INITIAL RESULTS FROM THE WISCONSIN TOKAPOLE

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I. Introduction

This paper describes a new plasma confinement configuration at Wisconsin and preliminary measurements of the plasmas contained therein. The device is essentially a Tokamak embedded in what used to be called the Small Wisconsin Toroidal Octupole, and hence the name, Tokapole (Tokamak surrounded by a multipole). The device is an outgrowth of the ohmic heating studies on the octupole which were begun by Lencioni[1] in 1967 and continued most recently by Etzweiler[2]. The present configuration was made possible by the installation of stronger hoop supports which allow the octupole field to be pulsed routinely at its maximum (5 kV) amplitude, by the development of a 1 msec, 10 kW, 9 GHz ECRH source for preionization, and most importantly, by the installation of a new 48 turn toroidal field coil in November 1976, which allows toroidal fields as high as 5 kG on axis with a half sine wave period of 4 msec and an L/R time of ~ 5 msec. All of this was done without sacrificing any of the capabilities of the device as a toroidal octupole.

The present configuration is similar to one at Gulf General Atomic[3] in which a toroidal field of 430 gauss was added to the DC Octupole and a toroidal current of 4 kA was produced. In that experiment densities of 3×10^{11} cm^{-3} and conductivity electron temperatures of 27 eV were obtained.

Figure 1 shows some of the flux surface configurations which can be produced in a toroidal octupole with ohmic heating. Fig 1(a) is the case with no plasma current. Unlike a quadrupole in which a critical plasma current is required to alter the flux surface topology[4,5], an octupole with a degenerate field null is topologically altered with an infinitessimal toroidal plasma current. Fig 1(b) shows the flux surfaces for a case in which the plasma current is opposite to the hoop current. Fig 1(d) is the case obtained in the

GGA octupole, and is identical to 1(c) except for a rotation of 45° about the minor axis. Cases (b), (c), and (d) have closed flux surfaces which do not encircle a hoop and so resemble a Tokamak with a four node poloidal divertor. Such configurations are being considered for impurity control in fusion reactors (as in the UWMAK design study), but have been investigated experimentally to only a limited extent. In addition to the GGA octupole, the Princeton FM-1[6] and the Japanese JFT-2A device[7] have tested some aspects of poloidal divertor action, and the Princeton PDX device presently under construction is dedicated to a thorough test of poloidal divertors.

Also of interest in predicting Tokapole behavior is the proliferation of small, low field, research oriented Tokamaks as indicated in the table below:

Device	R (cm)	a (cm)	B_T (kG)	I(kA @ q=3)
MIT Rector	57	17.5 x 70	4	40
MIT Versator I	54	13	5.5	30
MIT Versator II	40	15 x 15	9	85
UCLA Microtor	30	10 x 12.5	10	60
UCLA Macrotor	100	44 x 76	5	200
RPI Rentor	45	15	5	42
Cal Tech Tokamak	45.7	16.2	4	40
UC-Irvine Tokamak	60	17	5	40

For comparison, the Wisconsin Tokapole has a maximum toroidal field of 5 kG, major radius R = 43 cm and a minor radius determined by the distance from the minor axis to the surface of the hoops of a = 13 cm. For a safety factor at the edge of q = 3, a plasma current of 32 kA would be expected.

For equilibrium, a Tokamak requires a programmed vertical field (and sometimes a horizontal field as well). Because of the thick aluminum walls, the

Tokapole cannot have a fast time varying vertical field, unless the coils were put inside the vacuum system. On the other hand, the L/R time of the hoops and walls is much longer than the duration of the experiment and so image currents could be expected to help provide the equilibrium. However, the hoop current is generally opposite to the plasma current, and so there could be a problem with the equilibrium unless the plasma current becomes large enough to reverse the currents in the hoops.

II. Diagnostics

The most significant quantity in diagnosing the tokapole plasma is the total toroidal plasma current. This measurement is made difficult by the fact that the hoops carry a toroidal current that varies in time and that is often much larger than the plasma current. The quantities that can be easily measured are the poloidal gap voltage (V_{PG}) and the current in the 60 turn primary winding of the iron core ohmic heating transformer (I). These quantities can be used to infer a plasma current by considering the electrical circuit of figure 2(a) in which the ohmic heating transformer has been assumed to have a unity turns ratio. In the absence of plasma current ($I_p = 0$), the circuit consists of an inductance (the hoops) and a small series resistance driven by a capacitor bank (C) through an ignition switch. If the plasma current is assumed to flow on the octupole separatrix (see fig 1(a)), the plasma links 1/3 of the total flux (since there is twice as much common flux as private flux in the unperturbed octupole) and hence can be considered as a current source in parallel with 1/3 of the total system inductance. The plasma is mostly resistive with some inductance, but that should not matter in the calculation. Then the poloidal gap voltage can be written as

$$V_{PG} = IR + \frac{2}{3} L \frac{dI}{dt} + \frac{1}{3} L \frac{d}{dt} (I - I_p).$$

Integrating and solving for I_p gives

$$I_p = 3[I - \frac{1}{L} \int V_{PG}\, dt + \frac{R}{L} \int I\, dt]. \tag{1}$$

Figure 2(b) shows a circuit that electronically performs the required mathematical operation and yields at its output a signal proportional to the plasma current. The actual circuit used is somewhat more complicated because the resistance R is not quite constant in time (it results largely from skin currents flowing in the hoops, walls, and primary windings) and because the toroidal field coil links the transformer core and produces a signal proportional to the toroidal field which must be bucked out. In practice the circuit can be nulled to about 1% in the absence of plasma which is not quite adequate for reliable plasma current measurements. Furthermore, the null condition is dependent on the poloidal field amplitude and magnetic history of the iron core. Consequently, measurements are usually made by taking an experimental pulse without plasma (by turning off the pulsed gas source), and then taking a pulse with plasma and subtracting the two signals to get I_p (t) By this technique currents accurate to about +1 kA are obtained. The calibration can be checked by removing the signal which bucks the toroidal field coil current in which case this known current looks to the circuit like a plasma current, except larger by a factor of 3 because it links all of the core flux.

The plasma current produces a poloidal field that opposes the octupole field in the vicinity of the hoops (see fig 1(b)). If the toroidal plasma current is assumed to be contained within a circular surface of radius a centered on the toroid's minor axis, then a can be estimated by equating the field due to the plasma ($\mu_o I_p/2\pi a$) to the field due to the hoops (which varies as a^3 giving the result:

$$a = [10^5 \, I_p/I_H]^{1/4}, \tag{2}$$

where a is in cm and I_H is the total current in the four hoops. The actual shape of the current channel is probably not circular, but rather more like a square or diamond as shown in fig 1. There is good reason to believe that the plasma current is largely contained within radius a since at larger radii the flux lines encircle the hoop and so the conductivity should be strongly limited by mirrors[2] and obstacles (hoop supports and probes).

The Tokamak safety factor q can also be estimated if it is assumed that the toroidal plasma current density is constant over the circular cross section of radius a. Again, this is a reasonable assumption since the electric field and mirror ratio are nearly constant for $r < a$ and since the confinement is probably not adequate to produce strong gradients of density or temperature within the current channel. The resulting q is independent of r and is given by

$$q = \frac{5a^2 B_T}{I_p R}$$

where a is in cm, B_T is the toroidal field on axis in kG, I_p is the toroidal plasma current in kA, and R is the major radius (43 cm).

The electron temperature can be estimated by again assuming a constant current density, but over a square cross section with diagonal 2a, a voltage per turn equal to 1/3 of the poloidal gap voltage (as in fig 2(a)), and a resistivity given by the Spitzer formula[8] with Z = 1:

$$T_e = [25.4 \, I_p/a^2 \, V_{PG}]^{2/3}$$

where I_p is in amps, a is in cm, V_{PG} is in volts, and T_e is in eV. This temperature is often referred to as the "conductivity temperature" and is always lower than the real temperature by a factor $Z_{eff}^{2/3}$ which for most Tokamaks is in the range of 1-5 (or higher if very dirty).

Finally, this temperature can be used in conjunction with a Langmuir probe to estimate the plasma density. To avoid placing a probe directly in the current channel (which reduces the toroidal current by ~20%), the probe was placed in the upper outer "bridge" (the narrow region behind a hoop in the upper right hand corner of fig 1(b)). The probe has a special shape[9] which enables it to take a volume averaged density for the case without a plasma current channel. With the current channel, the density is characteristic of the divertor scrape off region only, but it may be that the particle lifetime in the divertor region (~1 msec) exceeds the time required for a particle to escape from the current channel and so the densities would not be very different. In any case the density is estimated using the conductivity temperature T_e and the ion saturation current density J_{SAT} as follows:

$$n = \frac{10^{10}\, J_{SAT}}{\sqrt{T_e}}$$

where J_{SAT} is in mA/cm^2, T_e is in eV and n is in cm^{-3}. Actually, the probe is operated as a floating double probe by returning the current not to the grounded tank wall, but rather to a second larger electrode in the bridge a short distance away, thereby reducing the probe's sensitivity to the floating potential changes which are typically 15-20 volts. Further corrections are made for the fact that the 45 volt probe bias may not be large compared with either kT_e/e or the voltage drop across the 10Ω resistor used to measure J_{SAT}. The complete formula with these corrections is

$$n = \frac{10^{10}\, J_{SAT}}{\sqrt{T_e}\,\{1 - \exp[(0.00265\, J_{SAT} - 45)/T_e]\}} . \qquad (5)$$

III. Typical Results

The Tokapole can be operated in either of two modes: 1) If the toroidal field is applied first and then the octupole is pulsed on near the peak of B_T, the plasma current flows in the same direction as the hoop current, and the configuration of fig 1(b) is obtained. 2) If the octupole field is turned on before the toroidal field, and the toroidal field reaches its peak while the octupole field is decaying, the plasma current flows opposite to the hoop current. A third mode in which the toroidal field is pulsed on near the peak of the octupole field results in large poloidal currents and a topology as in fig 1(a). Although all three cases have been studied, only case 1) will be discussed here because 1) it most closely resembles a Tokamak, 2) it produces the highest plasma current, and 3) it is the most extensively studied.

Figure 3 shows the time dependence of the magnetic fields for this case. Although the toroidal field has been operated at 5 kG peak on axis for a 4 msec half period, most of the experiments have been done with a 3 kG field and a 2 msec half period. This is because for fields above 3 kG, the 9 GHz ECRH preionization is ineffective and because the falling toroidal field apparently helps to heat the plasma and keep the conductivity and hence the toroidal current from decaying. Also shown is the ECRH preionization power and the poloidal gap voltage which is determined almost entirely by the hoops. Recall that the single turn voltage at the position of the plasma is $V_{PG}/3$. The Tokapole, unlike a Tokamak, is characterized by a fixed toroidal electric field, and the plasma current is determined by the plasma properties. In a Tokamak, the plasma current is fixed and the voltage varies with the plasma. Figure 3 also shows a typical case of the plasma current as measured by the method previously described. The current reaches a peak of ~ 25 kA at a time of ~ 800 μsec after the beginning of the ohmic heating pulse. At the time of peak current the plasma radius is a ~ 10 cm, the safety factor is q ~ 1, the conductivity electron temperature is ~ 25 eV and the average density is ~ 5 x 10^{12} cm^{-3}. Since this density is measured in the bridge region, the peak density on the axis of the current channel is probably ≳ 10^{13} cm^{-3}. The poloidal field produced by the plasma should be ~ 500 gauss at a radius of 10 cm.

The rapid determination of these quantities was made possible by a computer program developed by Alan Biddle in which analog signals representing I_p, V_{PG}, J_{SAT}, and B_T are digitized and used to calculate the quantities of interest during the minute or so between experimental pulses. A sample of the computer printout is shown in fig 4. Three experimental pulses are shown, the first case is under normal conditions, the second is with the toroidal field reduced to 1.5 kG and the third is with the poloidal gap voltage reduced to half its normal value. In addition to the values at the time of peak current, the program also calculates the conductivity temperature and electron density at 200 μsec intervals throughout the discharge. These quantities along with a and q are plotted in figure 5 as a function of time for the normal (B_T = 3 kG) discharge.

With the assistance of Rich Groebner, some attempt was made to estimate the actual electron temperature using the H_e singlet-triplet line ratio method[10]. The Tokapole was run with He gas at less than optimum conditions giving plasma currents of 5-10 kA and conductivity temperatures of ~ 15 eV. The measured electron temperature was typically 20-35 eV giving a Z_{eff} of ~2-3. It is not known whether the Z_{eff} is the same for the higher current hydrogen discharges, but the value obtained is certainly within reason.

IV. Flux Plots

With measurements of the toroidal plasma current, it is possible to calculate more precisely the flux surfaces which result. For this purpose a computer code (SIPLOT) was used. The code calculates flux surfaces for a linear octupole with 4 equal filimentary currents to represent the hoops and 12 filamentary image currents to represent the walls. The dimensions are scaled to closely approximate the toroidal octupole. Figure 6 shows one quadrant of an octupole without plasma current. The poloidal flux contours are at intervals of ~ 1/10 of the total flux (called Dories). Figure 7 shows a case with a filimentary current down the axis with a magnitude of 10% of the total current in the four hoops (corresponding very nearly to the 25 kA case discussed earlier). In figure 8 the plasma current is also 10% of the hoop current but in the opposite direction. Figure 9 is a more realistic case in which the current is 10% of the hoop current but distributed with a constant current density over a circular surface of radius a centered on the axis. Figure 10 is the same as figure 9 but with the current in the opposite direction. Figure 11 is the same as figure 9 but with a plasma current equal to 20% of the hoop current. Figure 12 is the same as figure 9 but redrawn with all four quadrants, and it probably represents the best estimate of the Tokapole flux plot.

