

OTHER USES OF THE ICRH COUPLING COIL

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November 1975

PLP 663

Plasma Studies

University of Wisconsin

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It has been suggested that other non-sinusoidal waveforms could be fed across the terminals of the ICRH coupling coil in the small octupole to produce different types of plasma heating. This note examines several such possibilities.

An experimentally easy waveform to produce would be a voltage that rose abruptly to some value V_0 and then decayed exponentially with a long time constant, τ . Such a waveform could be produced by switching a charged capacitor across the coil terminals by means of an ignition. For a sufficiently large capacitor the circuit would be overdamped and would produce the desired waveform. Such a waveform would contain a fourier spectrum of frequencies from $\omega \sim 1/\tau$ up to infinity. Thus for τ large, both electron and ion cyclotron heating should occur throughout the volume of non-uniform field in the octupole.

In order to compare the volume-averaged heating rates for such a pulse with, for example, the ICRH rate using sine waves, we assume that the coupling coil imposes a maximum zero-to-peak voltage that is allowed across its terminals, and we ask how a waveform of $V = V_0 \sin \omega t$ compares with one that has $V = V_0 e^{-t/\tau}$ ($t > 0$). We further assume that τ is very large, so that the pulse is essentially a step function.

$$V = \begin{cases} 0 & t < 0 \\ V_0 & t \geq 0 \end{cases} .$$

For sine waves, it is well known that the ion (or electron) energy is given by

$$\bar{W} = \frac{GmE^2}{B_0^2} (\omega t) \quad (1)$$

where G is a geometric factor $\lesssim 0.1$ in the octupole, m is the particle (electron or ion) mass, $\overline{E^2}$ is the spatially averaged mean square electric field, B_0 is the field at cyclotron resonance, and ω is the rf frequency. Note that the energy rises linearly in time as expected for a stochastic process.

For the step function case, we calculate the heating by noting that a particle will accelerate for about one cyclotron radian before it begins to gyrate, and so the heating is strongly peaked near the zero field axis. Another way to say this is that particles near the zero field axis are non-adiabatic and hence are most effected by the fast rising electric field. The volume-averaged heating rate is determined from

$$\begin{aligned}\overline{W} &= \frac{1}{V} \int \frac{1}{2} m \left(\frac{eE}{m\omega_c} \right)^2 dV \\ &= \frac{e^2 \overline{E^2}}{2m} \int_{r_0}^a \frac{r dr}{\omega_c^2} \approx \frac{4mE^2 a^4}{B_0^2 r_0^4}.\end{aligned}$$

$$\text{But } t = \frac{1}{\omega_c} = \frac{ma^3}{eB_0 r_0^3} \Rightarrow \frac{a}{r_0} = (\omega t)^{1/3},$$

so

$$\overline{W} \approx \frac{4m\overline{E^2}}{B_0^2} (\omega t)^{4/3}. \quad (2)$$

Comparing this to Eq. (1) shows that the coefficient and the time dependence imply a greater heating for the step function than for the sine wave provided the E fields (i.e., coil voltage) are the same. It is only necessary that the step function remain on for several cyclotron radians ($\omega t > 1$). In this case, $\omega = eB_0/m$, where B_0 is the field near the wall in the octupole. Note also that since $\overline{W} \propto m^{-1/3}$, most (91%) of the energy goes into the electrons rather than the ions.

If a toroidal field were added to the octupole, we would expect a runaway condition with average energies given by

$$\bar{W} = \frac{1}{2} \left(\frac{eE}{M} t \right)^2$$

or

$$\bar{W} = \frac{m\bar{E}^2}{2B_0^2} (\omega t)^2 \quad (3)$$

This produces an even larger heating for $\omega t > 1$, but since $\bar{W} \propto m^{-1}$, nearly all (99.95%) of the energy goes into the electrons.

In order to test these predictions, a computer code (SPEMS2) was used to follow the trajectories of 100 non-interacting charged particles in an ideal linear octupole field with filamentary currents, and a spatially constant E field. The average energy of the 100 particles as a function of time was determined for the three cases discussed above, and the results are shown in the figure. The scaling of energy with time and the approximate magnitude of the energy agree well with the predictions (Eqn. 1, 2, and 3).

In the octupole, the E field is not spatially uniform and the field on axis about about 1/20 of the field at the surface of the coil (which is $V_0/2.7$ volts/meter), or about 185 volts/meter for 10 kV across the coil. This considerably exceeds the voltages normally used for toroidal ohmic heating experiments. The coupling coil can be represented approximately by an $0.7 \mu\text{H}$ inductance in series with a resistance of 0.06Ω . A 10 kV $60 \mu\text{F}$ capacitor could be used to dump into the coil and it would produce a slightly underdamped ($Q = 1.8$) wave with a $40 \mu\text{sec}$ period, and a peak current of about 63 kA. Using the impulse approximation and assuming that the coil weighs 10 kg it would only jump 0.01 cm off the floor as a result of image currents in the wall.

The approximate maximum energy gained by a particle is given by

$$W_{\text{MAX}} = \frac{1}{2} m \left(\int \frac{eE}{m} dt \right)^2 \approx 31.7 \text{ eV}$$

for a particle on axis. Note that since the maximum energy is $[\int E dt]^2 \propto LCV^2$, the particle energy is determined solely by the energy stored in the capacitor and is approximately 10 eV/kjoule. This rather small value, together with the fact that most of the energy goes into a few electrons on axis has led us to be rather pessimistic about the future of this type of experiment. To say it another way, in order to get 100% coupling to the plasma would require a density of $\sim 2 \times 10^{15} \text{ cm}^{-3}$ and at those densities there would be difficulty getting the field to penetrate the plasma. On the other hand it may happen that instabilities or parametric effects come into play at low densities and give rise to other interesting effects such as anomalous heating.

