OHMIC HEATING RATE IN A TOROIDAL OCTUPOLE

J. C. Sprott

August 1975

PLP 643

Plasma Studies

University of Wisconsin

Consider a toroidal octupole field B_P with superimposed orthogonal toroidal field B_T and both fields varying in time. We wish to estimate the power P delivered to the plasma as a function of time via ohmic heating assuming the vacuum fields are unperturbed by any currents in the plasma. As a starting point,

$$P = \int \sigma E_{\parallel}^2 \, dV,$$

where σ is the dc conductivity, $E_{\parallel}$ is the component of $\vec{E}$ parallel to $\vec{B}$ $(= \vec{B}_P + \vec{B}_T)$, and the integration is over the volume of the plasma. Now $E_{\parallel}$ consists of two parts:

$$E_{\parallel} = \frac{E_T B_T}{B} + \frac{E_P B_P}{B} .$$

The toroidal electric field is given in terms of the poloidal flux ψ by

$$E_T = -\frac{1}{2\pi R} \frac{d}{dt} (\psi - \psi_{HOOP})$$

where R is the major radius.

The poloidal electric field can be approximated by

$$E_P \simeq \int \frac{dB_T}{dt} dA / \oint d\ell,$$

where $\oint d\ell$ is the length of a (closed) poloidal field line and the area integral is evaluated in the poloidal plane between the hoops and the flux surface on which E_P is to be calculated.

A fortunate property of an octupole is that in poloidal flux space most of the plasma is located very near the separatrix, and so it is reasonable to expect that electric fields calculated on the separatrix would represent an appropriate average for determining the power absorbed by the plasma. We further make a large aspect ratio approximation so that the variation of the toroidal field over the volume of the plasma is negligible. Then

$$E_T = -\frac{1}{2\pi R_o}\frac{d}{dt}(\psi_{SEP} - \psi_{HOOP}) = -C_1\dot{B}_{Po}$$

and

$$E_P = \frac{dB_T}{dt}\int_{SEP} dA/\oint d\ell \ldots = C_2\dot{B}_T.$$

The constants C_1 and C_2 can be determined with a bit of work directly from the flux plot. For the small octupole,

$$C_1 \simeq 0.0135 \text{ meters}$$

and $$C_2 \simeq 0.02 \text{ meters.}$$

Now since we have used an approximation in which B_T, E_T, and E_P are constant in space, the volume integral involves only the spatial dependence of B_P which we take to be

$$B_P = \left(\frac{r}{a}\right)^3 B_{Po}.$$

Therefore the power can be written as

$$P = 4\pi^2 R_o \int_0^a \frac{(E_T B_T + E_P r^3 B_{Po}/a^3)^2}{B_T^2 + r^6 B_{Po}^2/a^6}\,\sigma r dr.$$

It is reasonable to assume that the conductivity is constant in space and to remove it from the integral. The remaining integral is just barely double with an integral table and a very ambitious graduate student. Lacking the latter, a satisfactory approximation can be obtained by evaluating the integral in the limits α ($\equiv B_{Po}/B_T$) $\ll 1$ and $\alpha \gg 1$, and fitting the intermediate case with a smooth analytic function to get the result:

$$P = 4\pi^2 R_o\, a^2\, \sigma \left(\frac{E_T^2}{2 + 4\,\alpha^{2/3}} + \frac{2E_T E_P\, \alpha}{5 + \alpha^{5/3}} + \frac{E_P^2\, \alpha^2}{8+2\alpha^2} \right)$$

Finally we wish to obtain an expression for the conductivity which we assume to arise from both electron-ion and electron-neutral (H_2) collisions. The electron-ion (Spitzer) resistivity is

$$\eta_{SP} = \frac{5.2\text{x}10^{-5}\, Z_{eff}\, \ell n\Lambda}{T_e^{\;3/2}} \cong \frac{6.24\text{x}10^{-4} Z_{eff}}{T_e^{\;3/2}} \text{ ohm-meters}$$

where T_e is in eV and Z_{eff} is any anomaly factor that may be present. The electron-neutral resistivity is

$$\eta_N \cong 6.29\text{x}10^{-6} n_o/n_e \text{ ohm-meters,}$$

and the total conductivity is

$$\sigma = \frac{1}{\eta_{SP}+\eta_N} = (Z_{eff}/1600\ T_e^{\;3/2} + 6.29\text{x}10^{-6} n_o/n_e)^{-1}$$

Putting it all together gives

$$\frac{dP}{dV} = \left(\frac{C_1^{\;2}\, \dot{B}_{Po}^2}{1+2\alpha^{2/3}} - \frac{4\, C_1 C_2 \dot{B}_{Po} \dot{B}_T \alpha}{5+\alpha^{5/3}} + \frac{C_2^{\;2} \dot{B}_T^2 \alpha^2}{4 + \alpha^2} \right)$$

$$/(Z_{eff}/1600\ T_e^{\;3/2} + 6.29\text{x}10^{-6} n_o/n_e) \text{ watts/m}^3.$$

Note that the middle (cross) term implies that the heating is enhanced whenever one field is rising while the other is falling ($\dot{B}_{Po}\ \dot{B}_T < 0$).

