AN ADMITTANCE PROBE

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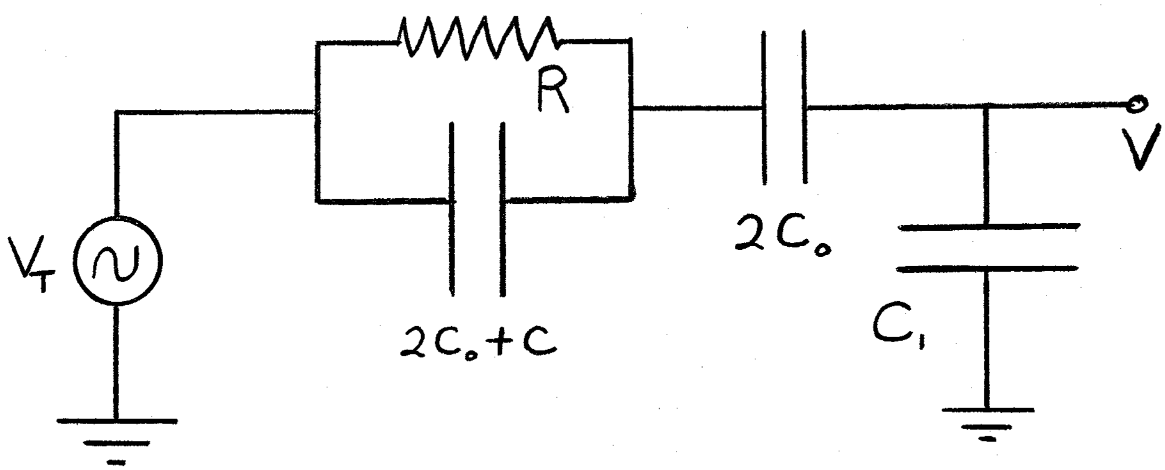
Introduction

This paper describes a type of probe which was developed to study the effective capacitance and resistance of plasma sheaths. In PLP 61, predictions are made about the sheath resistance and capacitance of a floating electrostatic probe under certain ideal conditions. The admittance probe has been used with plasmas in the toroidal octupole to test the predictions of PLP 61. In certain instances this type of probe may prove a useful diagnostic tool, supplementing the usual Langmuir probe. Another interesting application of the admittance probe is the measurement of hoop displacement in the toroid when the magnetic field is pulsed.

The admittance probe is simply a conducting electrode in contact with the plasma and connected across one leg of a capacitance bridge. The circuit which is used with the probe is shown in Figure 1. The plasma sheath is represented by a parallel RC circuit in series with a time dependent voltage source V_f, equal to the floating potential. The sheath is effectively connected across one leg of a capacitance bridge which is driven by a low level ($V_o \lesssim 1$ volt) sine wave generator at frequency f. The bridge is balanced so that there is no output voltage when no plasma is in contact with the probe tip. A time dependent R(t) or C(t) then unbalances the bridge and produces a modulated RF output signal which is amplified by a runed RF amplifier, detected, and observed on an oscilloscope. The Fourier component of V_f at frequency f is small enough ($<< V_o$) that it can be neglected.

Theory of Operation

It is essential to understand how the amplitude and phase of the RF output voltage depend on R and C. By Thevenin's theorem, the bridge circuit can be redrawn as follows:



where $V_T = -\frac{1}{2}V_o\left[\frac{1+j\,\omega\,RC}{1+j\,\omega\,R(2C_o+C)}\right]$.