Note that for the case of plasma current in the same direction as the hoop current (fig 12), there is very little magnetic flux between the center of the current channel and the surface of the hoops (<1 Dory), and so the confinement may not be very good, especially at high temperatures. The absolute containment zone (banana orbit size) of protons with energy greater than about 25 eV would extend from the minor axis to the surface of the hoops. A particle might still be confined for the time required to ∇B drift across the toroidal field, however. At 25 eV and B_T = 3 kG, this time is ~ 600 μsec.

V. Comparison with SIMULT

A zero dimensional computer code (SIMULT) has been used to successfully predict the results of a wide variety of experimental cases in the Wisconsin toroidal octupole. The program includes a subroutine (OHMIC) for ohmic heating but does not take into account the modification of the flux surfaces by the plasma current. Nevertheless, a blind application of the program (Jan. 5, 1977 version) resulted in a prediction not wildly out of agreement with experiment (at the time of peak current):

Quantity	Experiment	SIMULT
Toroidal Current (I_p)	25 kA	?
Density (n)	5×10^{12} cm^{-3}	2.5×10^{12} cm^{-3}
Electron Temp (T_e)	24 eV	4 eV
Ion Temp (T_i)	?	3 eV
Ion Sat Current (J_{SAT})	1000 mA/cm^2	500 mA/cm^2
Neutral Density (n_o)	?	2×10^{12} cm^{-3}

The calling program (MAIN) along with the printout of the results for SIMULT are included in the Appendix. A graph showing the time dependence of the simulation results is shown in figure 13. Figure 13 should be compared with the experimental results in figure 5. The results suggest an underestimation of the heating due to the toroidal current which is just what would be expected from neglecting the existence of the current channel. Future work will include refinements of SIMULT to more accurately represent the Tokapole cases.

VI. Experimental Scalings

With so many parameters available, there are numerous possible scalings that could be studied. Neutral H_2 gas is pulsed in a short burst to a pressure of about 10^{-4} torr about 16 msec before the fields are applied. The pumping speed is ~1000ℓ/sec, and the pressure does not decay appreciably during the experiment. At higher pressures the electron temperature drops, and at lower pressures the density drops, both of which decrease the plasma current.

All of the measurements described here were done with constant and near optimal gas pressure. At moderately low pressures, occasional pulses exhibit runaway behavior as evidenced by a large plasma current ($\gtrsim$ 20 kA) which peaks early in time (~ 200 μsec) with relatively small densities ($\sim 10^{10}$ - 10^{11} cm^{-3}). This regime is diffucult to reproduce, and consequently, the existence of high energy electrons (as evidenced by x-rays or synchrotron radiation) has not been verified. Figure 14 shows the peak plasma current as a function of pressure (actually puff valve dial setting with 600 torr manifold pressure) with and without the 9 GHz ECRH preionization. The effect of the preionization is to enable the plasma to be formed at lower pressures.

The timing of the onset of the ohmic heating and the ECRH preionization can be varied and the plasma current is found to be optimal for the case shown in fig 3, although the timing is not very critical. All of the cases discussed are with this timing. The plasma current always peaks about 700-800 μsec after the beginning of the ohmic heating pulse for all cases considered.

Of more interest is the scaling of the plasma current and other parameters with toroidal field strength and ohmic heating voltage. Figure 15(a) shows the variation of the peak plasma current with ohmic heating voltage for B_T = 3 kG, measured in units of the voltage to which the poloidal field bank is charged. (5 kV on the bank gives about 20 volts/turn at the plasma at the time of peak current.) Figure 15(b) shows the variation of the peak current with toroidal field for B_p = 5 kV, again measured in units of the voltage to which the toroidal field bank is charged. (5 kV on the bank gives about 3 kG on axis at the time of peak field.) The two curves are remarkably similar and each give a quadratic dependence of plasma current on the respective field. Assuming $I_p \propto B_p^2 B_T^2$, the scaling of other parameters can be inferred from equations (2) - (4):

$$a \propto B_p^{1/4}\, B_T^{1/2} \propto I_p^{1/8}$$

$$q \propto B_p^{-3/2}$$

$$T_e \propto B_p^{1/3}\, B_T^{2/3} \propto I_p^{1/6} .$$

Note that a and T_e depend only on plasma current and that the dependence is very weak in both cases. In figure 16 a and T_e are plotted as a function of I_p for many different combinations of B_T and B_p, and the results are consistent with the above scalings. Note also that q is independent of B_T and depends only on the ohmic heating voltage. Coincidentally, at the maximum available ohmic heating voltage (B_p = 5 kV) q is almost exactly equal to unity. In figure 17, q is plotted as a function of B_p for B_T = 5 kV (3 kG) and as a function of B_T for B_p = 5 kV, and the results are consistent with the above scalings.

The other interesting scaling is the density as a function of B_p and B_T which is shown in figure 18. Although the scaling is more complicated than a simple power law, the data can be reasonably well fit with

$$n_e \propto B_p^{8/3}\, B_T^{4/3} .$$

Actually this scaling is a result of some hindsight because it happens to be just what is required to give a constant energy confinement time defined by

$$\tau_E = \frac{n_e T_e V}{I_p V_{PG}/3} ,$$

where V is taken as the total volume of the toroid (not the volume of the current channel) which is 3×10^5 cm^3. From the fit to the data, the energy confinement time is τ_E = 8 μsec. A statistical analysis of the 33 data points in figure 18 gives $\tau_E = 11.4 \pm 6$ μsec. This time is about equal to the transit time of a few eV neutral across the plasma. Note that τ_E is much shorter than the 800 μsec required for the current to build up, and so the plasma is apparently in a quasi-steady state. In fact, the decay of the current after the peak in figure 3 can be accounted for entirely by the decay in B_T and V_{PG} ($I_p \propto B_T^2 V^2_{PG}$), and so if the toroidal field and poloidal gap voltage were left on longer, I_p would be nearly constant. However, the ohmic heating transformer is being operated near its volt-second limit (but without cocking), and so lengthening the pulse by a large factor does not appear feasible.

The ohmic heating power absorbed by the plasma is given by

$$P_{OH} = I_p V_{PG}/3 \propto B_p^3 B_T^2,$$

and is about 500 kW with 5 kV on both capacitor banks. Since τ_E = const, the energy stored in the plasma has the same scaling and is ~ 5 joules for 5 kV on both banks. Note that since the toroidal field dominates the poloidal field over most of the volume, the plasma β defined by

$$\beta = \frac{2\mu_o n\, kT_e}{B_T^2}$$

is independent of B_T, varies as B_p^3, and has a maximum value of ~ 0.01%. The "poloidal beta" is larger, but still only ~ 1%.

For the typical Tokapole parameters, the mean free path of a thermal H_2 neutral is $\lesssim$ 1 cm and so the interior of the plasma is probably burned out of thermal neutrals, and is replenished by ~ 7 - 8 eV Franck Condon neutrals. This conclusion is further supported by the fact that the H_β light peaks early in the discharge and is falling sharply at the time of peak current.

A lower limit on the ion temperature can be obtained by assuming that the ions are heated only by classical collisions with the electrons and that the ion lifetime is equal to the energy confinement time. Such a calculation gives $T_i > 0.5$ eV.

VII. Summary and Future Plans

The typical operating parameters and scaling of the Tokapole are summarized in figure 19 for a toroidal field of 3 kG on axis and the maximum available ohmic heating voltage of 20 volts/turn.

The most significant question raised by the results is why the energy confinement time is as short as 10 μsec (and constant), and the closely related question of why the conductivity electron temperature is only 25 eV. The most likely explanation is the excitation of impurity atoms released from the walls by plasma bombardment. The energy confinement time and electron temperature do not appear to be strongly influenced by the base pressure, however. Most of the experiments were done with base pressures of $\sim 2 \times 10^{-7}$ torr, but the results at 2×10^{-5} torr (a few hours after being up to air) are not significantly different. It will be important to measure the energy loss by impurity radiation and to attempt more aggressive discharge cleaning techniques. If the losses cannot be accounted for by radiation, then there might be a problem with the equilibrium or stability of the Tokapole configuration, and this will be studied both theoretically and experimentally. Because of the relatively low temperatures, the brief duration of the discharge, the short electron mean free path, and the experimental fact that a 1/4" diameter probe extended all the way across the midplane depresses the current by only 20%, there is hope that probes can be used to measure the spatial and temporal evolution of all quantities of interest. The first priority item is to get an independent measurement of the toroidal plasma current using local magnetic probes.

Of course we have every intention of operating the Tokapole with a 5 kG toroidal field, and a KU-band microwave system (~ 10 kW for 1 msec at 15.5-17.5 GHz) has recently been installed on the machine for ECRH preionization. If the scaling laws continue to extrapolate, we would expect to achieve $I_p \simeq 70$ kA, $T_{ec} \simeq 30$ eV and $n \simeq 8 \times 10^{12}$ cm^{-3} with $P_{OH} \cong 1.4$ MW. Later it might be possible to go to even higher toroidal fields with the acquisition of mroe capacitors (72 kG at present) provided there is interest in doing so.

The Tokapole configuration also offers the possibility of various types of rf heating. The first priority in such studies will be fast wave ion cyclotron resonance heating. The density and radius are such that toroidal eigenmodes ought to exist and apparatus is presently under construction which should be capable of supplying ~ 1 MW of rf power at either the fundamental or second harmonic of the ion cyclotron resonance frequency.

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APPENDIX

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*   *   *   *   *   *  UNIVAC 1110 TIME/SHARING EXEC --- MULTI-PROCESSOR SYSTEM
RUNID:C00705    PROJECT:02980          USER:4126810219           FILE:PR*000CC


@RUN  SPROTT,2980,4126810219,1M
@ASG,T TEMP.
@ASG,TH 20.,T,U5709
@COPY,G 20.,TEMP.
FURPUR-MACC 4.00 RLIB82 01/17-10:19:11
4126810219*SIMULT(0) COPIED ON 01/05/77 AT 11:45:20
     7 BLOCKS COPIED.
EOF ENCOUNTERED ON INPUT TAPE
@FREE 20.
@FOR,IS TEMP.MAIN,.MAIN
FORTRAN-MACC 1.17S-01/17/77-10:19:52 (,0)         MAIN
                      DIMENSION IOPT(5),P(21),VAL(27,101),VALM(7,2)
                      IOPT(1)=5
                      IOPT(2)=1
                      IOPT(3)=0
                      IOPT(4)=1
                      IOPT(5)=0
                      P(1)=2.2
                      P(2)=10.0
                      P(3)=90.0
                      P(4)=0.0
                      P(5)=2.45
                      P(6)=2000.0
                      P(7)=1000.0
                      P(8)=0.005
                      P(9)=0.0035
                      P(10)=50.0
                      P(11)=3.0
                      P(12)=-0.0005
                      P(13)=2.0
                      P(14)=0.005
                      P(15)=0.006
                      P(16)=40.0
                      P(17)=-0.0005
                      P(18)=0.0005
                      P(19)=0.0
                      P(20)=1.0E4
                      P(21)=3000.0
                      CALL SIMULT(IOPT,P,VAL,VALM)
                      END