The above expression was incorporated as an electron heating term in program SIMULT (see PLP 607, 556) in an attempt to model the experiments being done by Etzweiler on the small octupole (see PLP 636, 623). On that experiment, the poloidal field pulse is a 5 msec half sine wave and the toroidal field pulse is a 2 msec damped half sine wave given approximately by

$$B_T = 1.77\ B_{TMAX} \sin(1571\ t) \exp(-654\ t)(t > 0),$$

which can be initiated at an arbitrary time relative to the poloidal field pulse. The simulation was run with B_T = 1.25 kG (corresponding to 2.0 kV on the B_T bank), B_{Po} = 4.8 kG (corresponding to 2.0 kV on the B_P bank), p = $1\text{x}10^{-5}$ torr (gauge) and Z_{eff} = 1. The 500 watt cw 2.45 GHz microwaves were turned off and the toroidal field pulse turned on at 3.5 msec into the 5 msec poloidal field pulse. The electron temperature rises abruptly to ~ 4 keV when the toroidal field comes on while the density rises more slowly to ~ $1.5\text{x}10^{12}$ cm^{-3}. The neutrals are fairly well burned out (n_e/n_o ~ 10). The conductivity is limited by electron-neutral collisions and a computer simulation with Z_{eff} = 10 looks almost identical to the Z_{eff} = 1 case. The ion temperature always remains less than 1 eV and in fact drops to ~ 0.1 eV during the ohmic heating because of poorer coupling to the electrons and enhanced losses. The dominant loss mechanism is thermal flow to obstacles (taken as 90 cm^2). The neutral density rises about a factor of two as a result of electron impact desorption which Groebner estimates to be

$$\Gamma_n/\Gamma_e \simeq 0.0072\ T_e^{0.53}.$$

The results of the simulation are reasonable except for the 4 keV electron temperature which is probably much too high. It does suggest the importance of improving our soft x-ray diagnostics, however. There are a number of effects that could reduce the temperature in the experiment: impurities, turbulent enhancement of the resistivity, poor penetration of the electric fields, instabilities, anamolous losses, electron runaway, etc. In particular, for 4 keV and $1.5\text{x}10^{12}\text{cm}^{-3}$ the Dreicer critical runaway electric field is only 0.04 volts/meter which is surely exceeded in the experiment. Lencioni (PLP 276) has experimentally measured the plasma resistivity in this runaway regime, and he finds

$$\eta_L = 10^8\sqrt{T_e}/4n \qquad \text{(ohms/meter)}$$

where T_e is in eV and n is in cm^{-3}. If we add this resistivity to η_{SP} and η_N, the simulation predicts a much more credible value of $T_e \sim 100$ eV and $n \sim 7\times10^{11} cm^{-3}$. The detailed results are shown in Figure 1. The simulations throughout the rest of this paper include the Lencioni term to account for electron runaway.

It is, of course, desirable to have some comparisons of the simulation with experimental results. The one quantity that is easy to measure experimentally is the volume-averaged ion saturation current to a Langmuir probe in the bridge (see Strait, PLP 566). The ion saturation current can not be interpreted in the usual way, however, because we anticipate that kT_e/e may considerably exceed the usual probe bias voltage of 45 volts. However, if plasma losses are governed by thermal flow to grounded obstacles in the plasma (i.e. hoop supports), we expect the floating potential V_f to be near zero. In fact experimental measurements show $|V_f| \lesssim 10$ volts for the case considered here. For $V_f = 0$, the current to a probe biased to $-V_B$ is given by

$$I = I_{o_i}\ (1 - \exp(-eV_B/kT_e)),$$

so that in the limit $kT_e/e >> V_B$, the probe current obeys the rather unusual relation

$$I \propto n/\sqrt{T_e}$$

provided $T_e > T_i$. Program SIMULT was modified to predict the current density to a Langmuir probe biased to - 45 volts for arbitrary T_e, assuming $V_f = 0$. The predicted current is compared with the observed current in Fig. 2. The agreement is pretty good considering all the approximations that are involved. Recall also that the secondary electron emission coefficient for a probe may exceed unity at these electron temperatures, and so detailed agreement is even less likely.

To further test the simulation, the peak ion current was measured as a function of the four easily varied parameters: B_T, B_{Po}, τ (toroidal field turn-on time), and p (filling pressure). The experimental results are compared with the computer simulation in Figs. 3-6. The agreement is not too impressive and shows a tendency for the simulation to overestimate the heating, especially at low values of magnetic field.

Finally, as an aside, the change in ion gauge pressure reading from before to immediately after the pulse was recorded as a function of initial pressure and the result is shown in Fig. 7. Note that over the intermediate range of pressures the ohmic discharge acts as a vacuum pump and produces as much as a 40% reduction in background pressure. The simulation always predicts a positive Δp because it is assumed that every ion that strikes a surface recombines and returns as a neutral. This is clearly an erroneous assumption and needs to be replaced with something more realistic.

