A QUANTITATIVE ACCOUNT FOR THE ANOMALOUS FALLOFF OF THE ECRH HEATING RATE WITH ELECTRON DENSITY

by

J. C. Sprott and K. L. Wong

May 1974

PLP 560

Plasma Studies

University of Wisconsin

In PLP 559, the physical mechanism of the anomalous falloff of the electron cyclotron heating rate has been discussed. It is the purpose of this note to estimate the fraction of the r.c.p. wave that can reach the resonance zone and give a more quantitative account for the scaling of the heating rate with electron density.

Assume the r.c.p. wave propagates along straight field lines,

$$E(x) = E_o e^{ikx}$$

then,

$$-\frac{dE}{E} = ikdk + ikdx$$

$$\ln \frac{E_1}{E_2} = i \int_{x_1}^{x_2} (xdk + kdx)$$

$$= i \int_{x_1}^{x_2} (k + \frac{xdk}{dx}) \; dx \tag{1}$$

where

$$k = \frac{\omega}{c} (1 - \frac{\omega_p^2}{\omega(\omega-\omega_{ce})})^{1/2} = \frac{\omega}{c} (1 - \frac{\omega_p^2}{\omega\alpha x})^{1/2}$$

$$\alpha = \frac{e}{m} |\nabla_{\parallel} B|$$

$$x_1 = \frac{\omega_p^2}{\omega\alpha} = \text{position at which } k^2 = 0$$

$$x_2 = 0 = \text{position at which } k^2 = -\infty$$

$$= \text{position of the resonance zone}$$

E_1 = electric field at x_1

E_2 = electric field at x_2

$$\ln \frac{E_1}{E_2} = i \int_{x_1}^{0} \frac{\omega}{c} (1 - \frac{\omega_p^2}{\omega\alpha x})^{1/2} dx + i \int_{x_1}^{0} \frac{\omega}{2c} \frac{\omega_p^2}{\omega\alpha x} (1 - \frac{\omega_p^2}{\omega\alpha x})^{-1/2} dx$$

$$= i \frac{\omega}{c} \left(- \frac{ib\pi}{2} \right) + i \frac{\omega}{2c} (-ib\pi)$$

$$= \frac{\omega_p^2 \pi}{c\alpha}$$

$$= 2\pi^2 \frac{B_o}{\lambda|\nabla_{\parallel}} \frac{\omega_p^2}{\omega^2}$$

$$E_2 = E_1 e^{\left(-2\pi^2 \frac{B_o}{\lambda|\nabla_{\parallel} B|} \frac{\omega_p^2}{\omega^2}\right)} \qquad (2)$$

Since the heating rate is proportional to the square of the electric field,

$$G = G_o e^{\left(-4\pi^2 \frac{B_o}{\lambda|\nabla_{\parallel} B|} \frac{\omega_p^2}{\omega^2}\right)} \qquad (3)$$

where G_o is the theoretical heating rate with a uniform, isotropic electric field. In our experiment,

