

NUMERICAL CALCULATIONS OF SINGLE
PARTICLE ORBITS IN A LINEAR OCTUPOLE

by

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This note describes a computer code SPEMS (Single Particle Equation of Motion Solver) that calculates the trajectory of a single, non-relativistic particle in a linear octupole magnetic field (with B_z). It can be easily adapted to any (time dependent) electromagnetic field. It uses the UW MACC subroutine DEPC which is a general differential equation solver using the predictor-corrector method. The predictor-corrector method was found to be slightly faster and more accurate than the Runge-Kutta method (DERK).

The equations that are solved are the Lorentz force equations in cartesian coordinates:

$$\begin{aligned}\dot{V}_X &= V_Y B_Z - V_Z B_Y \\ \dot{V}_Y &= V_Z B_X - V_X B_Z \\ \dot{V}_Z &= V_X B_Y - V_Y B_X ,\end{aligned}$$

where

$$\begin{aligned}V_X &= \dot{X} \\ V_Y &= \dot{Y} \\ V_Z &= \dot{Z} .\end{aligned}$$

For the sake of simplicity, e/M has been set equal to one, which is equivalent to measuring the magnetic field in units of cyclotron radians/sec. These six equations are solved simultaneously from $t = 0$ to t_{\max} with arbitrary initial conditions, $X_0, Y_0, Z_0, V_{X0}, V_{Y0}, V_{Z0}$.

The linear octupole field is produced by four infinite current filaments at $X = \pm 1$, $Y = \pm 1$, with currents in the +Z direction and return currents at infinity. The four currents are equal and have a magnitude chosen so that near the axis ($X = Y = 0$), the fields satisfy

$$B_X^2 + B_Y^2 = (X^2 + Y^2)^3 .$$

The field parallel to the currents (B_Z) can be specified arbitrarily.

The program prints the X, Y, and Z coordinates, the relative energy (V^2/V_0^2), and magnetic moment ($V^2 B_0^2 / V_0^2 B$) of the particle at 100 equally spaced intervals between $t = 0$ and t_{\max} and it graphs the projection of the particle's position on the X-Y plane at the same intervals using the UW MACC subroutine GRAPH2.

The program is typically able to follow a particle for 500 gyroradians in 10 seconds of 1110 computing time at a cost of about \$1.00 with an accuracy (in energy) of 1%.

Attached is a complete of FORTRAN listing with output for the case referenced by Kerst in PLP 542.

.SPEMS

-MACC 1.10S-02/26/74-10:30:32

SPEMS

```
1. C PROGRAM SPEMS - SPROTT FEB 24, 1974
2. DIMENSION YO(6),YF(7),YI(101,6)
3. DIMENSION XG(101),YG(101)
4. EXTERNAL DERIVS
5.
6. C SPECIFY PARAMETERS
7. X=0.5
8. Y=0.0
9. Z=0.0
10. VX=0.05
11. VY=VX
12. VZ=VX
13. TMAX=500.0
14. TP=TMAX/100.0
15. EPS=1.0E-3
16. HMIN=1.0E-5*TMAX
17. HSTART=0.1*TMAX
18.
19. C SOLVE EQUATIONS
20. UO=VX*VX+VY*VY+VZ*VZ
21. CALL RCAL (X,Y,Z,0.0,RX,RY,RZ)
22. PMUO=((VX*BY-VY*BX)**2+(VY*BZ-VZ*BY)**2+(VZ*BX-VX*BZ)**2)
23. 2/(RX*RX+BY*BY+RZ*BZ)**1.5
24. WRITE (6,200)
25. 200 FORMAT (1H0,5X,1HI,9X,4HTIME,10X,1HX,12X,1HY,12X,1HZ,11X,2HKE,11 X,
26. 22HMU)
27. YO(1)=X
28. YO(2)=Y
29. YO(3)=Z
30. YO(4)=VX
31. YO(5)=VY
32. YO(6)=VZ
33. CALL DEPC (6,0.0,YO,TMAX,YF,DERIVS,EPS,0.0,HMIN,TMAX,HSTART,2,TP,
34. 22H.,YI,101,6,NPOINT,NOTIFY,$400)
35. 400 CONTINUE
36. DO 600 I=1,NPOINT
37. T=TP*FLOAT(I-1)
38. U=(YI(I,4)**2+YI(I,5)**2+YI(I,6)**2)/UO
39. CALL BCAL (YI(I,1),YI(I,2),YI(I,3),T,BX,BY,BZ)
40. PMU=((VX*BY-VY*BX)**2+(VY*BZ-VZ*BY)**2+(VZ*BX-VX*BZ)**2)
41. 2/(BX*BX+BY*BY+RZ*BZ)**1.5/PMUO
42. WRITE (6,500) I,T,YI(I,1),YI(I,2),YI(I,3),U,PMU
43. 500 FORMAT (1H,16,6F13.4)
44. XG(I)=YI(I,1)
45. YG(I)=YI(I,2)
46. 600 CONTINUE
47.
48. C GRAPH RESULTS
49. NPOINT=-NPOINT
50. CALL GRAPH2 (XG,'R',YG,'R',NPOINT,'6X6','AUTO','PARTICLE TRAJECTOR
51. 2Y TN A LINEAR OCTUPOLF..','X POSITION..','Y POSITION..','*')
52. CALL GRPHND
53. END
```