      END OF COMPILATION:          NO  DIAGNOSTICS.
@MAP,IXN
MAP27-4 RLIB64 01/17-10:19:54
END MAP
@XQT
```

AUXILIARY HEATING: OHMIC ECRH

STEP	TIME (SEC)	DENSITY (10**9/CC)	TE (EV)	TI (EV)	DNEUTRAL (10**9/CC)
0	-.0005	.0010	.0250	.0250	3348.8000
10	-.0004	.0004	.1918	.0460	3348.8336
20	-.0004	.0004	1.0473	.0424	3348.8336
30	-.0003	.0033	110.2035	.0536	3348.7965
40	-.0003	.3504	22.3170	.0490	3346.8360
50	-.0002	1.1782	6.5923	.0323	3347.4023
60	-.0002	1.7569	5.8965	.0299	3347.0599
70	-.0001	2.0514	7.9035	.0300	3345.8629
80	-.0001	98.4067	10.2187	.0439	3136.4561
90	-.0000	125.6600	3.5893	.1297	3248.7475
100	.0000	337.3491	7.5005	.1373	2930.0737
110	.0001	671.6818	5.3285	.3166	2837.8374
120	.0002	1005.7027	4.9349	.6134	2668.1297
130	.0002	1251.2404	5.0154	1.0232	2495.5697
140	.0003	1448.0250	4.7776	1.4227	2400.0392
150	.0003	1609.4514	4.5254	1.7694	2333.8236
160	.0004	1741.0221	4.3315	2.0501	2279.5861
170	.0004	1849.2392	4.1859	2.2642	2233.7000
180	.0005	1940.1537	4.0828	2.4207	2193.0843
190	.0005	2019.5529	4.0215	2.5330	2154.5061
200	.0006	2093.2518	4.0040	2.6154	2114.4608
210	.0007	2167.4103	4.0335	2.6815	2068.9853
220	.0007	2248.9177	4.1129	2.7422	2013.3569
230	.0008	2345.8943	4.2457	2.8063	1941.6403
240	.0008	2468.3621	4.4368	2.8808	1846.0002
250	.0009	2629.1332	4.6969	2.9725	1715.6385
260	.0009	2844.9088	5.0509	3.0907	1534.9980
270	.0010	3137.0609	5.5619	3.2517	1280.7378
280	.0010	3527.8648	6.4127	3.4899	918.3302
290	.0011	4000.0239	8.2554	3.8944	432.7729
300	.0011	4323.4209	12.7439	4.6629	91.6910
310	.0012	4379.7162	18.6440	5.6486	40.7524
320	.0013	4392.4191	23.4149	6.4935	35.8609
330	.0013	4419.3294	26.7601	7.1557	35.6024
340	.0014	4462.7778	28.7189	7.6694	36.5186
350	.0014	4525.6330	29.3362	8.0734	38.0262
360	.0015	4605.2577	28.7050	8.4026	39.9939
370	.0015	4698.4272	26.9971	8.6862	42.4711
380	.0016	4799.9987	24.4549	8.9439	45.6887
390	.0016	4903.1766	21.3545	9.1801	50.2817
400	.0017	5002.5295	17.9480	9.3699	58.5100
410	.0018	5108.2083	14.3374	9.4197	82.1280