Now assume that we choose C_o and ω (= $2\pi f$) such that $\frac{1}{2\omega\,C_o} << R$. Then we can disregard the R in the above circuit, and the output voltage becomes

$$V = \frac{C_o}{C_o+C_1}\,V_T = -\,1/2\,\left[\frac{C_o}{C_o+C_1}\right]\left[\frac{1+j\omega RC}{1+j\omega R(2C_o+C)}\right]V_o\quad.$$

But since $\frac{1}{2\omega C_o} \ll R$, this equation can be simplified to

$$V = -\tfrac{1}{2} \left[\frac{C_o}{C_o + C_1} \right] \left[\frac{1 + j\omega RC}{j\omega R(2C_o + C)} \right] V_o \quad .$$

The amplitude of V is

$$|V| = \tfrac{1}{2}\left[\frac{C_o}{C_o+C_1}\right]\left[\frac{1+\omega^2R^2C^2}{\omega^2R^2(2C_o+C)^2}\right]^{\frac{1}{2}} |V_o| \quad ,$$

and the phase of V with respect to V_o is

$$\Theta = \tan^{-1}\left(\frac{1}{\omega RC}\right)$$

Hence by measuring $\frac{|V|}{|V_o|}$ and Θ, one can determine the sheath resistance and capacitance using the above equations since C_o, C_1, and ω are known.

The admittance Y of the sheath is defined as the reciprocal of the sheath impedance, or

$$Y = \frac{1}{R} + j\omega C = \frac{1 + j\omega RC}{R} \quad ,$$

which has a magnitude

$$|Y| = \left[\frac{1 + \omega^2R^2C^2}{R^2} \right]^{\frac{1}{2}} \quad .$$

Making the additional assumption that $C \ll 2C_o$, we obtain the particularly nice result,

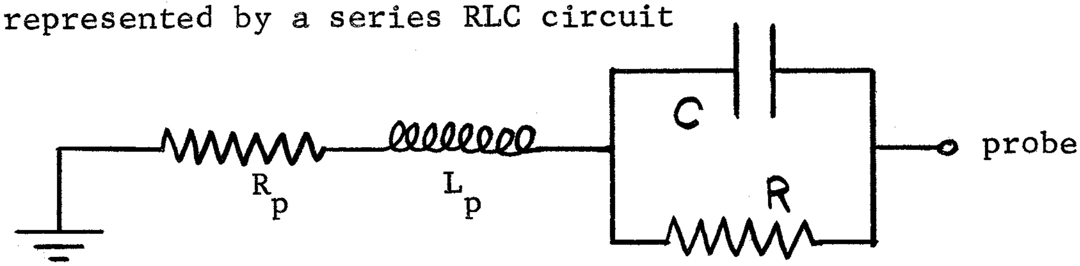
$$|V| = \frac{1}{4\omega}\left[\frac{1}{C_o+C_1}\right] |V_o| \; |Y| = \alpha \, |Y| \quad .$$

In summary, the main result of this derivation is that, for the admittance probe in Figure 1, the output voltage at a given frequency is proportional to the sheath admittance, provided $R \gg \frac{1}{4\pi f C_o}$ and $C \ll 2C_o$. By choosing C_o sufficiently large, both conditions are easily met at any frequency . The sensitivity, however, decreases as C_o is increased, so it is advantageous to use a high frequency and make C_o as low as possible.

It should be pointed out that as the frequency f approaches the electron plasma frequency,

$$f_{pe} = \frac{1}{2\pi} \left[\frac{4\pi n e^2}{m} \right]^{\frac{1}{2}} = 9 \times 10^3 \sqrt{n\,(cm^{-3})},$$

the inductance and resistance of the plasma outside the sheath can no longer be neglected. At frequencies near or above f_{pe}, the impedance between the probe tip and ground can best be represented by a series RLC circuit



where C is the same sheath capacitance as used before. The sheath resistance R can be neglected since in general $R \gg \frac{1}{\omega_{pe} C}$.

The series resonance

$$\omega_o = [L_p C]^{-\frac{1}{2}}$$

is expected to occur near the electron plasma frequency, and this is the basis of operation of the resonance probe[1].