In conclusion, we have derived an expression for the volume-averaged power absorbed by the plasma in an octupole due to ohmic heating when an arbitrarily time varying poloidal and toroidal field are present. Computer simulations show an unreasonably high electron temperature (4 keV) if the resistivity is classical. However, since the electric field exceeds the critical field for electron runaway, it is more reasonable to use a resistivity such as that measured experimentally by Lencioni, in which case the simulation give a more believable value of 100 eV. The calculation is available as a function subprogram (OHMIC) which can be called from SIMULT. The intent is to continually refine the [illegible]

The intent is to continually refine the subprogram as more detailed information on the conductivity becomes available from the experiment.

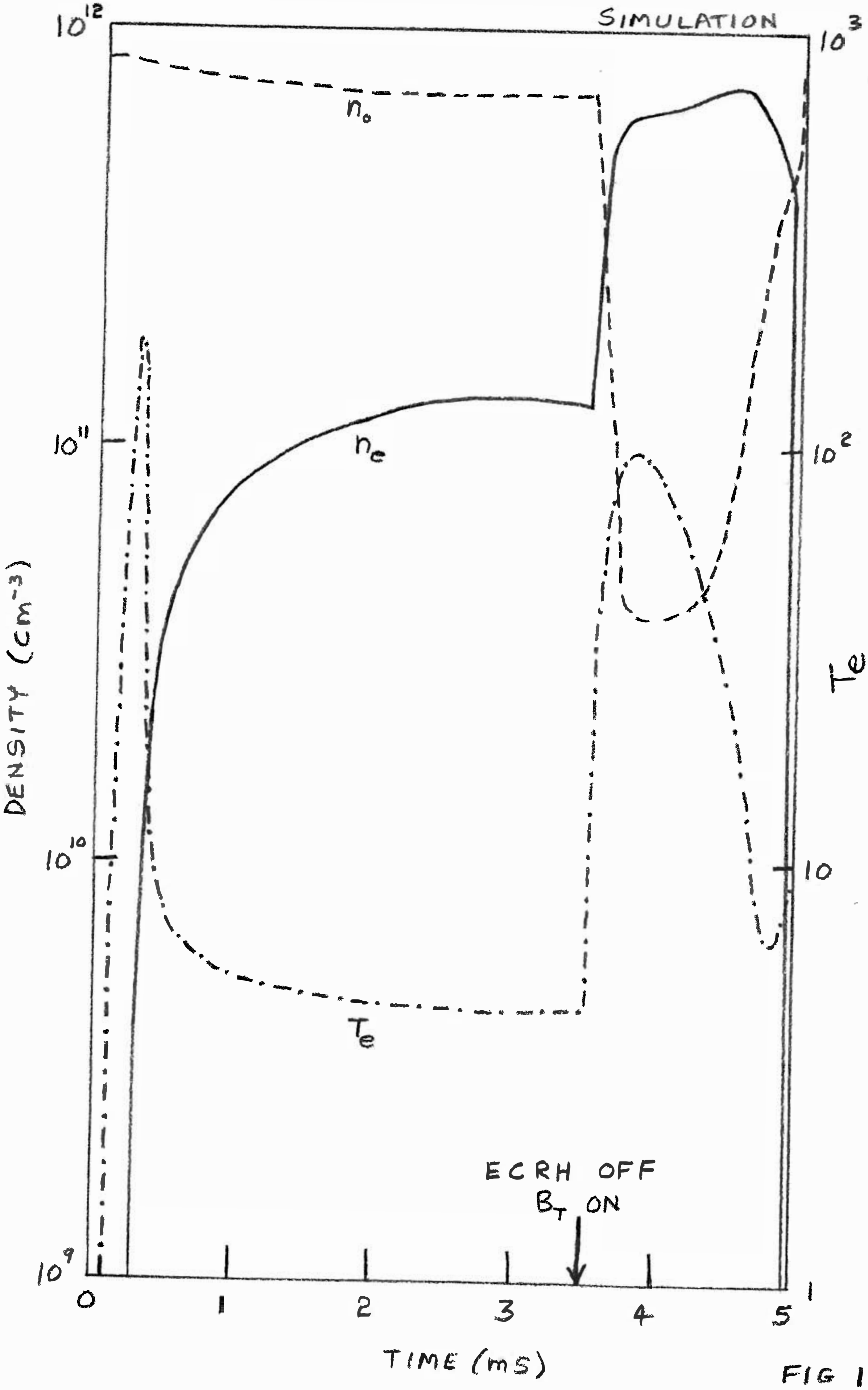
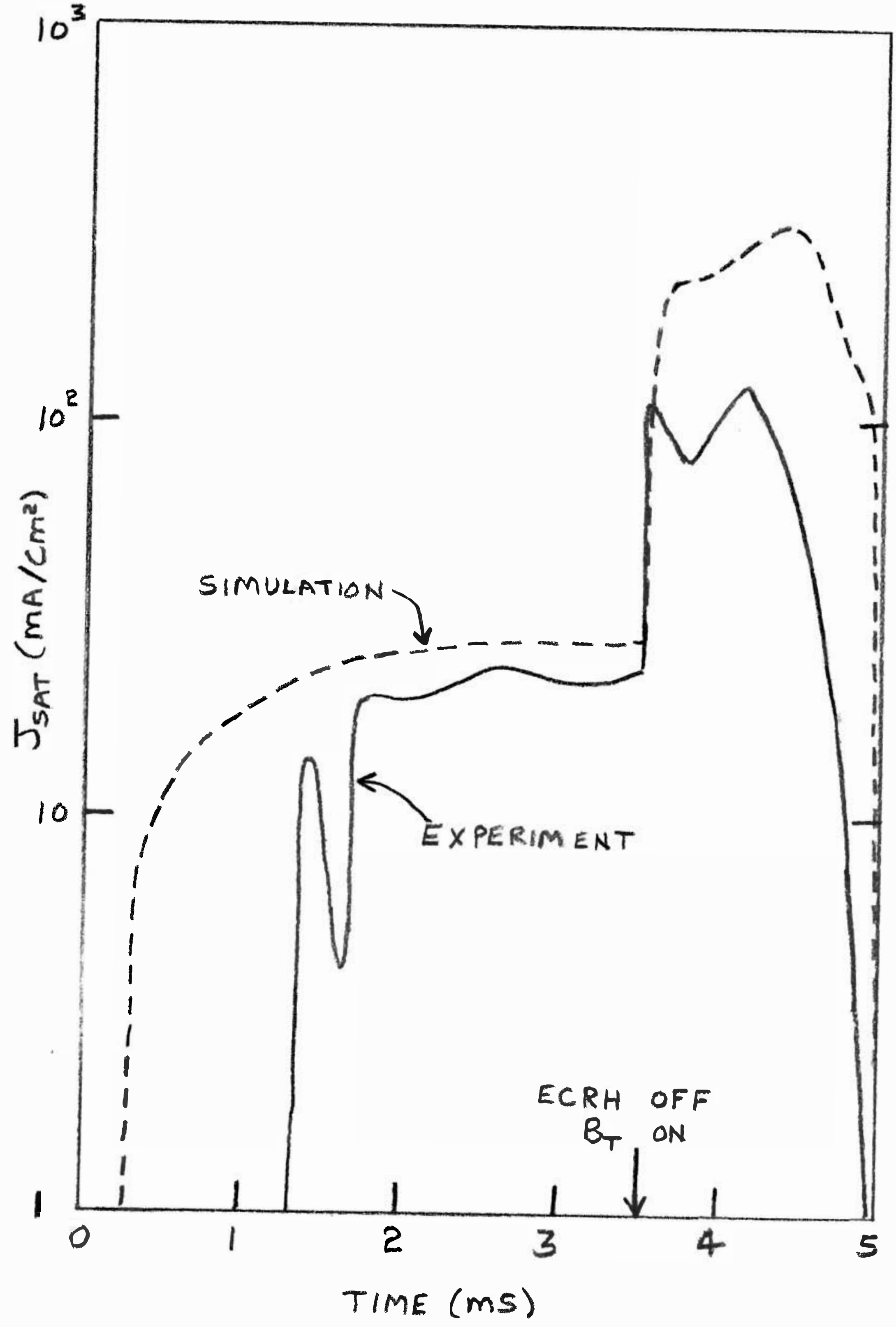
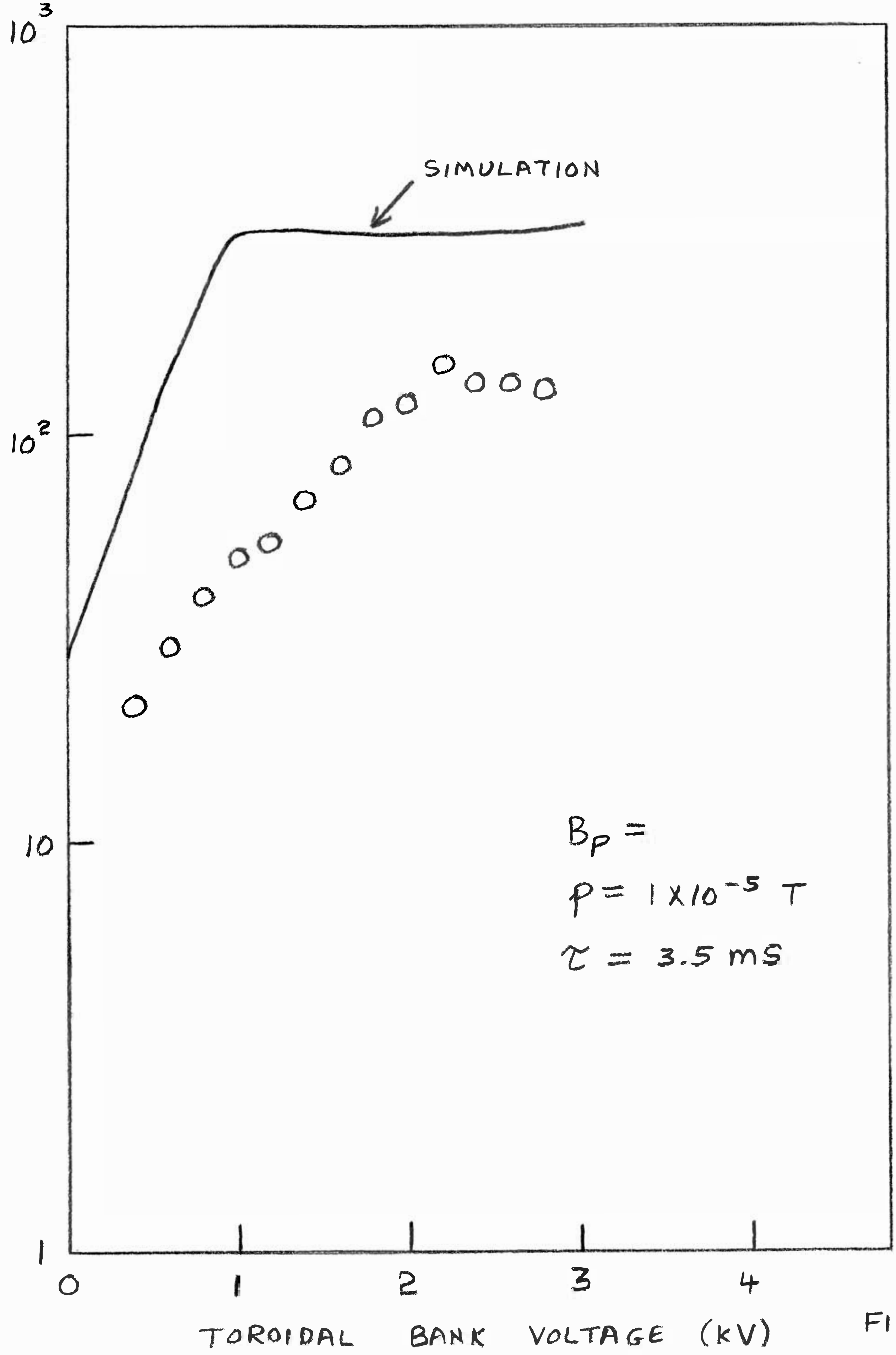