$B_o = 3.24$ kG

$\lambda = 3.2$ cm

$|\nabla B| \sim 0.5$ kG/cm

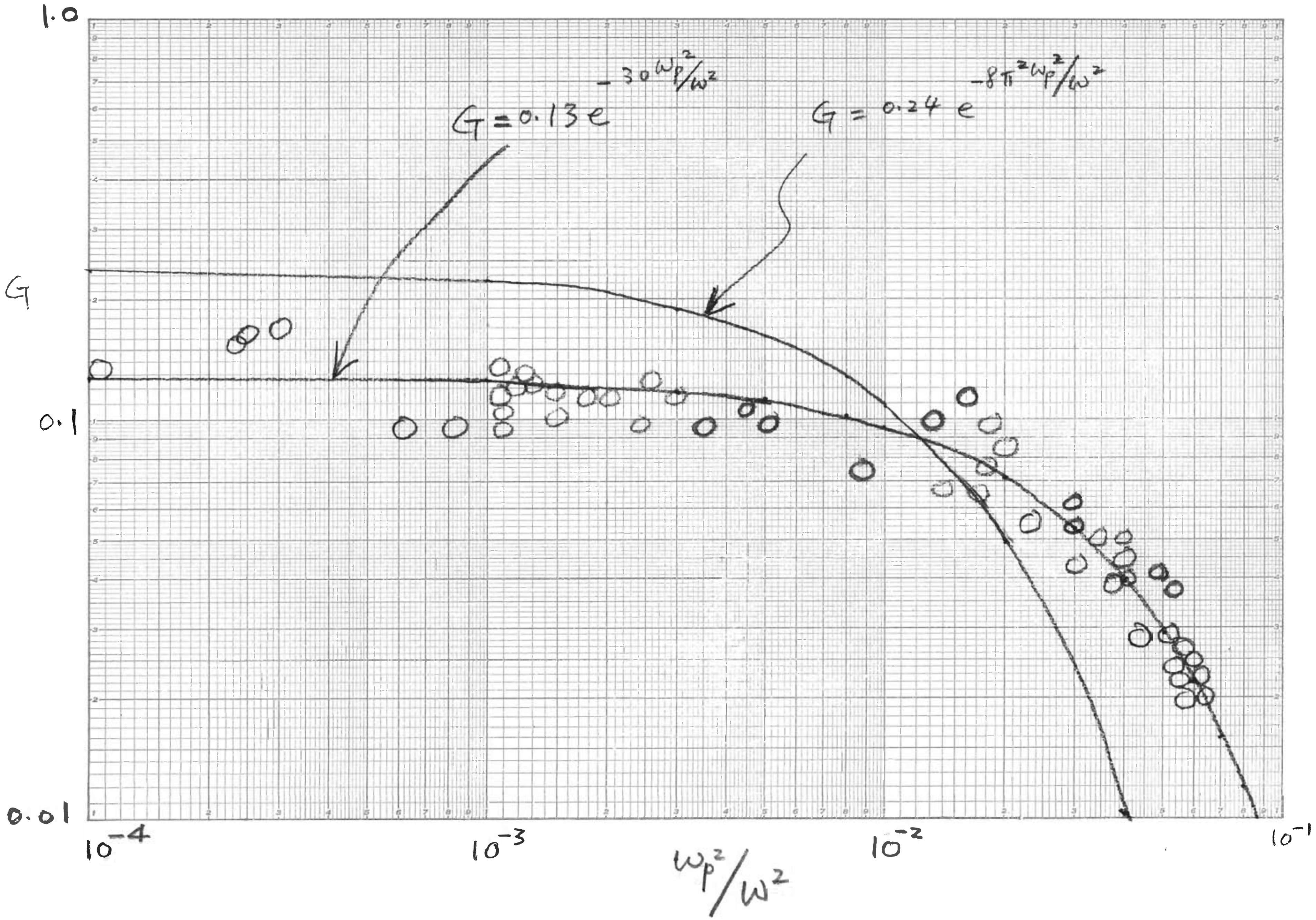
$G_o \simeq 0.24$

So Eq. (2) gives

$$G = 0.24 e^{\left(-8\pi^2 \frac{\omega_p^2}{\omega^2}\right)} \qquad (4)$$

This curve is shown in Fig. 1. We don't expect a very nice fit to the data points since it is derived from the straight field line assumption, and in our experiment, the radii of curvature of the field lines are comparable to the wavelength. Moreover, the wave number k varied rapidly (k^2 from 0 to $-\infty$) over a distance shorter than the wavelength. So Eq. (2)

is not exactly correct. However it gives a qualitative behavior of the heating rate as a function of electron density. The curve which has the best fit to the data is $G = 0.13 \exp(-30\omega_p^2/\omega^2)$ as shown in Fig. 1. It should be noted that if our density measurements are a factor of two too high, then the corrected data points will fit the curve $G = 0.24 \exp(-60\omega_p^2/\omega^2)$ which is very close to Eq. (4).


1.0
G = 0.13 e
- 30 ωp²/ω²
G = 0.24 e
-8π² ωp²/ω²
G
0.1
0.01
10⁻⁴
10⁻³
10⁻²
10⁻¹
ωp²/ω²