Since the admittance probe result was derived only for a parallel RC, it is not clear that the result holds for this more complicated situation. In fact, it does as the following derivation shows. Consider an arbitrary complex admittance Y between the probe electrode and ground. Y may be any combination of R's, L's, and C's that we choose, or it may be some function which cannot be represented at all by a finite number of circuit elements. Applying Thevenin's theorem gives

$$V_T = -\tfrac{1}{2}V_o \left[\frac{Y}{Y+2j\omega C_o} \right].$$

If we assume that $|Y| << 2\omega C_o$, we obtain the result

$$V = -\tfrac{1}{2}\left[\frac{C_o}{C_o+C_1} \right] \left[\frac{Y}{2j\omega C_o} \right] V_o,$$

whose magnitude is

$$|V| = \frac{1}{4\omega}\left[\frac{C_o}{C_o+C_1} \right] |V_o| \; |Y| = \alpha |Y| .$$

Hence for an arbitrary impedance between the probe electrode and ground, the admittance probe is linear, provided

$$|z| = \frac{1}{|Y|} >> \frac{1}{4\pi f C_o} .$$

The response time of the probe is limited only by the bandwidth of the RF amplifier which may be many MHZ, but in no case can it exceed 1/f. Note that the input resistance of the probe is infinite so that the probe remains at the floating potential, provided V_f does not change rapidly. If the floating potential has high frequency components, the probe follows V_f up to a frequency of about

$$f_c = \frac{1}{2\pi RC_o} .$$

Although the probe can accurately measure sheath admittance at frequencies much higher than f_c, the dependence of sheath capacitance and resistance on probe voltage is rather complicated, and its use as a diagnostic tool is limited to frequencies below f_c. If C_o is made as small as possible, namely just equal to the capacitance of the probe cable, f_c is the same as the cutoff frequency for the conventional biased double Langmuir probe.

Derivation of Sheath Capacitance

An expression for the sheath capacitance of a plane probe at the floating potential in a Maxwellian plasma was derived in PLP 61. In that derivation, it was assumed that both ions and electrons respond without appreciable delay to any fluctuating voltage imposed on the probe. The results of PLP 61 are stated below and an alternate derivation for the sheath capacitance is given for the case where the time variation of potential is too rapid for the ions to follow. The two cases are divided roughly by the ion plasma frequency defined as

$$f_{pi} = \frac{1}{2\pi} \sqrt{\frac{4\pi n e^2}{M}} = 210 \sqrt{n(cm^{-3})} \ H_z.$$

Case I ($f \gg f_{pi}$): Following a treatment by Crawford and Grard[2], the sheath is assumed to have a thickness x given by Child's law for a space-charge-limited ion diode:

$$x = \frac{2}{3} \left(\frac{\varepsilon_o}{j_i}\right)^{\frac{1}{2}} \left(\frac{2e}{M}\right)^{1/4} V^{3/4} .$$

ji is the ion current density and V is the potential of the probe with respect to the plasma. If the voltage V has a time derivative $\frac{dV}{dt}$, the sheath edge moves with a velocity

$$\dot{x} = \frac{1}{2}\left(\frac{\varepsilon_o}{ji}\right)^{\frac{1}{2}}\left(\frac{2e}{M}\right)^{1/4} V^{-1/4}\frac{dV}{dt}.$$

This gives rise to a displacement current

$$i_D = neA\dot{x} = \frac{neA}{2}\left(\frac{\varepsilon_o}{ji}\right)^{\frac{1}{2}}\left(\frac{2e}{M}\right)^{1/4} V^{-1/4}\frac{dV}{dt},$$

where A is the area of the probe. The capacitance is defined by the relation

$$i_D = C\frac{dV}{dt}$$

or $$C(V) = \frac{neA}{2}\left(\frac{\varepsilon_o}{ji}\right)^{\frac{1}{2}}\left(\frac{2e}{M}\right)^{1/4} V^{-1/4}.$$