SIMULATION
10^{12}
10^{11}
10^{10}
10^{9}
10^{3}
10^{2}
10
1
n_o
n_e
T_e
DENSITY (cm^{-3})
T_e
ECRH OFF
B_T ON
0
1
2
3
4
5
TIME (mS)

FIG 1



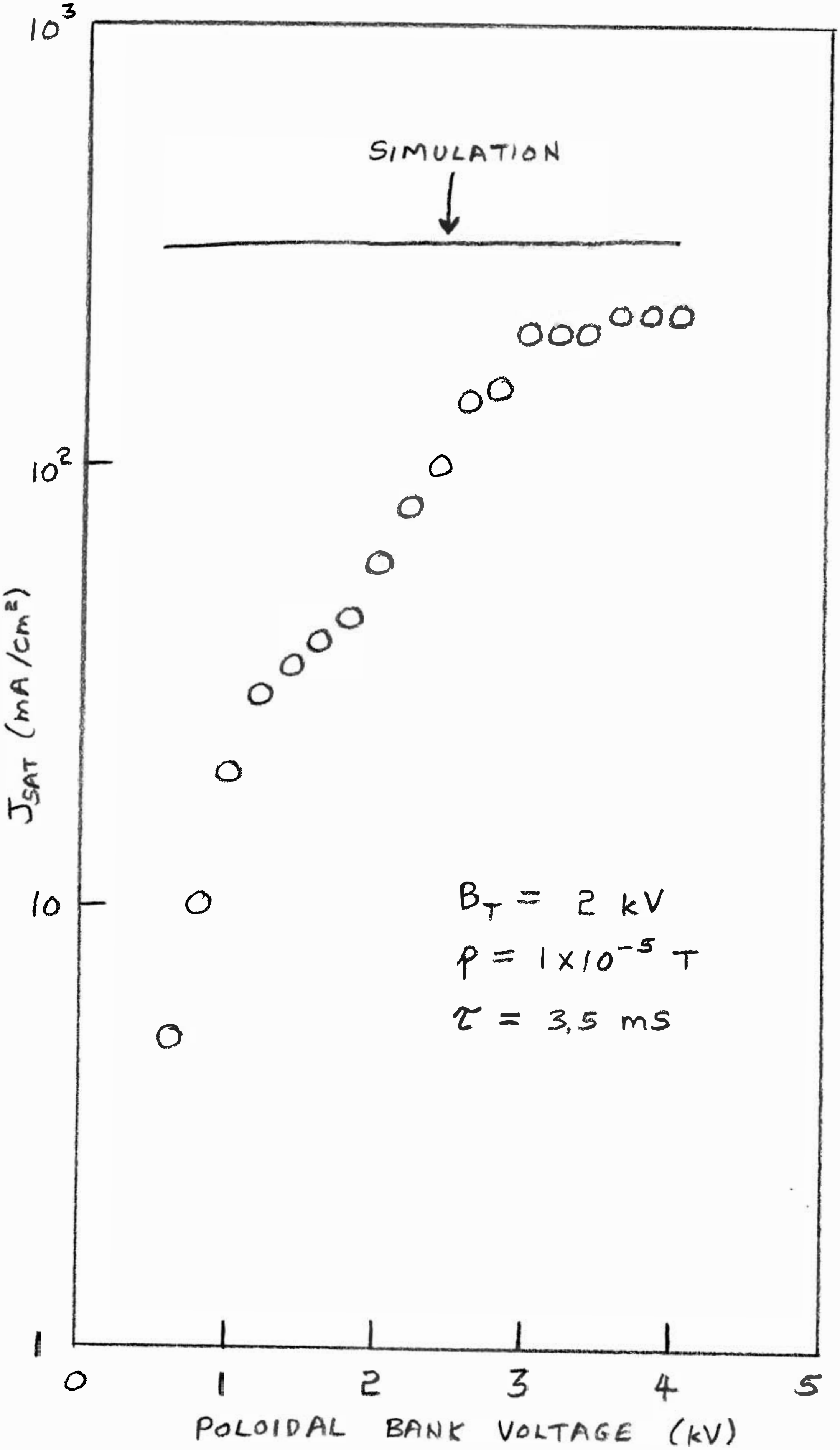
10^3
10^2
10
1
J_{SAT} (mA/cm²)
SIMULATION
EXPERIMENT
ECRH OFF
B_T ON
0
1
2
3
4
5
TIME (ms)

