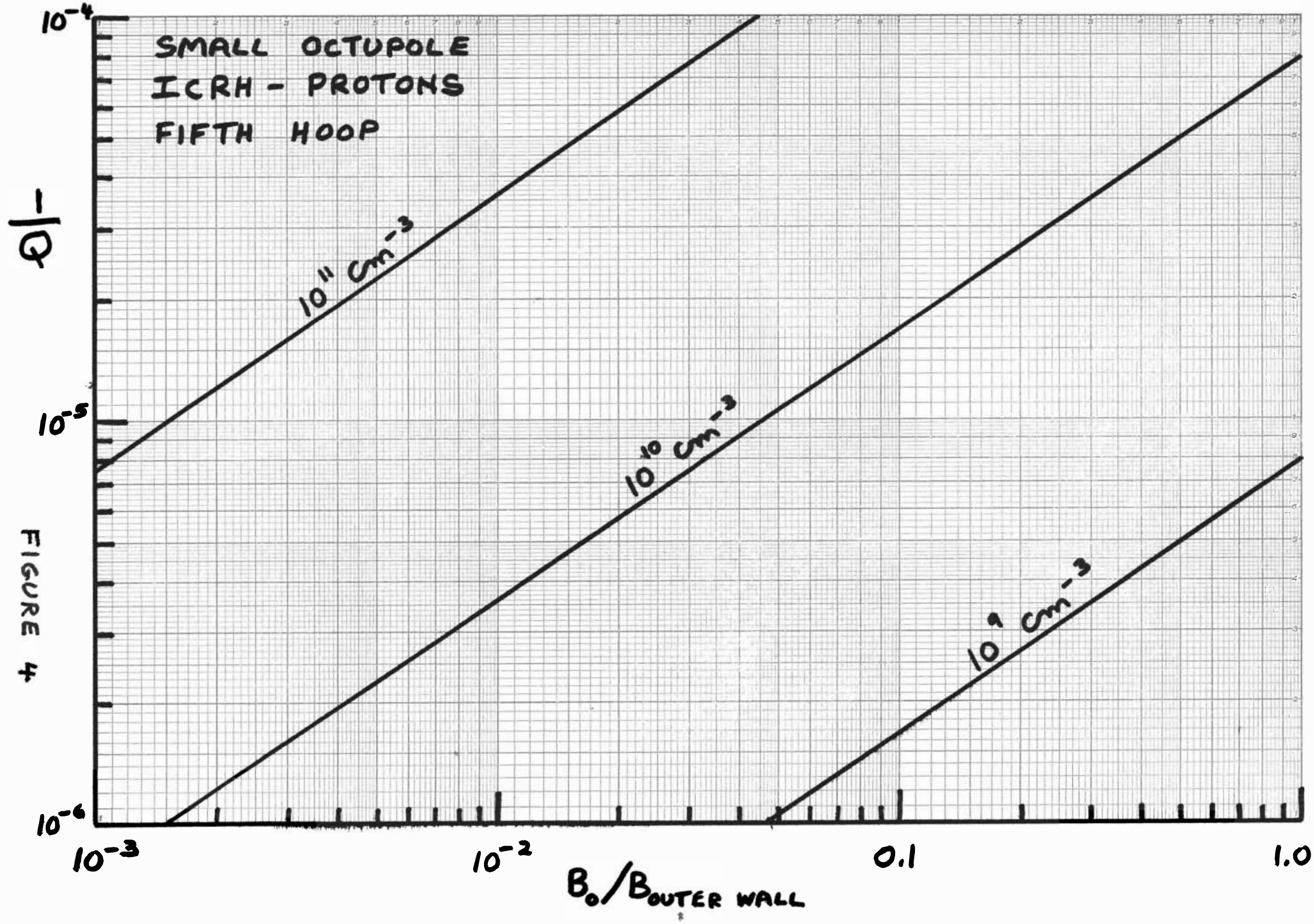

SMALL OCTUPOLE
ICRH - PROTONS
FIFTH HOOP
$\frac{1}{Q}$
$10^{-4}$
$10^{-5}$
$10^{-6}$
$10^{11}$ cm$^{-3}$
$10^{10}$ cm$^{-3}$
$10^{9}$ cm$^{-3}$
$10^{-3}$
$10^{-2}$
0.1
1.0
$B_0/B_{OUTER\ WALL}$

FIGURE 4