END OF COMPILATION: NO DIAGNOSTICS.

qFOR,IS ,DERIVS

FORTRAN-MACC 1.10S-02/26/74-10:30:38

DERIVS

```
00101      1.      SUBROUTINE DERIVS(T,Y,DY,STORE,ITEST)
00102      2.      DIMENSION DY(1),Y(1),STORE(1)
00103      3.      CALL BCAL (Y(1),Y(2),Y(3),T,BX,BY,BZ)
00104      4.      DY(1)=Y(4)
00105      5.      DY(2)=Y(5)
00106      6.      DY(3)=Y(6)
00107      7.      DY(4)=Y(5)*BZ-Y(6)*BY
00110      8.      DY(5)=Y(6)*BX-Y(4)*BZ
00111      9.      DY(6)=Y(4)*BY-Y(5)*BX
00112     10.      DO 300 I=1,6
00115     11.          300 STORE(I)=Y(I)
00117     12.      RETURN
00120     13.      END
```

END OF COMPILATION: NO DIAGNOSTICS.

S ,BCAL

N-MACC 1.10S-02/26/74-10:30:44

BCAL

```
1.      SUBROUTINE BCAL (X,Y,Z,T,BX,BY,BZ)
2.      BX=(1.-Y)/((X-1.)**2+(Y-1.)**2)-(1.+Y)/((X-1.)**2+(Y+1.)**2)
3.      2+(1.-Y)/((X+1.)**2+(Y-1.)**2)-(1.+Y)/((X+1.)**2+(Y+1.)**2)
4.      BY=(X-1.)/((X-1.)**2+(Y-1.)**2)+(X-1.)/((X-1.)**2+(Y+1.)**2)
5.      2+(X+1.)/((X+1.)**2+(Y-1.)**2)+(X+1.)/((X+1.)**2+(Y+1.)**2)
6.      BZ=1.0
7.      RETURN
8.      END
```