420	.0018	5306.7246	9.9917	8.9962	231.5243
430	.0019	5665.8087	4.2857	6.7466	1398.4117
440	.0019	5619.1786	2.9356	3.8512	1682.3129
450	.0020	5496.0841	2.2159	2.4799	1786.8306
460	.0020	5375.1663	1.6513	1.7636	1850.2384
470	.0021	5273.0513	.9000	1.0662	1890.5209
480	.0021	5188.5430	1.2985	1.0418	1916.8055
490	.0022	5106.6673	1.2077	1.0037	1940.9919
500	.0022	5027.3833	.6941	.7021	1963.9882
510	.0023	4989.6240	.4892	.5711	1980.0527
520	.0024	2319.5853	.0686	.1046	1998.4298

SIMULT JAN 5,1977 VERSION

FIELD (KGAUSS)	POWER (WATTS)	JSAT (MA/SQCM)	DE	TE	TI
.0000	50.0000	.0000	RD	BR	NC
.0000	50.0000	.0000	OL	TC	NC
.0000	50.0000	.0000	OL	TC	NC
.0000	50.0000	.0012	RD	EX	TC
.0000	50.0000	.1492	OL	EX	NC
.0000	50.0000	.3143	OL	EX	NC
.0000	50.0000	.4435	OL	EX	NC
.0000	50.0000	.5978	OL	EX	NC
.0000	50.0000	32.3155	OL	EX	NC
.0000	50.0000	24.7592	OL	EX	NC
.0872	50.0000	95.8471	OL	EX	NC
.1820	50.0000	161.2158	OL	EX	NC
.2755	50.0000	232.3246	OL	EX	NC
.3677	50.0000	291.3884	OL	EX	NC
.4584	50.0000	329.1376	OL	EX	NC
.5476	50.0000	356.0545	OL	EX	NC
.6351	50.0000	376.8296	OL	EX	NC
.7209	50.0000	393.4714	OL	EX	NC
.8049	50.0000	407.7013	OL	EX	NC
.8870	50.0000	421.1883	OL	EX	NC
.9671	50.0000	435.6093	OL	EX	NC
1.0451	50.0000	452.6961	OL	EX	NC
1.1210	50.0000	474.3231	OL	EX	NC
1.1948	50.0000	502.6955	OL	EX	NC
1.2662	50.0000	540.7045	OL	EX	NC
1.3354	50.0000	592.5429	OL	EX	NC
1.4021	50.0000	664.8563	OL	EX	NC
1.4664	50.0000	769.1924	OL	EX	NC
1.5283	50.0000	928.2709	OL	EX	NC
1.5876	50.0000	1190.1352	OL	EX	TC
1.6443	50.0000	1558.1500	OL	EX	TC
1.6984	50.0000	1790.7464	OL	EX	TC
1.7498	50.0000	1886.9935	OL	TC	TC
1.7985	50.0000	1934.7281	OL	TC	TC
1.8446	50.0000	1968.2032	OL	TC	TC
1.8878	50.0000	1999.4274	OL	TC	TC
1.9283	50.0000	2030.8553	OL	TC	TC
1.9660	50.0000	2059.4442	OL	TC	TC
2.0009	50.0000	2076.6234	OL	EX	TC
2.0330	50.0000	2069.9695	OL	EX	TC
2.0622	50.0000	2024.4819	OL	EX	TC
2.0887	50.0000	1924.3943	OL	EX	TC
2.1122	50.0000	1725.2240	OL	EX	TC
2.1330	50.0000	1530.4729	OL	EX	NC