FIG 2



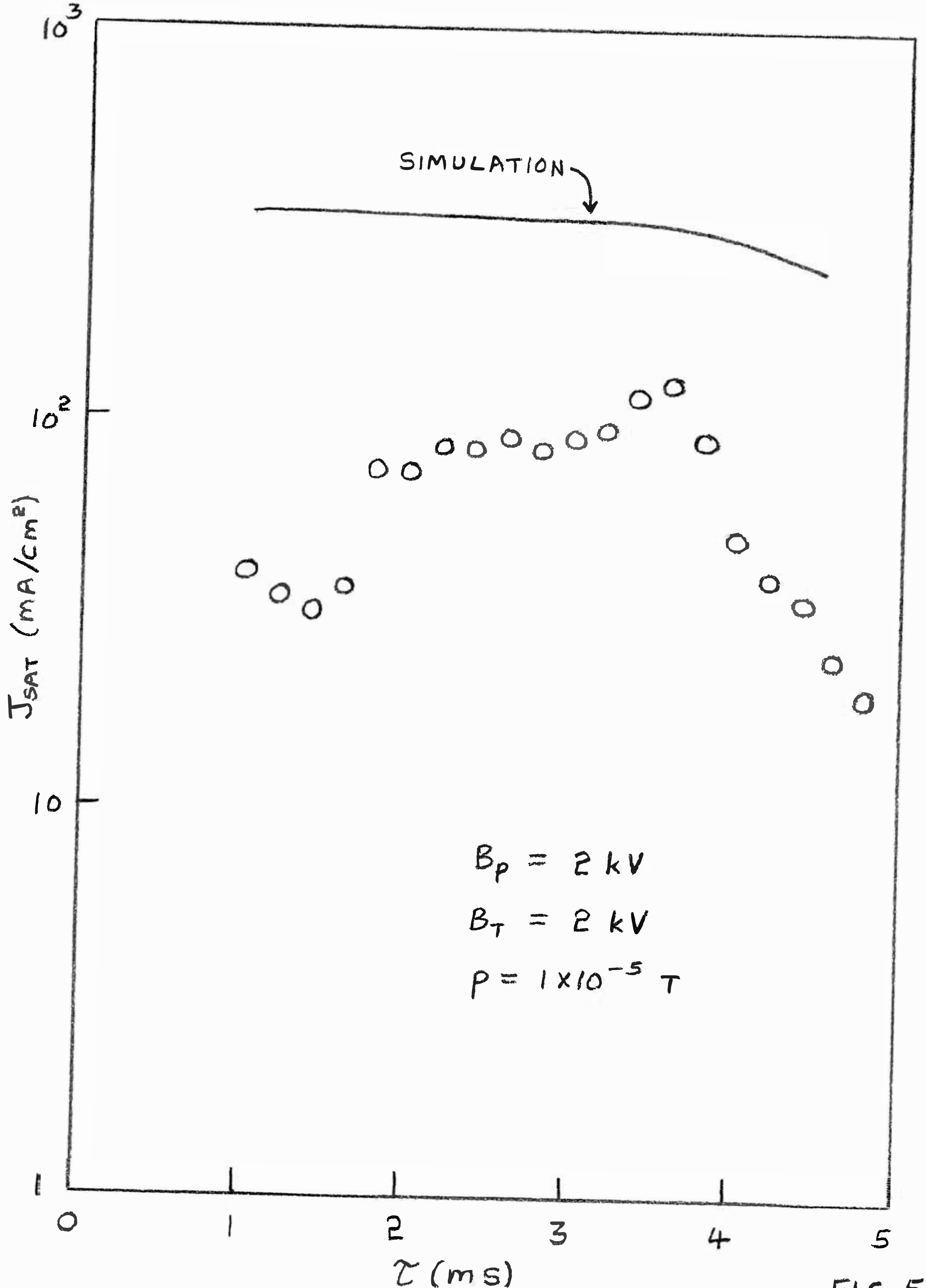
10^3
10^2
10
1
SIMULATION
$B_p =$
$p = 1 \times 10^{-5}$ T
$\tau = 3.5$ mS
0
1
2
3
4
TOROIDAL BANK VOLTAGE (kV)