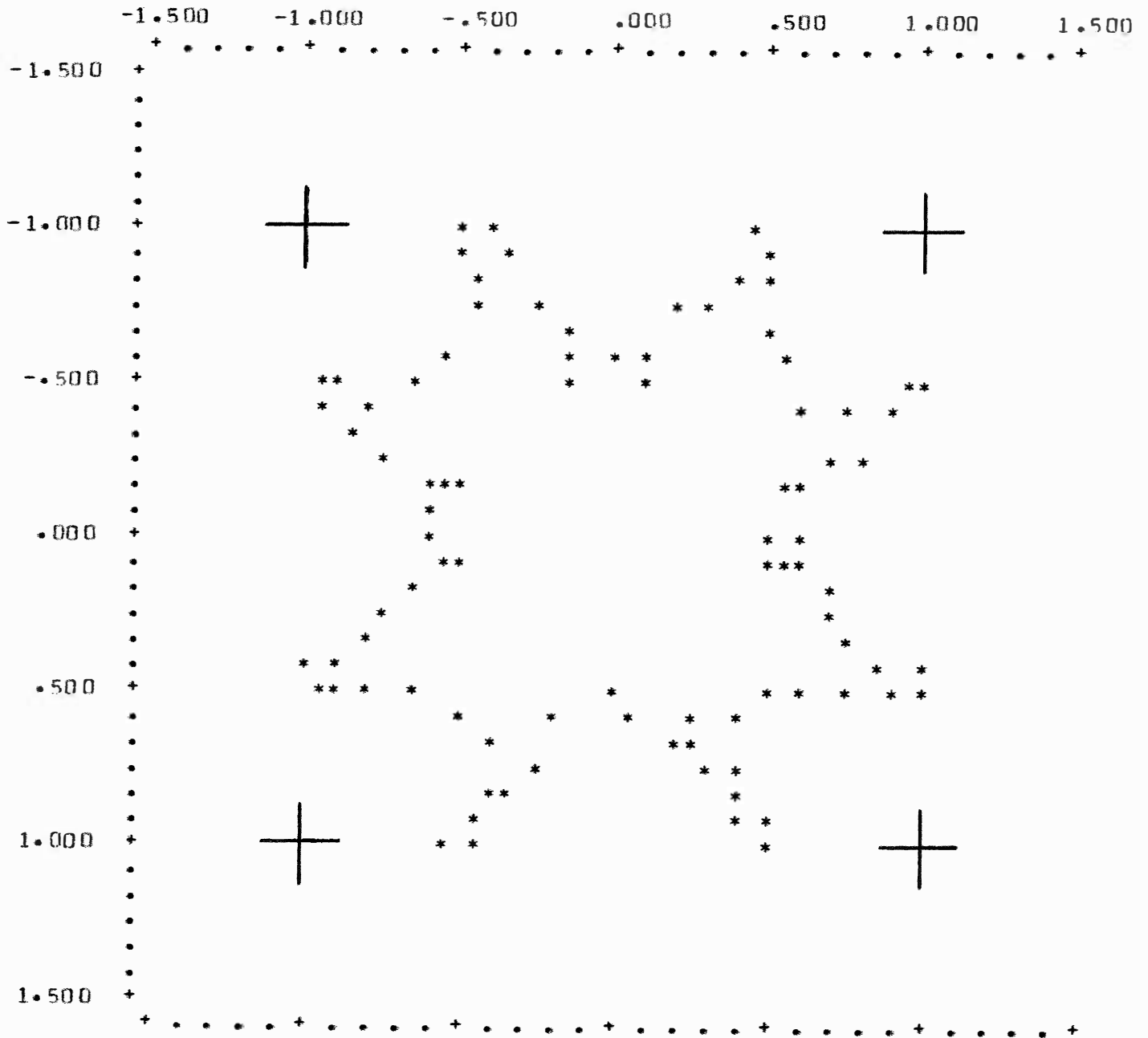
END OF COMPILATION: NO DIAGNOSTICS.