For $V_i \geq V_e$, ji can be written as

$$ji = ne\left[\frac{2eV_i}{M}\right]^{\frac{1}{2}}.$$

For $V_i < V_e$, the sheath criteron requires $V_i \approx 0.5\ V_e$. Substituting the above value of j_i into the equation for C(V) gives

$$C(V) = \frac{A}{2}\left[\varepsilon_o ne\right]^{\frac{1}{2}}\left[\frac{1}{V_i V}\right],$$

where V_i is the ion energy in eV. In terms of the Debye length,

$$\lambda_D = \sqrt{\frac{\varepsilon_o V_e}{ne}} = 744\sqrt{\frac{V_e(ev)}{n\ (cm^{-3})}}\ \text{cm}$$

where V_e is the electron energy, the capacitance is

$$C = \left[\frac{1}{2}\left(\frac{V_e}{\sqrt{V_i V}}\right)^{\frac{1}{2}}\right]\frac{\varepsilon_o A}{\lambda_D}.$$

An encouraging feature of this equation is the fact that the capacitance varies slowly with ion energy and probe potential, i.e., inversely as the fourth root. Hence one expects that an admittance probe would be well suited for measuring density in the presence of fluctuating plasma potentials.

In the usual application, the probe is operated at the floating potential, in which case

$$V = \tfrac{1}{2} V_e \log_e \left(\frac{V_e M}{V^* m}\right) ,$$

where V* is the electron or ion energy, whichever is greater. For $V_i \geq V_e$, the expression for the capacitance of a probe at the floating potential is

$$C = \tfrac{1}{2} \left[\frac{2V_e}{V_i \log_e (V_e M/V_i m)}\right]^{1/4} \frac{\varepsilon_o A}{\lambda_D} .$$

We now restrict discussion to a hydrogen plasma and consider two examples:

Example 1: $V_i = V_e = E.$

Numerically the capacitance is

$$C = .36 \frac{\varepsilon_o A}{\lambda_D} = 4.3 \times 10^{-5} \frac{A(cm^2) \sqrt{n(cm^{-3})}}{\sqrt{E(eV)}} \text{ pF.}$$

The sheath capacitance of a 1 cm^2 probe is plotted as a function of n and E in figure 2.

Example 2: $V_i = 40$ eV, $V_e = 10$ eV.

These values are characteristic of plasmas in the octupole.

Numerically,

$$C = .265 \frac{\varepsilon_o A}{\lambda_D} = 1.0 \times 10^{-5} A\ (cm^2) \sqrt{n(cm^{-3})}\ pF.$$

This result is plotted as a dotted line in figure 2.

Case II ($f \ll f_{pi}$): In PLP 61, an expression was derived for the sheath capacitance assuming that both ions and electrons respond to variations in probe voltage without delay. The central result of PLP 61 is stated below:

$$C(V) = \frac{\left(\frac{V_i}{V + V_i}\right)^{\frac{1}{2}} - \frac{1}{2}\exp(-V/V_e)}{2\left[\left(\frac{V_i}{V_e}\right)\left(\frac{V + V_i}{V_i}\right)^{\frac{1}{2}} - \left(\frac{V_i}{V_e} + \frac{1}{2}\right)\right]^{\frac{1}{2}}} \quad \frac{\varepsilon_o A}{\lambda_D}.$$

The same two examples as treated above are now considered.

Example 1: $V_i = V_e = E$.

For a floating probe, the potential is

$V = \frac{1}{2} E \log_e \left(\frac{M}{m}\right) = 3.76\ E.$

Numerically, the sheath capacitance becomes

$$C = .27 \frac{\varepsilon_o A}{\lambda_D} = 3.25 \times 10^{-5} \frac{A(cm^2)\sqrt{n(cm^{-3})}}{\sqrt{E(eV)}}\ pF.$$

This result is about a factor of ten smaller than that given in PLP 61, indicating a numerical error in that derivation. The corrected graph of C vs n for various energies is given in figure 3. This low value of capacitance means that it might be difficult to find a frequency for which

$$\frac{1}{RC} \ll \omega \ll \omega_{pe}.$$

If this condition cannot be met, the admittance probe cannot be expected to give an accurate measure of the sheath capacitance.