2.1509	50.0000	1146.6445	OL	EX	NC
2.1661	50.0000	900.1257	OL	EX	NC
2.1784	50.0000	742.3838	OL	EX	NC
2.1879	50.0000	566.2542	OL	TC	NC
2.1947	50.0000	614.9009	OL	IC	NC
2.1987	50.0000	583.6471	OL	IC	NC
2.2000	50.0000	438.1066	OL	TC	NC
2.1986	50.0000	392.1489	OL	TC	NC
2.1945	50.0000	78.0101	RE	TC	TC

530	.0024	62.8221	.0117	.0061	3714.6773
540	.0025	59.1992	.3697	.0907	3763.3901
550	.0025	58.5715	.3777	.1545	3763.3748
560	.0026	57.9503	.2429	.1499	3763.3621
570	.0026	56.9337	.1538	.1166	3763.3412
580	.0027	55.6560	.0922	.1037	3763.3174
590	.0027	54.4317	.0744	.1018	3763.2921
600	.0028	51.4142	.1674	.0863	3763.2629
610	.0029	47.7696	.0968	.0746	3763.2320
620	.0029	43.5413	.0580	.0683	3763.1988
630	.0030	30.1441	.0343	.0558	3763.1675
640	.0030	9.9865	.0203	.0598	3763.1348
650	.0031	4.9439	.3658	.0515	3763.0802
660	.0031	4.9092	3.3259	.0523	3762.7825
670	.0032	5.0054	3.4171	.0513	3762.5514
680	.0032	5.1245	3.4790	.0505	3762.3100
690	.0033	5.2706	3.5461	.0499	3762.0450
700	.0033	5.4487	3.6136	.0494	3761.7538
710	.0034	5.6628	3.6766	.0491	3761.4351
720	.0035	5.9139	3.7296	.0489	3761.0898
730	.0035	6.1998	3.7681	.0490	3760.7219
740	.0036	6.5137	3.7885	.0492	3760.3376
750	.0036	6.8444	3.7887	.0497	3759.9459
760	.0037	7.1770	3.7689	.0503	3759.5569
770	.0037	7.4943	3.7308	.0512	3759.1805
780	.0038	7.7936	3.7312	.0523	3758.7823
790	.0038	8.0717	3.6645	.0534	3758.4317
800	.0039	8.2957	3.5949	.0547	3758.1058
810	.0040	8.4604	3.5263	.0561	3757.8047
820	.0040	8.5657	3.4624	.0575	3757.5262
830	.0041	8.6142	3.4049	.0589	3757.2678
840	.0041	8.3885	2.6642	.0614	3757.3977
850	.0042	8.0710	2.5997	.0637	3757.3177
860	.0042	7.7370	2.5560	.0653	3757.2410
870	.0043	7.3911	2.5299	.0664	3757.1663
880	.0043	7.0361	2.5220	.0669	3757.0923
890	.0044	6.6741	2.5314	.0671	3757.0182
900	.0044	6.4095	3.5061	.0659	3756.6292
910	.0045	6.3333	3.5459	.0633	3756.3716
920	.0046	6.2386	3.5830	.0613	3756.1094
930	.0046	6.1216	3.6320	.0597	3755.8363
940	.0047	5.9767	3.6929	.0585	3755.5512