TIME	X	Y	Z	KE	MU
.0000	.5000	.0000	.0000	1.0000	1.0000
5.0000	.4734	-.0234	.2870	1.0002	1.0419
10.0000	.5391	-.0205	.5702	1.0003	.9857
15.0000	.6176	.0637	.8345	1.0004	.7954
20.0000	.6284	.1799	1.0918	1.0004	.6695
25.0000	.6011	.2483	1.3674	1.0004	.6998
30.0000	.6256	.2657	1.6454	1.0008	.6443
35.0000	.7270	.2980	1.8947	1.0016	.4028
40.0000	.8459	.3814	2.0867	1.0016	.1591
45.0000	.9368	.5057	2.1726	1.0017	.0928
50.0000	1.0194	.4929	2.1485	1.0019	.0148
55.0000	.9104	.4174	2.1580	1.0017	.0757
60.0000	.7361	.4008	2.0302	1.0016	.4142
65.0000	.5965	.4074	1.8081	1.0014	.7965
70.0000	.5233	.4758	1.5526	1.0020	1.0591
75.0000	.4972	.6123	1.3204	1.0023	1.3019
80.0000	.4934	.7676	1.1386	1.0022	1.3060
85.0000	.4870	.8899	.9984	1.0024	1.1054
90.0000	.4496	1.0109	.9823	1.0024	.9503
95.0000	.4929	.9797	1.0716	1.0023	.9204
100.0000	.4407	.8448	1.1594	1.0021	1.2658
105.0000	.3668	.7457	1.3570	1.0021	1.4582
110.0000	.2889	.7093	1.6064	1.0023	1.5024
115.0000	.1769	.6833	1.8546	1.0025	1.4965
120.0000	.0721	.6041	2.1032	1.0026	1.4023
125.0000	.0387	.5057	2.3749	1.0027	1.3052
130.0000	.0627	.4786	2.6616	1.0027	1.2976
135.0000	.0539	.5412	2.9455	1.0026	1.3417
140.0000	-.0361	.6080	3.2110	1.0029	1.3194
145.0000	-.1501	.6013	3.4715	1.0028	1.1735
150.0000	-.2055	.5583	3.7511	1.0029	1.0668
155.0000	-.2042	.5744	4.0349	1.0030	1.0741
160.0000	-.2196	.6763	4.2975	1.0036	1.0936
165.0000	-.2861	.8082	4.5176	1.0049	1.0429
170.0000	-.3758	.9077	4.6931	1.0049	.9294
175.0000	-.4996	.9464	4.7816	1.0050	.7090
180.0000	-.5084	.9970	4.7175	1.0055	.7473
185.0000	-.4079	.8989	4.6777	1.0053	.8521
190.0000	-.3827	.7357	4.5229	1.0053	.7345
195.0000	-.4338	.6242	4.2929	1.0056	.5354
200.0000	-.5566	.5566	4.0672	1.0063	.2989
205.0000	-.7064	.5068	3.8778	1.0076	.3407
210.0000	-.8443	.5000	3.7169	1.0076	.5507
215.0000	-.9219	.5203	3.5777	1.0077	.6305
220.0000	-.9978	.4477	3.5514	1.0077	.8580
225.0000	-.9780	.4458	3.6776	1.0075	.8474
230.0000	-.8641	.4016	3.8184	1.0075	.8282
235.0000	-.7857	.3092	4.0216	1.0075	.9728
240.0000	-.7167	.1943	4.2475	1.0077	1.1663
245.0000	-.6106	.1038	4.4912	1.0076	1.2437
250.0000	-.5110	.0860	4.7636	1.0076	1.2077
255.0000	-.4960	.1086	5.0509	1.0076	1.1769
260.0000	-.5595	.0815	5.3338	1.0074	1.2396
265.0000	-.6054	-.0218	5.5978	1.0077	1.3688