Example 2: $V_i = 40$ eV, $V_e = 10$ eV.

For a floating probe, the potential V is

$$V = \frac{1}{2} V_e \log_e \left(\frac{V_e M}{V_i m} \right) = 30.7 \text{ eV}.$$

The sheath capacitance is

$$C = .41 \frac{\varepsilon_o A}{\lambda_D} = 1.55 \times 10^{-5} A(\text{cm}^2) \sqrt{n(\text{cm}^{-3})} \text{ pF}.$$

Again this is smaller than the values derived in PLP 61. It is plotted as a dotted line in figure 3.

The results of the two cases considered are summarized below

Sheath Capacitance	$T_i = T_e = E$	Octupole
Case I ($f \gg f_{pi}$)	.36	.265
Case II ($f \ll f_{pi}$)	.27	.41

where capacitance is measured in units of $\frac{\varepsilon_o A}{\lambda_D}$.

In the octupole case, we expect the capacitance to be somewhat greater below the ion plasma frequency than above it. In an experiment by Crawford and Grard,[2] the capacitance was observed to be constant down to 0.2 f_{pi}, suggesting that RF ion motions are negligible, inclining us toward case I. Furthermore, since the sheath resistance almost certainly dominates the sheath capacitive reactance at the ion plasma frequency, case I is the more important for the admittance probe.

PRACTICAL CONSIDERATIONS

We now consider how the admittance probe might be applied to measurements in the octupole and some of the problems to be expected. First we ask what is the optimum size and shape for the probe electrode. If the probe area is too small, the admittance will be too small to measure. If it is too large, the plasma will be appreciably perturbed. A convenient compromise is about 1 cm^2. The shape of the electrode will be considered later.

For a 10^9 cm^{-3} plasma, the capacitance of a 1cm^2 probe is expected to be about 0.5 pF and the resistance about 10 KΩ. If C_o is made equal to the capacitance of the shielded cable, or about 100 pF, the requirement that $C_o << 2C_o$ is easily satisfied. The requirement that $R >> \frac{1}{4\pi f C_o}$ is satisfied to within 10% down to a frequency of 800 ^{o}KHz. Since the ion plasma frequency at 10^9 cm^{-3} is about 7 MHz, there exists a considerable range over which the sheath resistance can be accurately measured. The electron plasma frequency at this density is about 280 MHz.

As the wavelength of the bridge oscillator approaches the electrical length of the probe cable, the cable can no longer be treated as a lumped capacitance and it is difficult to balance the bridge. This problem can be overcome by substituting an identical length of cable for one of the other capacitors in the bridge. Alternately, the output of the AM detector can be fed into the AC input of an oscilloscope to subtract off the residual signal level. A 1 meter cable is one wavelength long at 200 MHz.

Because of the many factors which influence the overall gain of the system, the probe is best calibrated by attaching known capacitors from the tip to ground and measuring the corresponding output voltage. At low frequencies a resistor can be used for calibration, but since most resistors have a few tenth of a pF parallel capacitance, this method is inaccurate at high frequencies. For capacitors less than about 10 pF, the graph of C vs V should be approximately linear. A calibration curve for an admittance probe operating at 4 MHz is shown in Figure 4. The deviation at low capacitance is caused by stray capacitance which was estimated to be about 0.5 pF. The deviation at the other end of the curve is caused by the fact that $C/2C_o$ is no longer negligible. In the range of interest ($C < 5$ pF), the overall gain of the system is 0.34 volts/pF.