950	.0047	5.7949	3.7699	.0575	3755.2510
960	.0048	5.5610	3.8720	.0567	3754.9301
970	.0048	5.2487	4.0188	.0561	3754.5771
980	.0049	4.8059	4.2611	.0557	3754.1626
990	.0049	4.0974	4.7869	.0561	3753.5798
1000	.0050	2.3549	8.9672	.0616	3750.8484
MAXIMUM	VALUES	5674.0720	164.8421	9.4307	35.4972

FRACTIONAL CHANGE IN NEUTRAL PRESSURE = 12.0057 %

MAXIMUM RATIO OF NE / NO = 124.1873

@FIN

RUNID: C00705 PROJECT: 02980 USER: 4126810219

(5)

2.1878	50.0000	.7074	RE IC NC
2.1785	50.0000	3.7432	OL IC NC
2.1667	50.0000	3.7436	OL IC NC
2.1524	50.0000	2.9703	OL IC NC
2.1356	50.0000	2.3222	OL IC NC
2.1163	50.0000	1.8642	OL TC NC
2.0948	50.0000	1.8062	FD TC NC
2.0709	50.0000	2.1878	FD IC NC
2.0447	50.0000	1.5455	FD IC NC
2.0163	50.0000	1.1833	FD TC NC
1.9858	50.0000	.7408	FD TC NC
1.9532	50.0000	.2541	FD SR NC
1.9186	50.0000	.3110	FD IC NC
1.8820	50.0000	.9311	OL EX NC
1.8436	50.0000	.9623	OL EX NC
1.8033	50.0000	.9941	OL EX NC
1.7612	50.0000	1.0322	OL EX NC
1.7175	50.0000	1.0772	OL EX NC
1.6722	50.0000	1.1292	OL EX NC
1.6253	50.0000	1.1878	OL EX NC
1.5769	50.0000	1.2516	OL EX NC
1.5272	50.0000	1.3185	OL EX NC
1.4761	50.0000	1.3855	OL EX NC
1.4238	50.0000	1.4490	OL EX NC
1.3704	50.0000	1.5055	OL EX NC
1.3159	50.0000	1.5657	OL EX NC
1.2603	50.0000	1.6069	FD EX NC
1.2038	50.0000	1.6358	FD EX NC
1.1465	50.0000	1.6523	FD EX NC
1.0884	50.0000	1.6576	FD EX NC
1.0297	50.0000	1.6531	FD EX NC
.9703	50.0000	1.4240	FD EX NC
.9103	50.0000	1.3534	FD EX NC
.8500	50.0000	1.2864	FD EX NC
.7892	50.0000	1.2226	FD EX NC
.7282	50.0000	1.1621	FD EX NC
.6669	50.0000	1.1044	FD EX NC
.6055	50.0000	1.2481	FD EX NC
.5440	50.0000	1.2403	FD EX NC
.4826	50.0000	1.2281	FD EX NC
.4212	50.0000	1.2133	FD EX NC
.3600	50.0000	1.1945	FD EX NC
.2990	50.0000	1.1702	FD EX NC
.2383	50.0000	1.1380	FD EX NC
.1780	50.0000	1.0943	FD EX NC
.1181	50.0000	1.0317	FD EX TC
.0588	50.0000	.9323	FD EX TC
.0000	50.0000	.7285	FD EX TC
2.2000	50.0000	2077.7886	

10:19:10 LOAD U5709 9/7 20 -1 C00705

ITEM	AMOUNT	COST(DOLLARS)