5	270.0000	-.5729	-.1276	5.8624	1.0077	1.4044
6	275.0000	-.5213	-.1612	6.1460	1.0077	1.3682
7	280.0000	-.5453	-.1458	6.4315	1.0079	1.3863
8	285.0000	-.6537	-.1645	6.6973	1.0082	1.4781
9	290.0000	-.7806	-.2446	6.9259	1.0090	1.4966
0	295.0000	-.8893	-.3386	7.1180	1.0090	1.3196
1	300.0000	-.9686	-.4001	7.2822	1.0090	1.1037
2	305.0000	-.9569	-.4951	7.3723	1.0091	.9575
3	310.0000	-.9594	-.5171	7.2794	1.0093	.9099
4	315.0000	-.8644	-.4342	7.1753	1.0091	1.2398
5	320.0000	-.7250	-.4445	6.9986	1.0091	1.4010
6	325.0000	-.6096	-.5361	6.7979	1.0098	1.2366
7	330.0000	-.5041	-.6522	6.6042	1.0102	.7938
8	335.0000	-.4387	-.7795	6.4099	1.0107	.3377
9	340.0000	-.4697	-.9144	6.2631	1.0107	.0967
0	345.0000	-.5361	-.9537	6.1537	1.0110	.0946
1	350.0000	-.4419	-.9747	6.1299	1.0109	.0244
2	355.0000	-.3843	-.9479	6.2728	1.0109	.0374
3	360.0000	-.3106	-.8502	6.4457	1.0110	.1598
4	365.0000	-.2107	-.7397	6.6508	1.0109	.4219
5	370.0000	-.1439	-.6129	6.8924	1.0105	.7242
6	375.0000	-.1420	-.5235	7.1668	1.0107	.8720
7	380.0000	-.1531	-.5276	7.4545	1.0107	.8610
8	385.0000	-.0978	-.5866	7.7337	1.0106	.8066
9	390.0000	.0174	-.6004	7.9949	1.0107	.9199
0	395.0000	.1044	-.5409	8.2644	1.0106	1.0866
1	400.0000	.1113	-.4931	8.5509	1.0108	1.1195
2	405.0000	.0920	-.5371	8.8360	1.0108	1.0729
3	410.0000	.1305	-.6492	9.1002	1.0109	1.0503
4	415.0000	.2331	-.7495	9.3348	1.0112	1.1506
5	420.0000	.3433	-.8224	9.5512	1.0111	1.2265
6	425.0000	.4207	-.9197	9.7289	1.0112	1.1116
7	430.0000	.4352	-1.0081	9.8649	1.0112	.9658
8	435.0000	.4889	-.9549	9.9442	1.0109	1.0202
9	440.0000	.5181	-.9024	9.8344	1.0109	1.0647
0	445.0000	.4886	-.7927	9.6886	1.0107	1.2729
1	450.0000	.5250	-.6478	9.5153	1.0107	1.4347
2	455.0000	.5894	-.5051	9.3142	1.0109	1.4944
3	460.0000	.6753	-.4063	9.0841	1.0110	1.4279
4	465.0000	.8133	-.3882	8.8789	1.0113	1.2703
5	470.0000	.9661	-.4525	8.7824	1.0113	1.0274
6	475.0000	.9710	-.5389	8.7259	1.0118	.9374
7	480.0000	.9356	-.4307	8.7209	1.0114	1.0840
8	485.0000	.8738	-.3312	8.8686	1.0115	1.1425
9	490.0000	.7532	-.2475	9.0635	1.0115	1.1723
0	495.0000	.6238	-.1997	9.3037	1.0112	1.1841
1	500.0000	.5519	-.2007	9.5789	1.0113	1.2143

PARTICLE TRAJECTORY IN A LINEAR OCTUPOLE

Y POSITION



RUNID: CW1392 PROJECT: 02980

USER: 4126810219

ITEM	AMOUNT	COST(DOLLARS)
CPU TIME	00:00:08.577	\$0.32
FILE I/O REQUESTS	279	\$0.13
FILE I/O WORDS	270403	\$0.12
MEMORY USAGE	0.223	\$0.14
CARDS IN	82	\$0.02
PAGES PRINTED	5	\$0.07
JOB CHARGE	1	\$0.05
TOTAL COST		\$0.85

THE ABOVE DOLLAR AMOUNTS ARE APPROXIMATE AND ARE BASED ON RATES FOR WHIL
PROJ BALANCE \$2454.13

INITIATION TIME: 10:30:31-FEB 26,1974

TERMINATION TIME: 10:32:05-FEB 26,1974

PREVIOUS RUN TIME: 10:31:50-FEB 26,1974

EOF

5 PAGES COST

\$0.07

PROJ BALANCE

\$2454.09