FIG 3



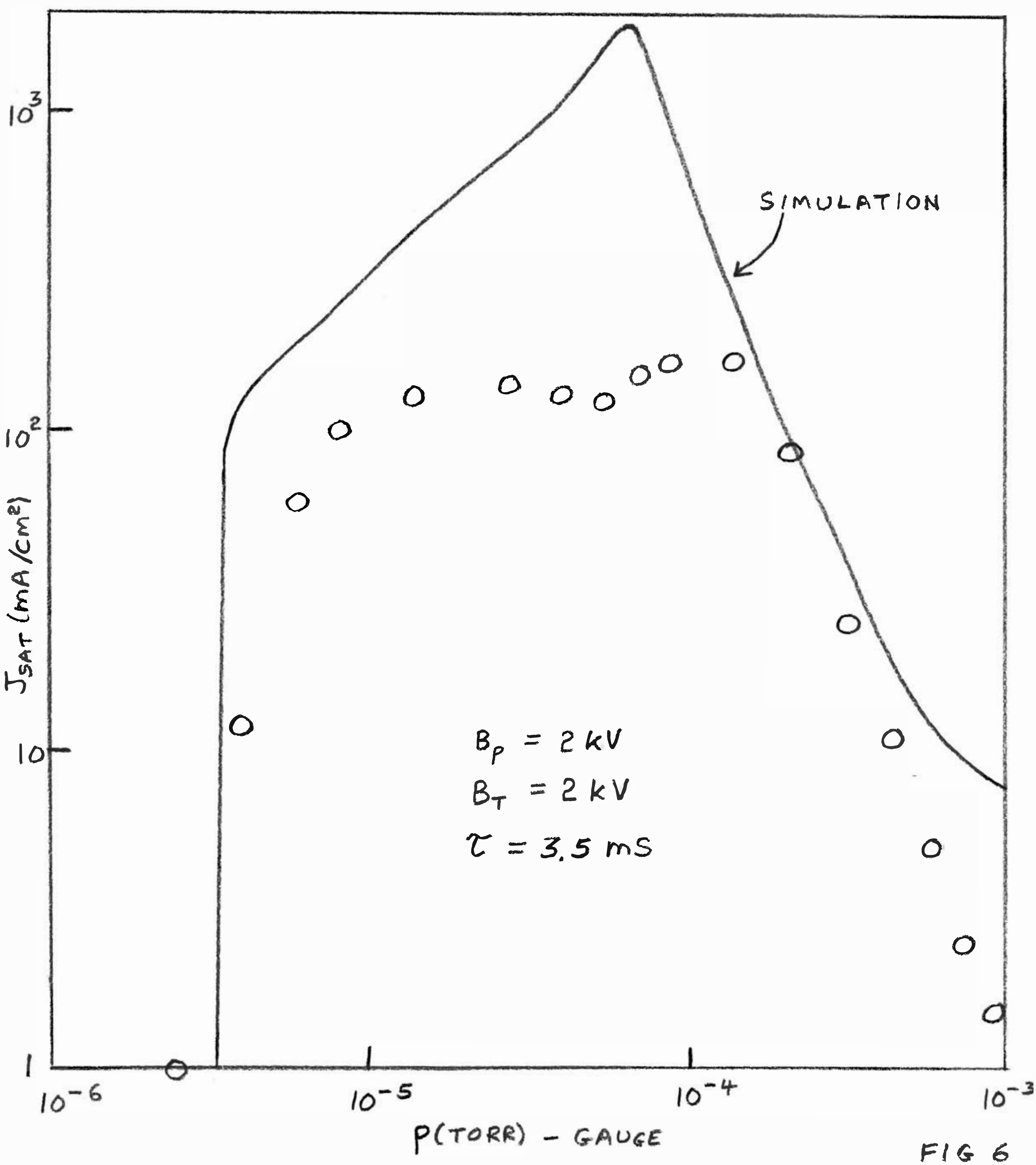
SIMULATION
$B_T = 2$ kV
$p = 1 \times 10^{-5}$ T
$\tau = 3{,}5$ ms
10^3
10^2
10
1
0
1
2
3
4
5
J_{SAT} (mA/cm^2)
POLOIDAL BANK VOLTAGE (kV)