CPU TIME	00:00:10.887	$0.41
FILE I/O REQUESTS	426	$0.19
FILE I/O WORDS	309254	$0.14
TAPE I/O REQUESTS	13	$0.00
TAPE I/O WORDS	12606	$0.02
SWAP REQUESTS	4	$0.00
SWAP WORDS	9216	$0.00
MEMORY USAGE	0.245	$0.14
CARDS IN	39	$0.05
PAGES PRINTED	3	$0.12
TAPE MOUNTS	1	$0.50
ER + CC	13	$0.12
JOB CHARGE	1	$0.20
TOTAL COST		$1.89

THE ABOVE DOLLAR AMOUNTS ARE APPROXIMATE AND ARE BASED ON RATES FOR ST

USER BALANCE $23.14

INITIATION TIME: 10:19:09 JAN 17,1977

TERMINATION TIME: 10:20:29 JAN 17,1977

PREVIOUS RUN TIME: 11:18:47 JAN 15,1977

EOF 4 PAGES COST $0.14 USER BALANCE $23.12 C0070

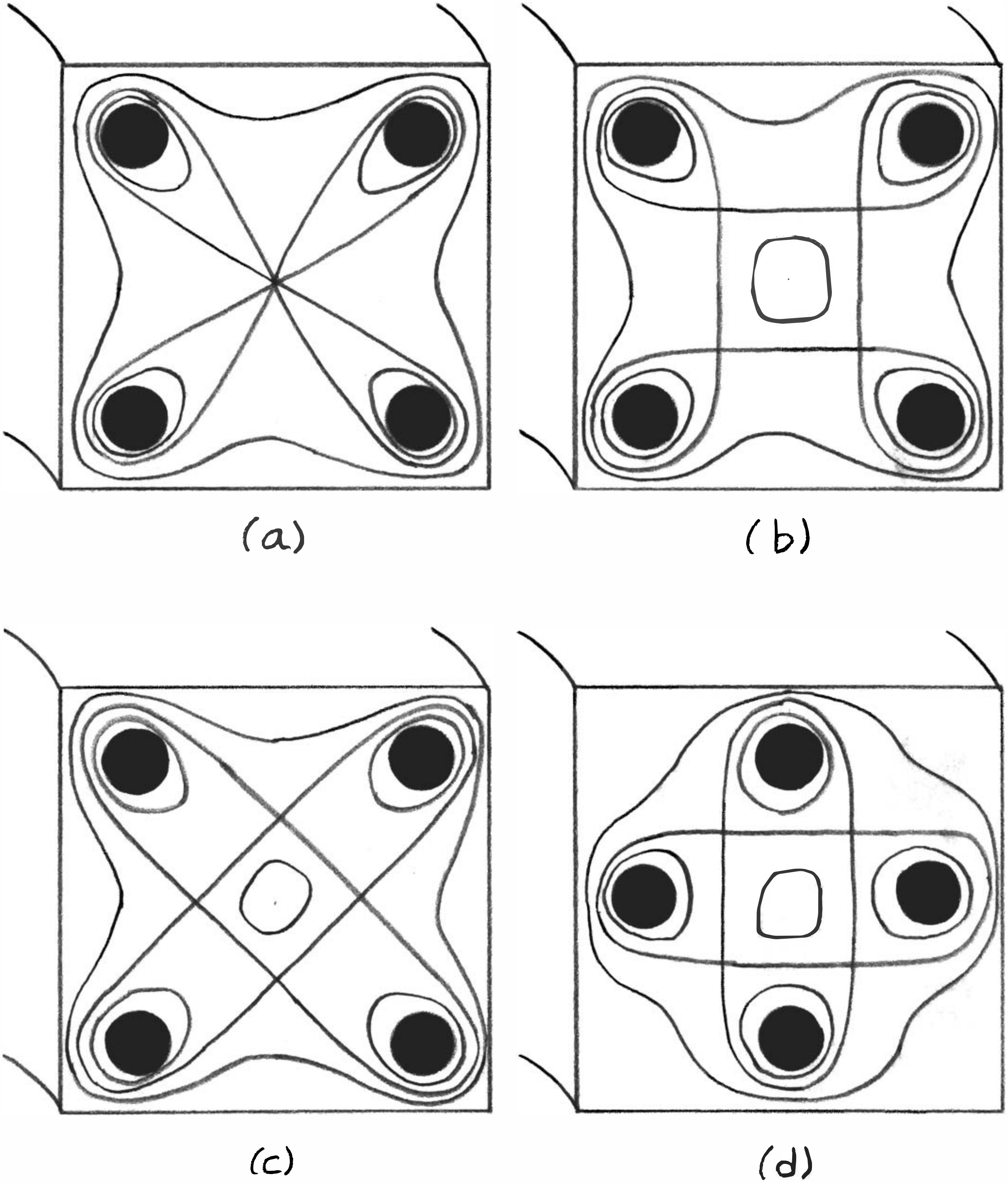
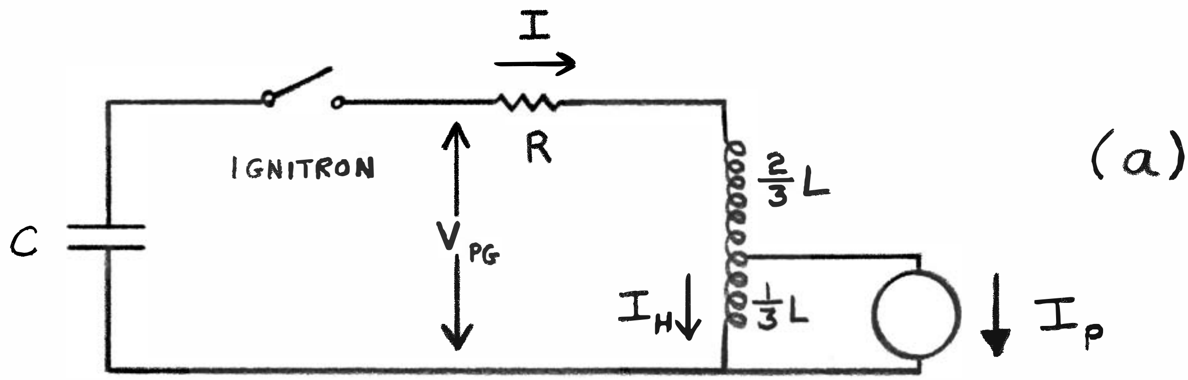


Figure 1



$$I_p = 3\left[I - \frac{1}{L}\int V_{PG}\,dt + \frac{R}{L}\int I\,dt \right]$$

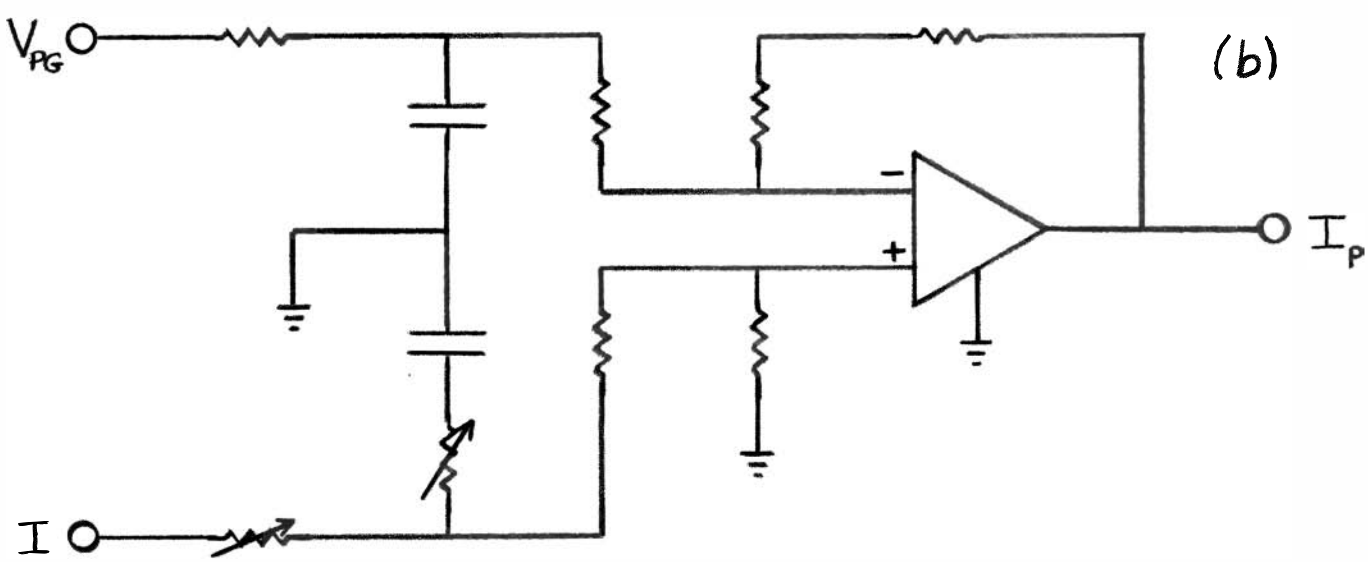
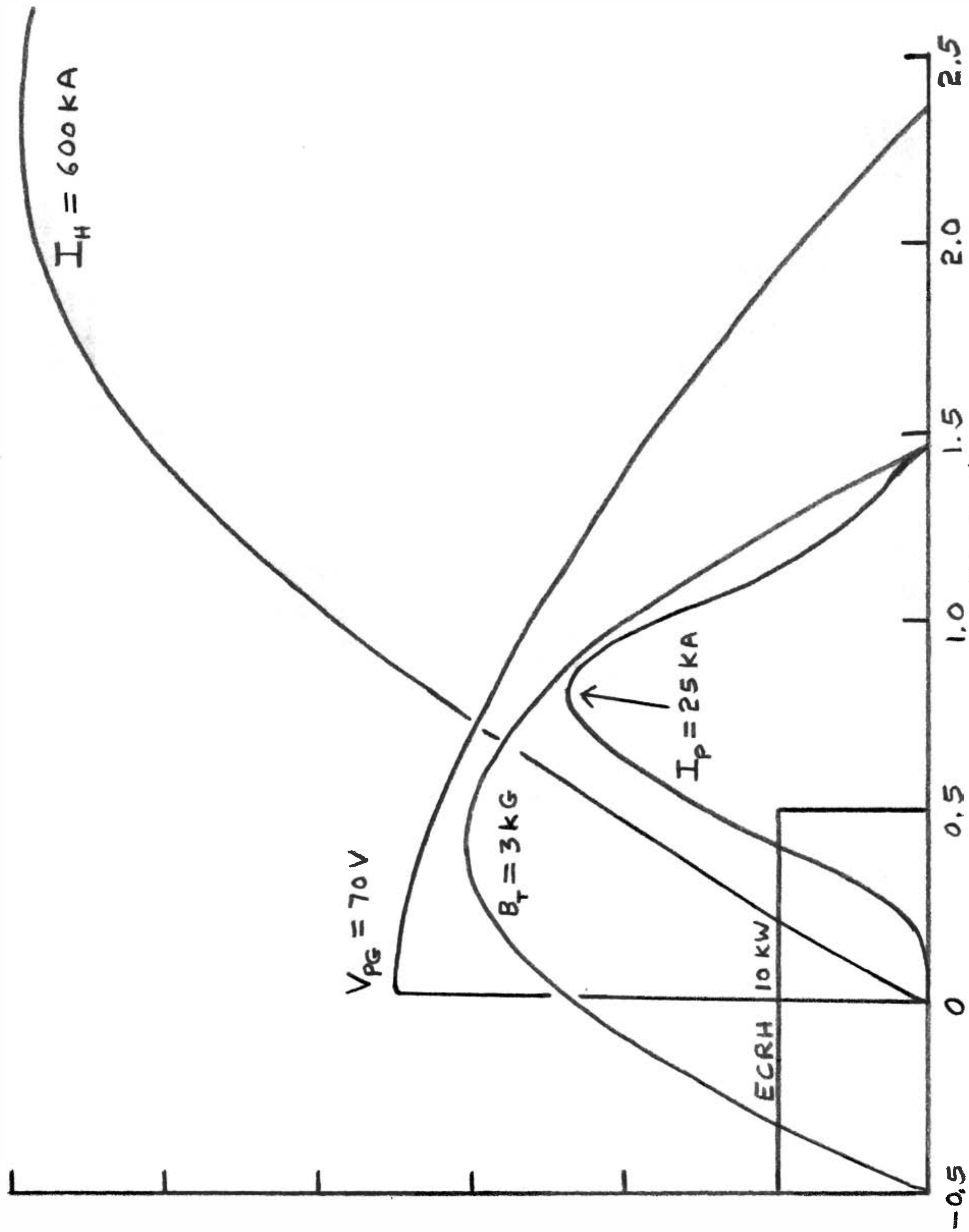


Figure 2



$I_H = 600$ kA
$V_{PG} = 70$ V
$B_T = 3$ kG
$I_p = 25$ kA
ECRH
10 KW
−0.5
0
0.5
1.0
1.5
2.0
2.5
TIME (msec)

Figure 3

18
IP= 24.090000 AT 750
TE= 23.754520
NE= .46802929E 13
A= 9.4693570
Q= 1.1127761

TIME	IP	JSAT	BT	VPG	IHOOP	TE	NE
200	2	17	2893	69	88	6	68
400	10	347	3030	66	169	14	1013
600	20	1444	2873	62	245	20	11367
800	23	1058	2463	57	317	24	4269
1000	15	811	1847	52	384	24	2648
1200	7	742	1114	46	444	21	2313
1400	4	486	351	40	497	20	1362

19
IP= 8.0600000 AT 700
TE= 15.956884
NE= .19737108E 13
A= 7.3036337
Q= 1.0150543

TIME	IP	JSAT	BT	VPG	IHOOP	TE	NE
200	1	0	1417	69	88	5	0
400	3	158	1495	66	170	9	534
600	7	439	1417	62	247	14	1299
800	7	738	1202	57	319	16	2313
1000	4	674	909	52	385	16	2068
1200	1	457	527	46	445	12	1399
1400	1	325	146	40	497	11	1027

20
IP= 5.3200000 AT 660
TE= 16.889399
NE= .59343544E 12
A= 7.8891216
Q= 3.6974020

TIME	IP	JSAT	BT	VPG	IHOOP	TE	NE
200	1	2	2854	35	45	5	10
400	3	48	3000	33	87	11	150
600	5	188	2825	31	126	16	515
800	5	241	2414	29	163	19	642
1000	5	439	1798	26	197	20	1211
1200	3	398	1065	24	228	20	1084
1400	1	205	293	21	255	16	565

Figure 4

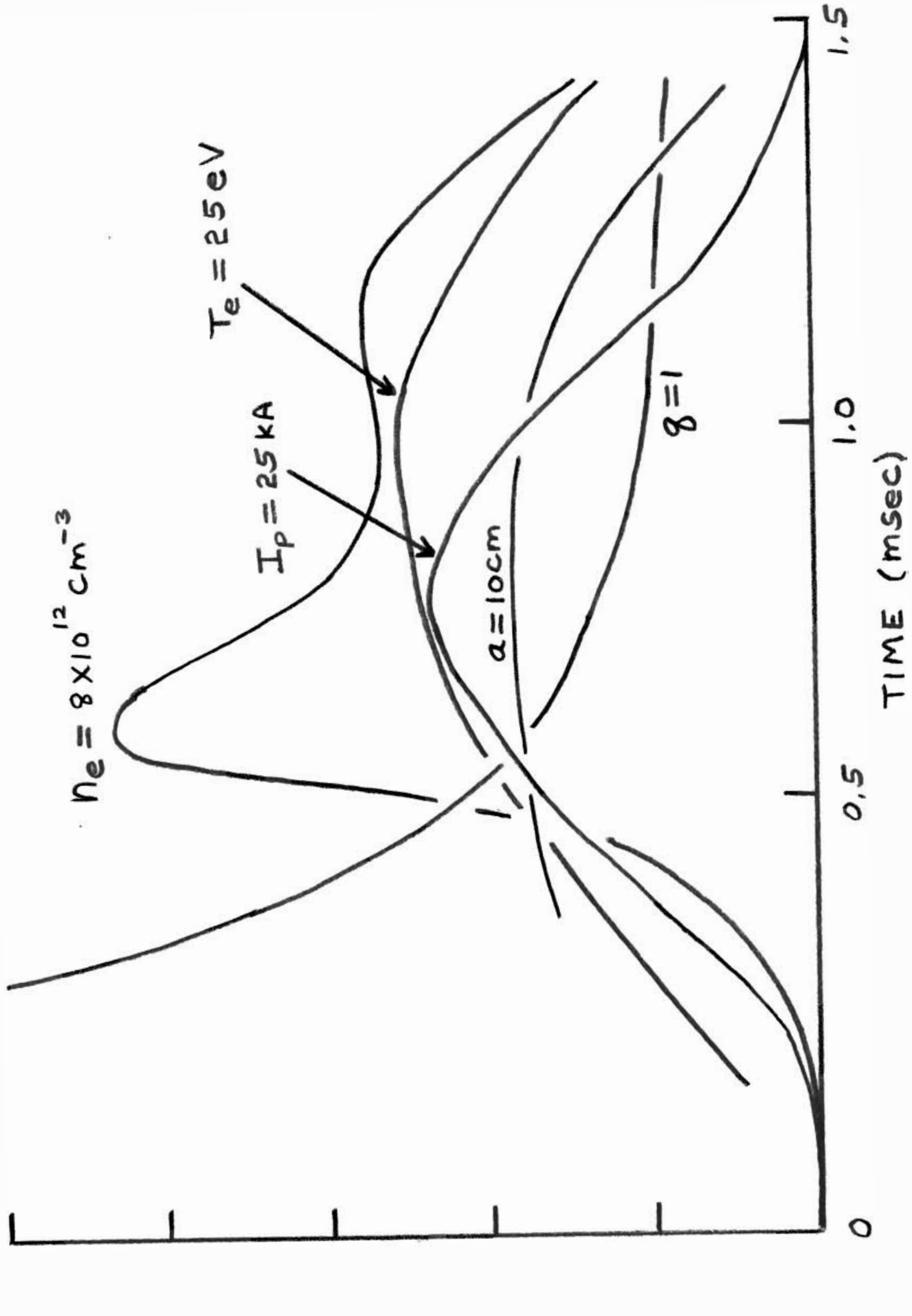


Figure 5