When sheath admittance is measured as a function of frequency, the circuit must be recalibrated at each frequency. This is most easily done by adjusting the RF generator voltage V_o to give some simple relation such as 1 volt/pF. Then the sheath admittance is found by multiplying the voltage by $2\pi f \times 10^{-12}$. To avoid continually removing the probe from the tank, the calibration capacitor can be placed in the bridge circuit.

One must be careful that the PF voltage at the probe tip does not become comparable to the plasma energy expressed in eV. If this happens, a rectifying action takes place, and the probe is shifted from its regular operating point at the floating potential. The probe voltage swing is approximately one half the peak-to-peak level of the RF generator, which is typically a few tenths of a volt. If the level is too low, the sensitivity will not be adequate and fluctuations of V_f may interfere.

The admittance probe can be used as a diagnostic tool in either of two ways: 1) at low frequencies, the output is proportional to the sheath conductance which varies linearly with the density n, or 2) at high frequencies, the output is proportional to the sheath capacitance which varies like the square root of n. The former is better for dense plasmas and the latter for tenuous plasmas. The low frequency case is a more sensitive function of n, while the high frequency case has better time resolution. In the octupole, either case should work.

Extension to Cylindrical Geometry

The most serious disadvantage of the admittance probe in tenuous plasmas is that planar geometry is achieved only by making the probe area very large. At a density of $10^9 cm^{-3}$ in the octupole, λ_D = 0.7 mm, and the capacitance sheath thickness is about 2 mm. Hence for a plane probe of 1 cm^2 surface area, the fringing effects cannot be ignored.

In practice, it is usually more convenient to use a cylindrical probe which is many Debye lengths long and recalculate the sheath resistance and capacitance in cylindrical geometry. Unfortunately, the detailed calculation of the capacitance is difficult and requires numerical methods. If the sheath thickness does not become too much larger than the probe radius, r, to a good approximation the sheath thickness for the cylindrical case is the same as for the planar case, and we can apply the standard equation for a long cylindrical capacitor:

$$C = \frac{2\pi\varepsilon_o \ell}{\log_e[(r+d)/r]} = \frac{.555\ell(\text{cm})}{\log_e[(r+d)/r]} \quad \text{pF}$$

Choosing a probe size of 1 cm long x 3 mm diameter gives a surface area of about 1 cm^2. The capacitance that one predicts for the two cases previously discussed is plotted as a function of density in Figure 5. It is clear that the capacitance variation is greatest at high densities, and a cylindrical admittance probe measuring capacitance is a poor diagnostic tool at low densities.

The sheath resistance calculated in PLP 61 is given by:

$$R = \frac{V_e}{neA}\left[\frac{2\pi M}{eV_i}\right]^{\frac{1}{2}} .$$

The area A which appears in this equation is the effective ion collection area of the probe. Several theories with varying degrees of mathematical complexity have been suggested[3] to determine A in cases where the sheath thickness is comparable to the probe radius. For our purposes it is probably a sufficiently good approximation to assume that the collecting area is given by

$$A = 2\pi\ (r + \lambda_D)\ \ell$$

or in the case of the previously considered probe dimensions,

$$A = 1 + \frac{1.57 \times 10^4}{\sqrt{n(cm^{-3})}}\ cm^2.$$

Using this value of A, the sheath resistance for a cylindrical probe 1 cm long x 3 mm diameter in the octupole can be calculated from the expression,

$$R = \frac{1.6 \times 10^{13}}{n(cm^{-3})A(cm^2)}\ \text{ohms}.$$

This resistance is plotted as a function of density in Figure 6 for the cylindrical and planar cases.

Experimental Results

To test the predicted behavior of the plasma sheath, the quiescent plasma present in the octupole between 200 and 1000μsec after the gun discharge was used. The center of the machine ($\rho = 0$) was used for all tests because the density is large and the magnetic field is zero there. The density at these times is in the 10^8 - 10^9 cm^{-3} range, and the potential and density fluctuations are very small. Tests were made with the admittance probe at port 8 ($\theta = +50°$), and with a single Langmuir probe of similar dimensions operating at saturation ion current at port 4 ($\theta = +147°$). The Langmuir probe served as a monitor to record density variations from shot to shot.