FIG 4



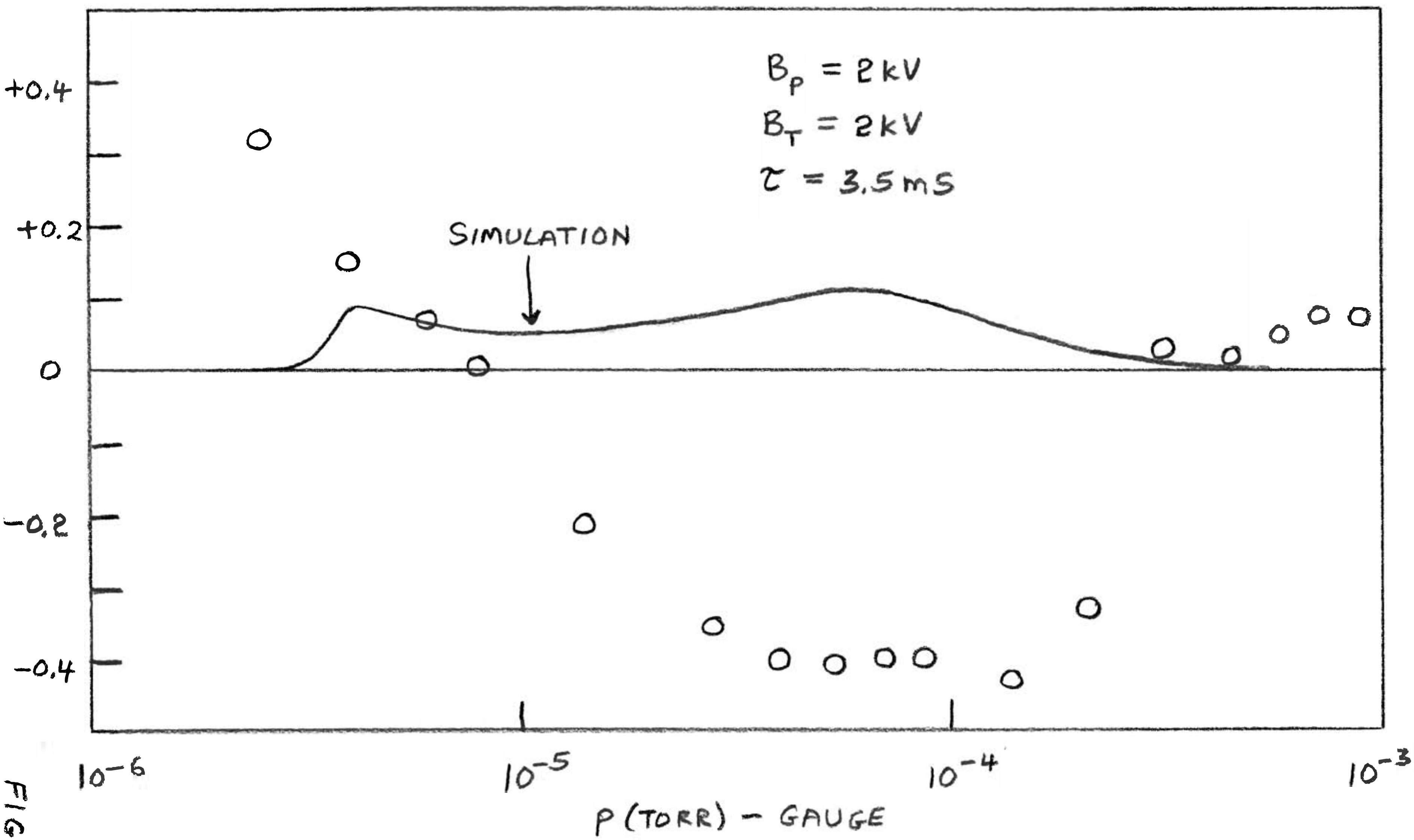
10^3
10^2
10
1
SIMULATION
J_{SAT} (mA/cm^2)
0
1
2
3
4
5
τ (ms)
B_P = 2 kV
B_T = 2 kV
$P = 1 \times 10^{-5}$ T

FIG 5



10^3
10^2
10
1
J_{SAT} (mA/cm^2)
SIMULATION
B_P = 2 kV
B_T = 2 kV
τ = 3.5 mS
10^{-6}
10^{-5}
10^{-4}
10^{-3}
P(TORR) - GAUGE

FIG 6



$B_P = 2\,kV$
$B_T = 2\,kV$
$\tau = 3.5\,mS$
SIMULATION
+0.4
+0.2
0
-0.2
-0.4
10^{-6}
10^{-5}
10^{-4}
10^{-3}
P (TORR) – GAUGE

FIG 7