Figure 6

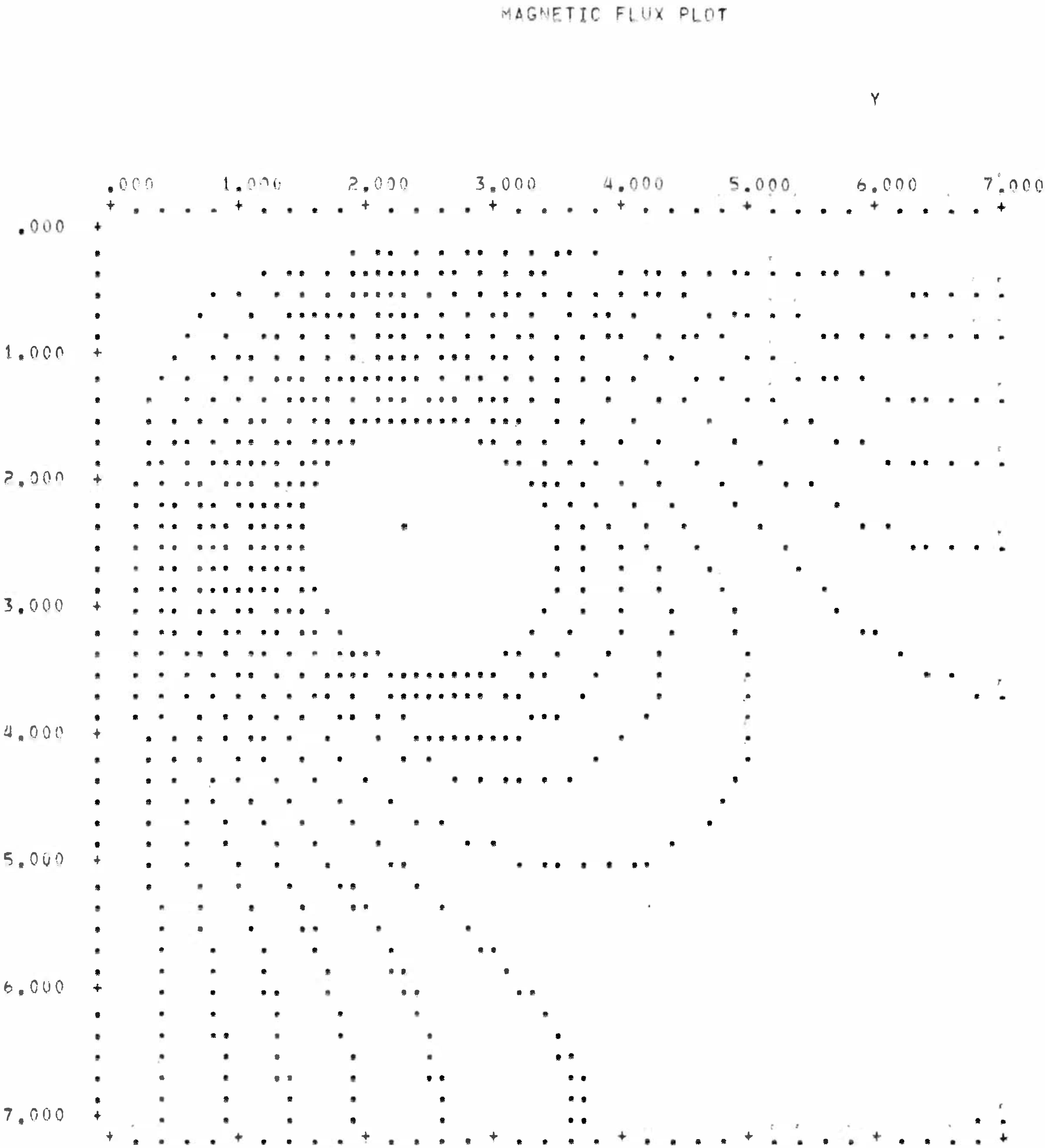
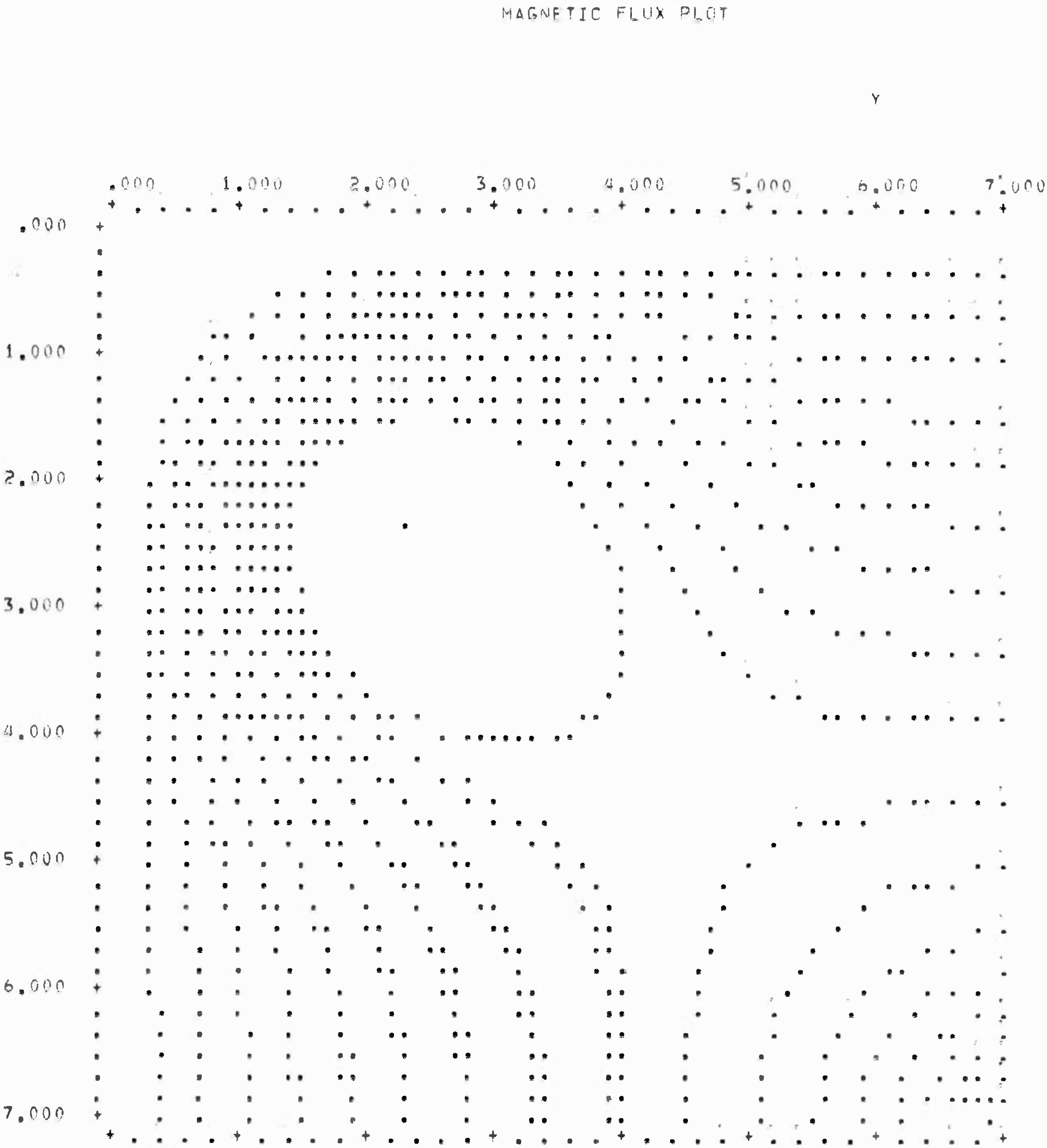

MAGNETIC FLUX PLOT
Y
.000
1.000
2.000
3.000
4.000
5.000
6.000
7.000
.000
1.000
2.000
3.000
4.000
5.000
6.000
7.000

Figure 7


MAGNETIC FLUX PLOT
Y
.000
1.000
2.000
3.000
4.000
5.000
6.000
7.000
.000
1.000
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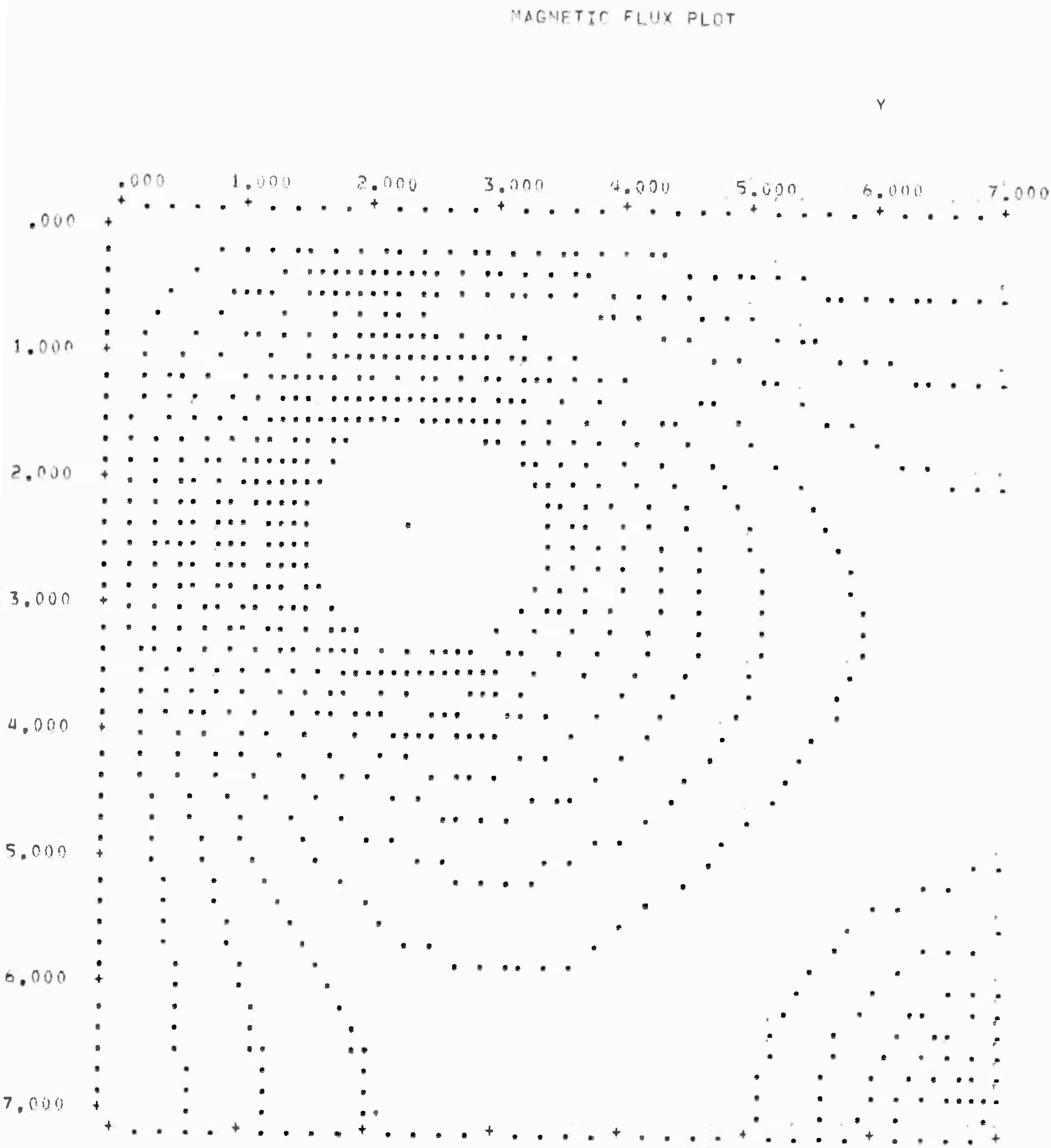
3.000
4.000
5.000
6.000
7.000

Figure 8



MAGNETIC FLUX PLOT
Y
.000
1.000
2.000
3.000
4.000
5.000
6.000
7.000
.000
1.000
2.000
3.000
4.000
5.000
6.000
7.000

Figure 9

MAGNETIC FLUX PLOT

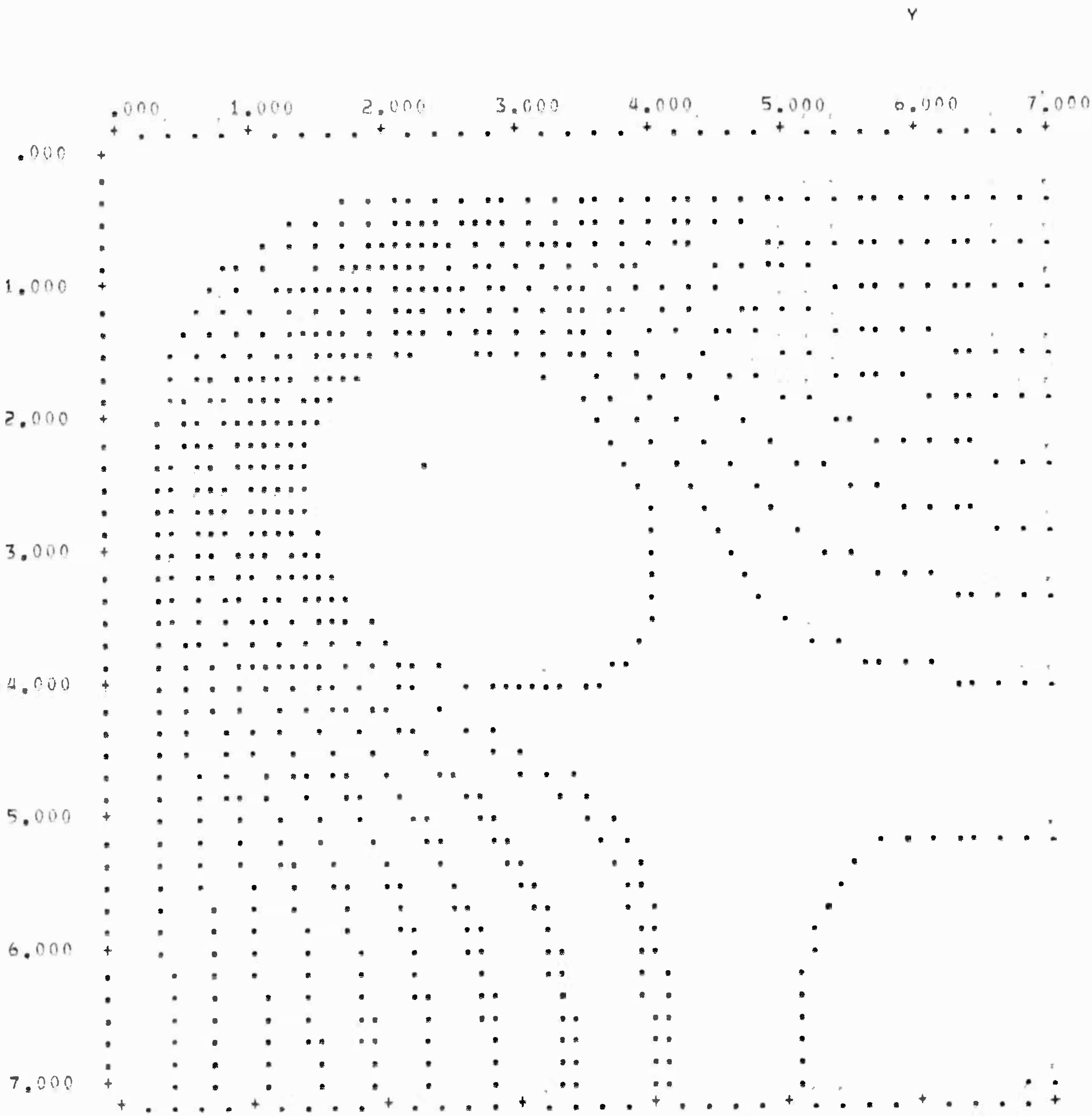


Figure 10

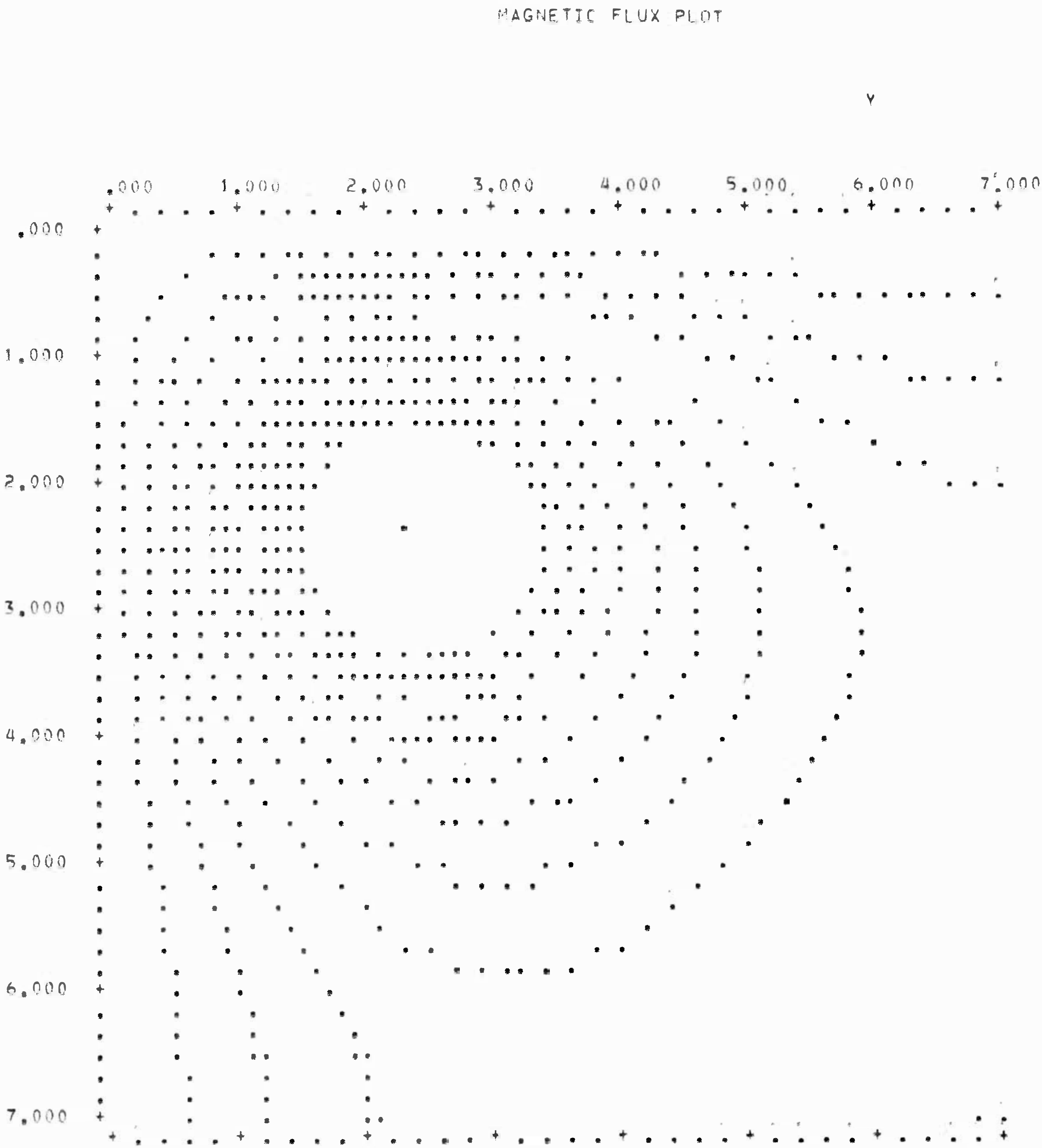
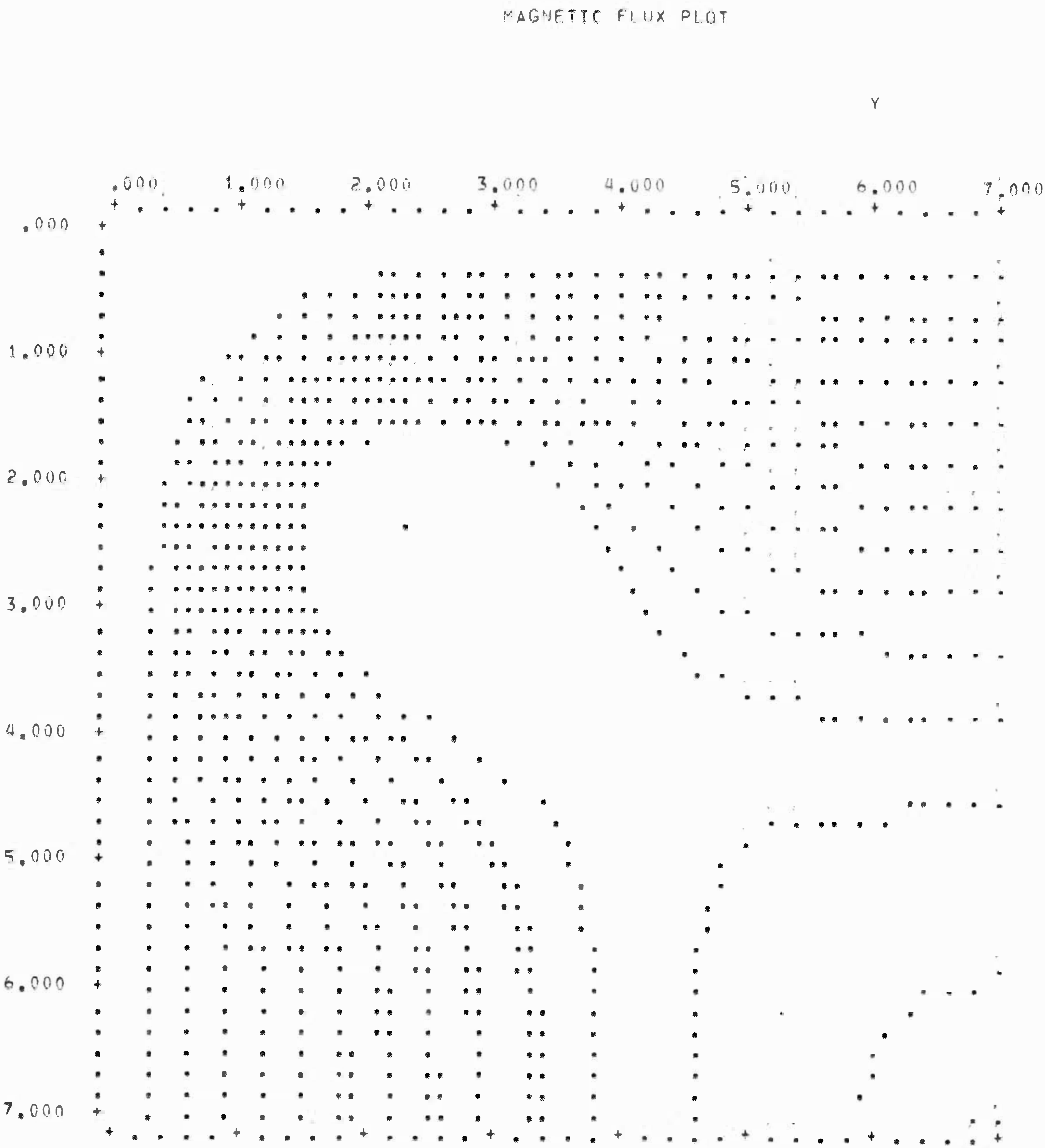
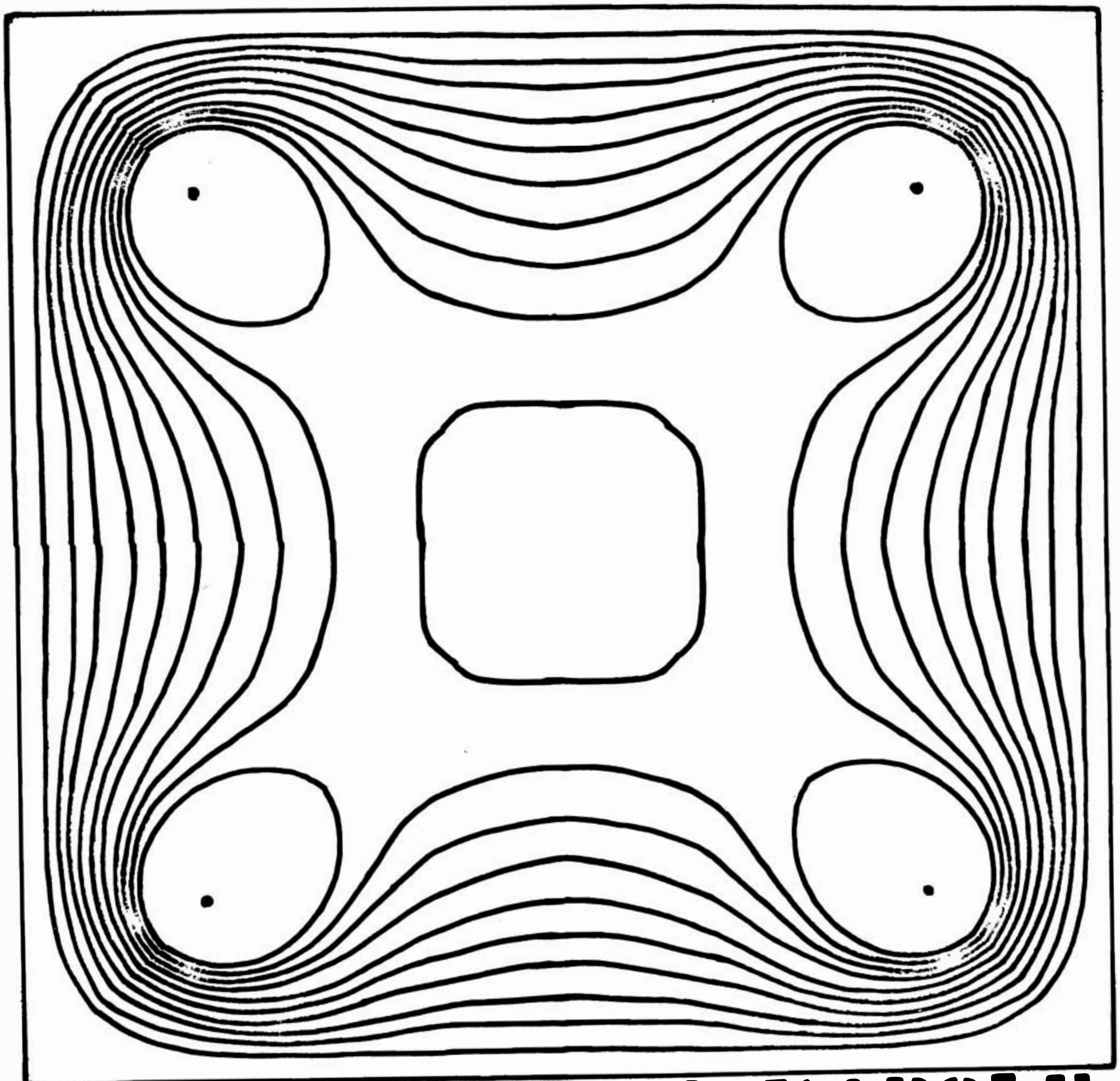

MAGNETIC FLUX PLOT
Y
.000
1.000
2.000
3.000
4.000
5.000
6.000
7.000
.000
1.000
2.000
3.000
4.000
5.000
6.000
7.000

Figure 11


MAGNETIC FLUX PLOT
Y
.000
1.000
2.000
3.000
4.000
5.000
6.000
7.000
.000
1.000
2.000
3.000
4.000
5.000
6.000
7.000



WISCONSIN TOKAPOLE

Fig. 12

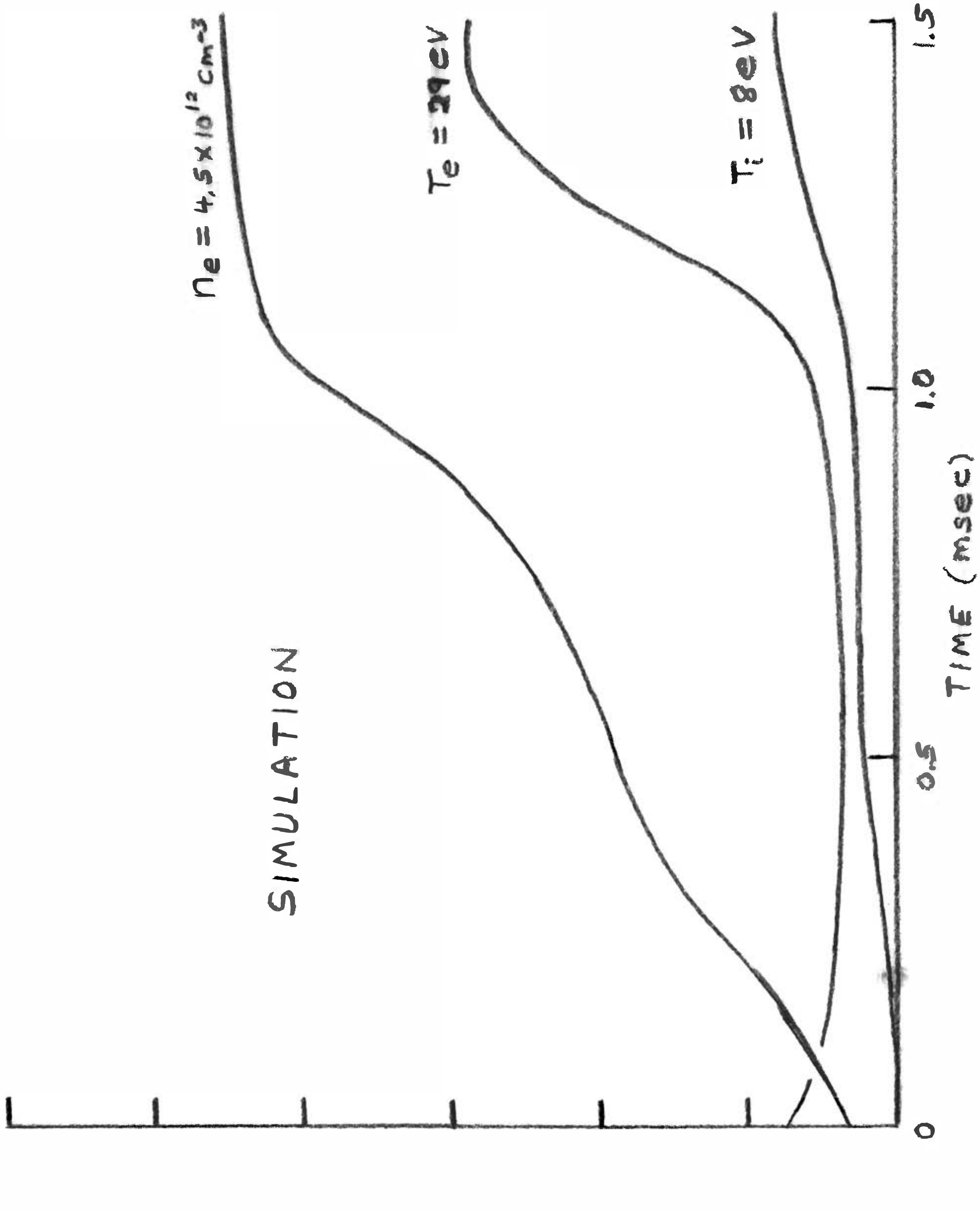
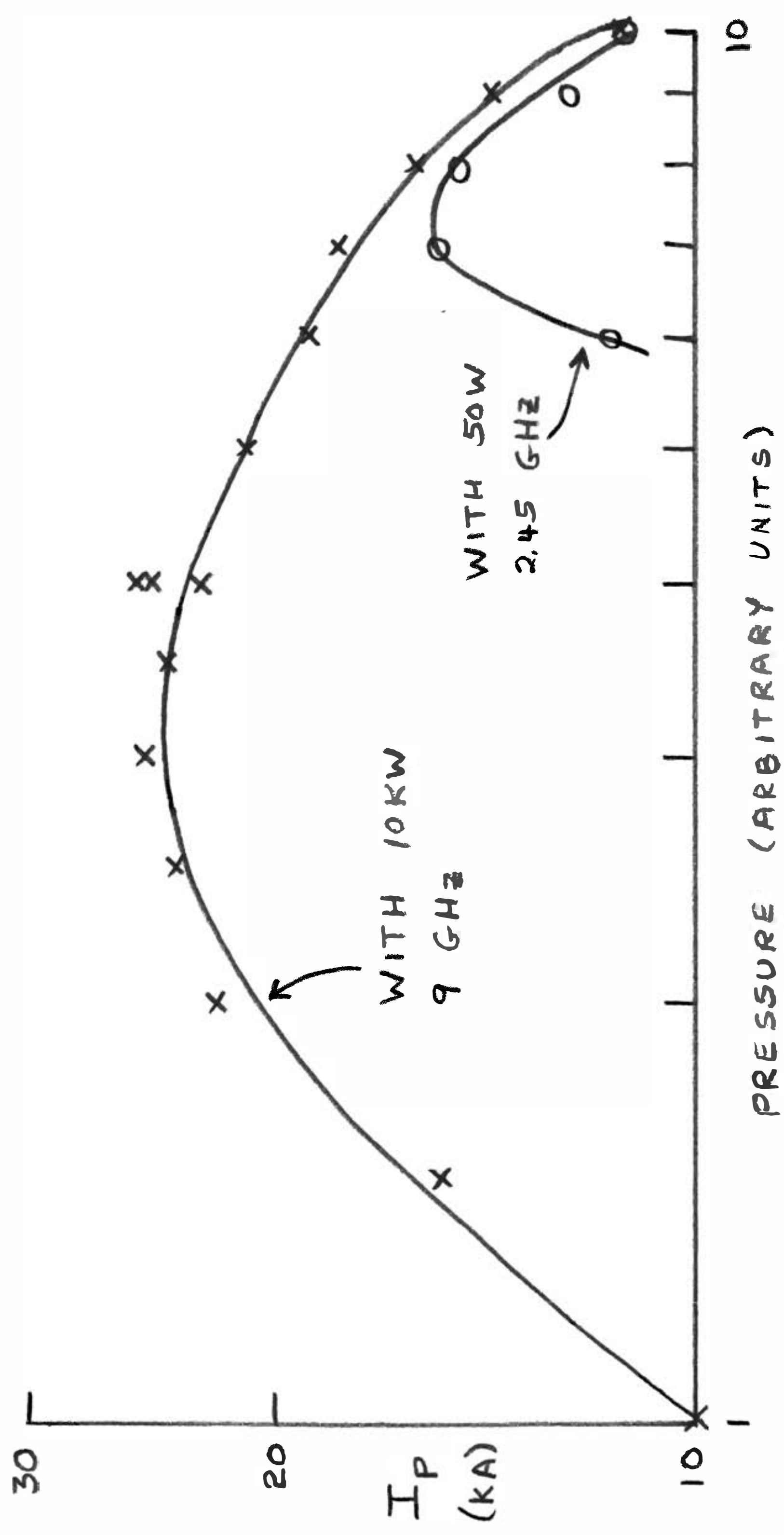

$n_e = 4.5 \times 10^{12}$ cm^{-3}
$T_e = 29$ eV
$T_i = 8$ eV
SIMULATION
TIME (msec)
0
0.5
1.0
1.5

Fig 13



30
20
10
I_P (kA)
1
10
PRESSURE (ARBITRARY UNITS)
WITH 10KW 9 GHz
WITH 50W 2,45 GHz

Fig 14

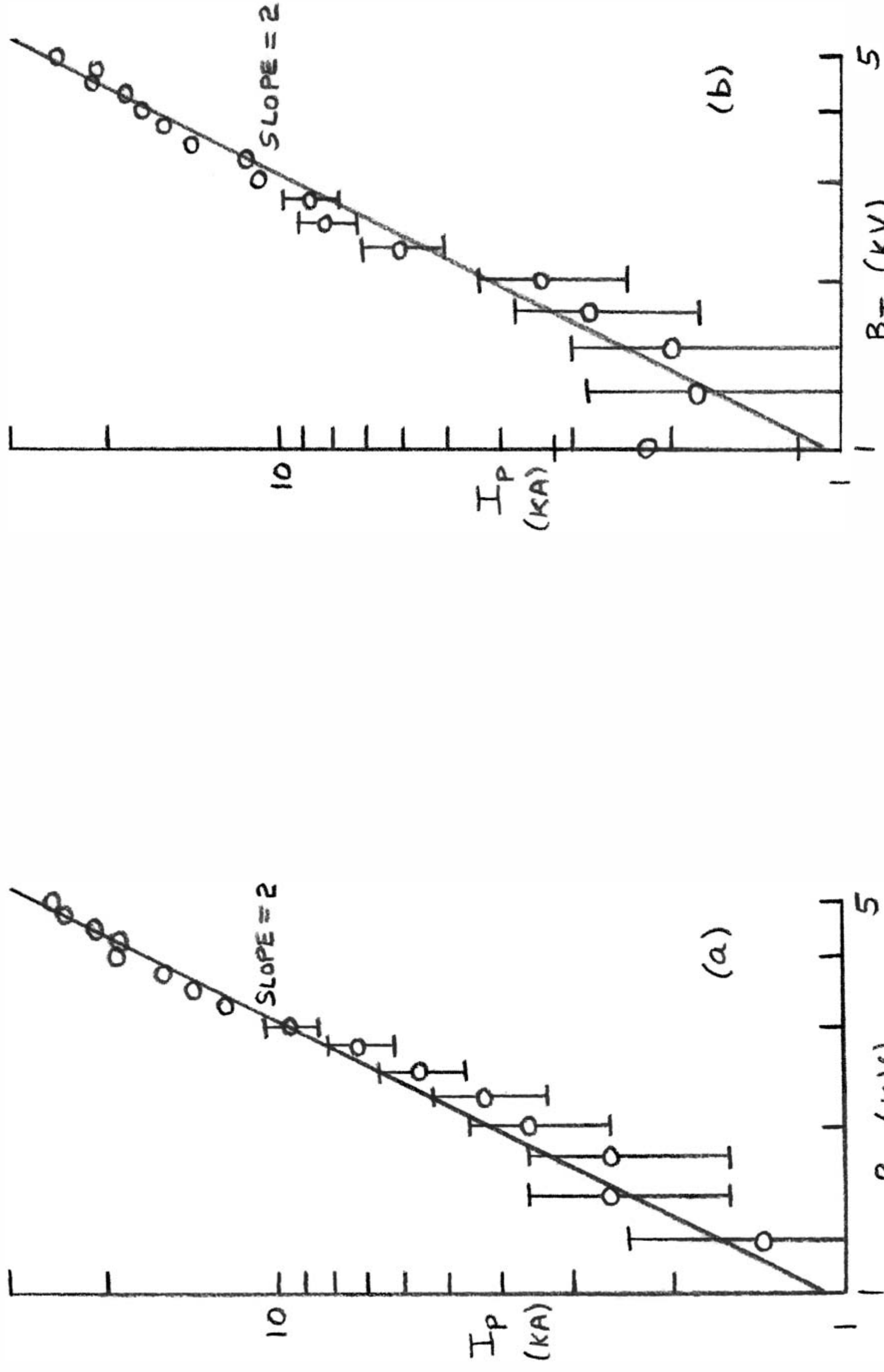


Fig 15

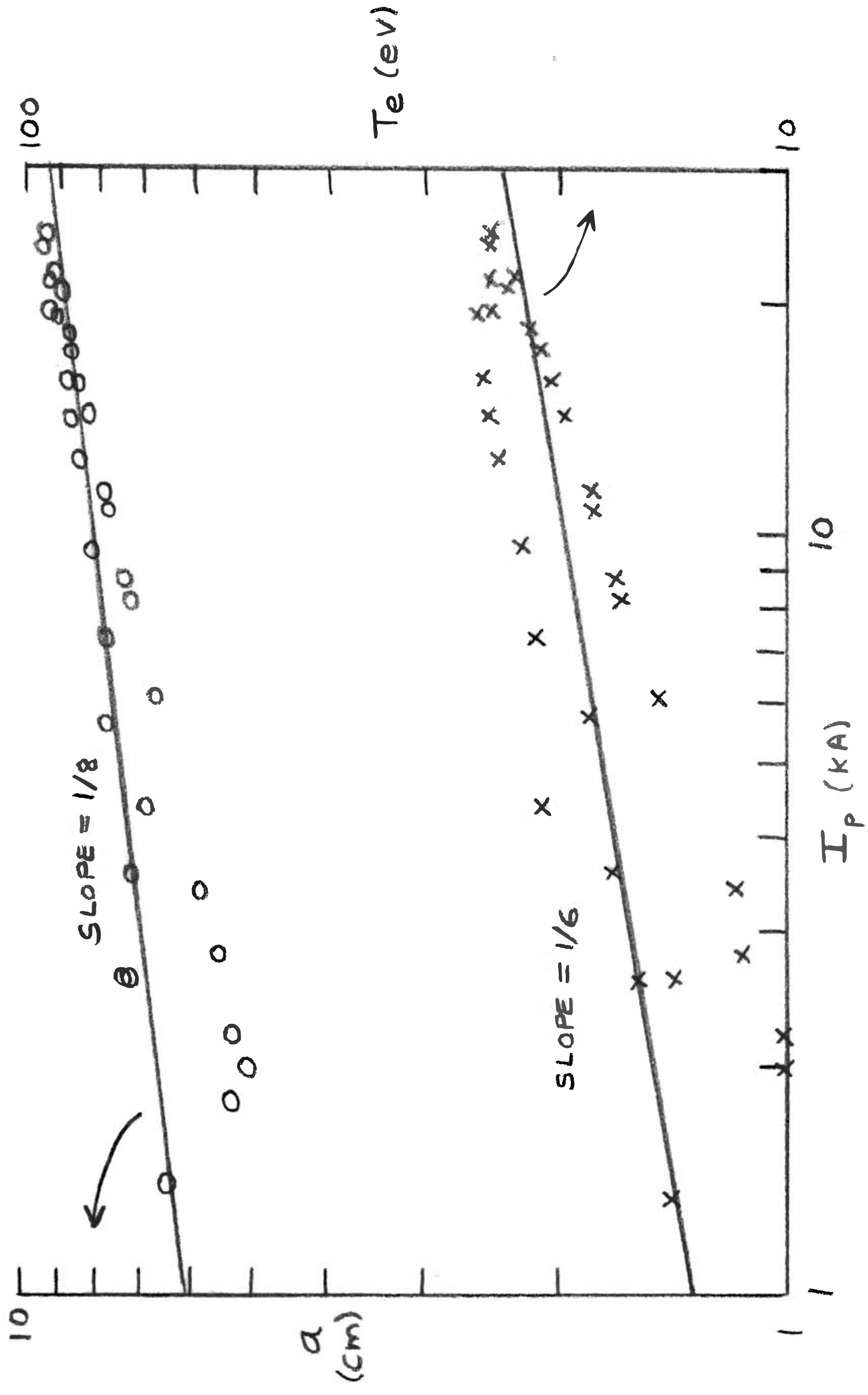


Fig 16

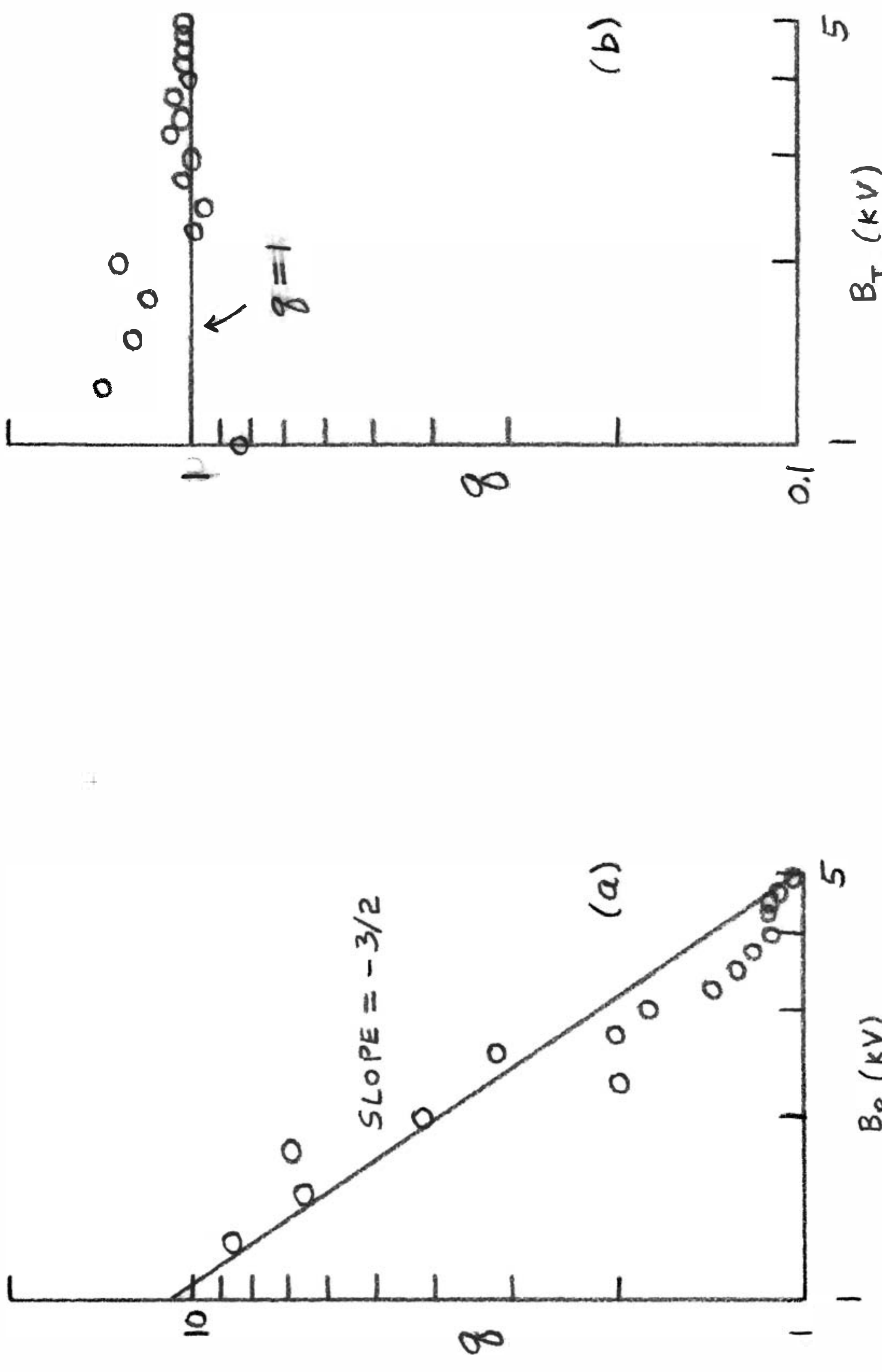


Fig 17

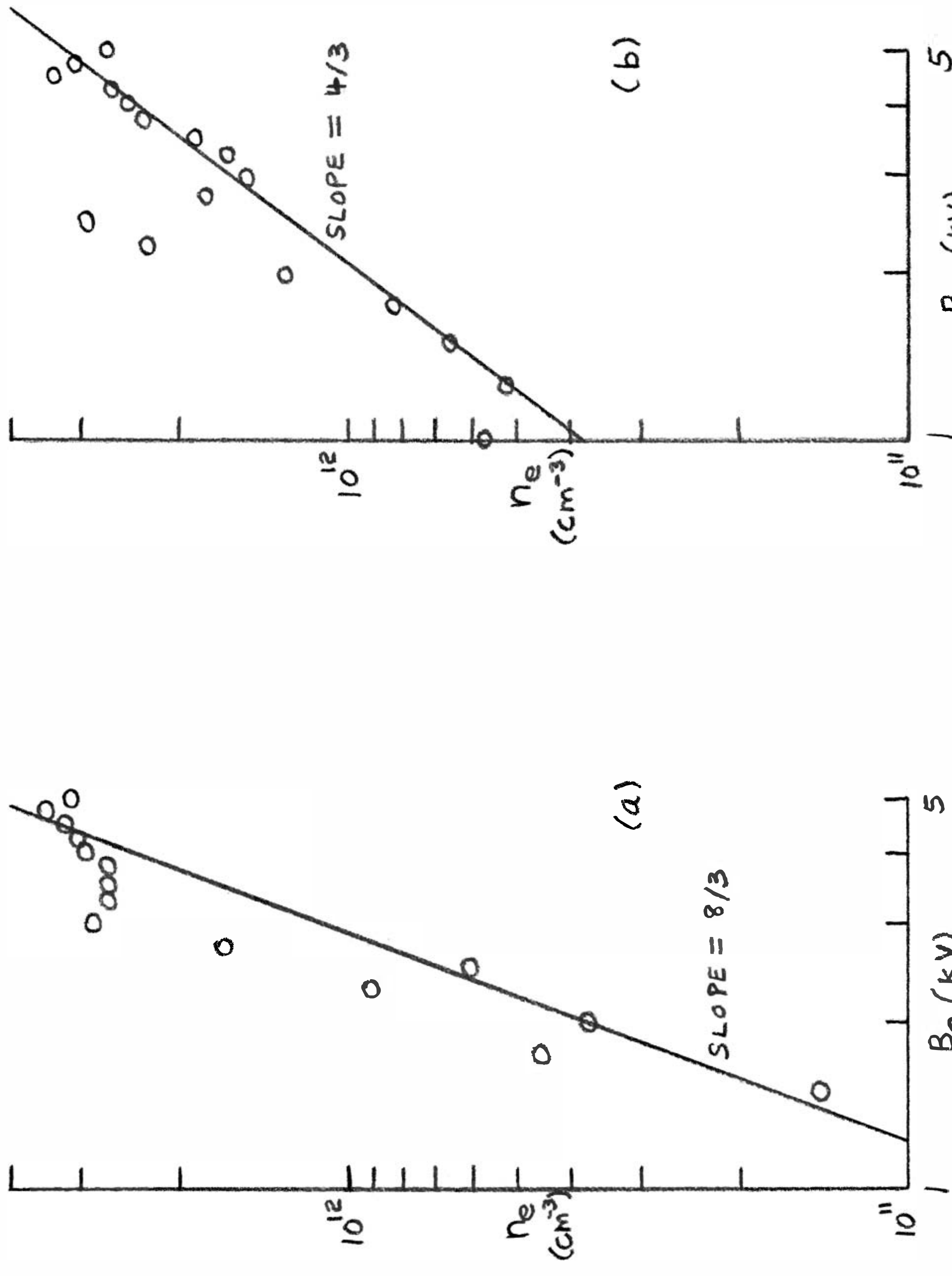

SLOPE = 4/3
(b)
B_T (kV)
n_e (cm^{-3})
10^{12}
10^{11}
1
5
SLOPE = 8/3
(a)
B_p (kV)
n_e (cm^{-3})
10^{12}
10^{11}
1
5

Fig 18

TYPICAL TOKAPOLE PARAMETERS AND SCALING

B_T = 3 kG (5 kG maximum)

I_p = 25 kA ($\propto B^2_p\ B^2_T$)

a = 10 cm ($\propto I_p^{1/8}$)

q = 1 ($\propto B_p^{-3/2}$)

T_{ec} = 25 eV ($\propto I_p^{1/6}$)

n_e = 5 x 10^{12} cm^{-3} ($\propto B_p^{8/3}\ B_T^{4/3}$)

τ_E = 10μ sec (constant)

P_{OH} = 500 kW ($\propto B_p^3\ B^2_T$)

Fig 19