A General Radio Bridge Oscillator (Type 1330-A) provided the RF signal which was fed to the bridge through a Pulse

Engineering (PE5158) pulse transformer. The bridge output was fed to a x100 wideband amplifier (Los Alamos design) and then into a parallel LC circuit with a Q which ranged between 10 and 100. The output was read directly as a modulated RF signal rather than as a detected signal to improve the frequency response and simplify calibration.

The first test was to measure the sheath impedance as a function of frequency. This was done by varying the frequency of the bridge oscillator, adjusting its amplitude to give 50 mvolts deflection for a 5 pF standard capacitor at the probe tip. Then the sheath admittance is given by

$$|Y| = 2\pi f(5 \times 10^{-12}) \frac{V}{.05}$$

where Y is in mhos and V in volts. The results are plotted in Figure 7 in terms of $|Z| = 1/|Y|$. The points were taken at times for which the density is the same, as determined by the Langmuir probe. At 200 KHz, $1/4\pi fC_o \approx 4K\Omega$ which is not small compared to the 12K sheath resistance, and hence below 200 KHz the points are inaccurate because the bridge is non-linear. The impedance appears to be approximately constant out to about 10 MHz where it falls sharply. The solid curve shows the calculated impedance of a parallel RC circuit with $R=12K\Omega$ and $C=2.3pF$. Within the experimental error, this curve represents a good fit.

From the Langmuir probe data, the density in Figure 7 was calculated as $3.3 \times 10^{-8} cm^{-3}$. Because of the uncertainty in ion velocity and distribution, and the difficulty of estimating the effective ion collection area of the probe when the sheath is thick, this value may be off by as much as a factor or 4. At 3.3 x $10^8 cm^{-3}$, the calculated sheath resistance is about 40KΩ and the sheath capacitance is about 0.5 pF. Both values differ from what is observed with the admittance probe by a factor of 4, suggesting that the Langmuir probe may be reading low. From the Langmuir probe data, the ion plasma frequency is calculated to be 3.8 MHz. The sheath capacitance appears to become important at frequencies slightly above the ion plasma frequency. No sign of a resonance is seen up to 30 MHz indicating that the electron plasma frequency is at least that high.

It was hoped that it would be possible to carry out admittance measurements up to at least the electron plasma frequency (~ 160MHz), but the difficulty of making a bridge work properly above 30 MHz was greater than anticipated. Besides, the phenomena that occur near the electron plasma frequency are not directly related to sheath properties and hence are outside the scope of this paper. These studies will be continued at a later date if time allows.

To test the feasibility of using the admittance probe as a diagnostic tool, the output voltage at a frequency of 300 KHz was compared to the saturation ion current of the Langmuir probe. Since the output voltage is proportional to the reciprocal of the sheath resistance which is proportional to the density, we expect the output to be a linear function of the Langmuir ion saturation current. Since the probes are of similar dimensions, the sheath expansions should be the same and thus not effect the comparison. Figure 8 shows the result. The experimental points were taken at 100μsec intervals and lie very well along a straight line. The slight deviation at low densities may be due to unequal sheath sizes. An embarrassing feature of this curve is the fact that it does not extrapolate to zero. This may be the result of a baseline shift caused by ground loops in the Langmuir probe circuit (The admittance probe is not sensitive to this.), or by some kind of unbalance in the bridge circuit.

It was hoped that a comparison between the admittance and Langmuir probes could be made at frequencies where the sheath is purely capacitive. The capacitive variation, however, was so slight (10% over a density change of 300%) that this could not be done. The slow variation in capacitance is consistent with the logarithmic variation expected for cylindrical geometry.

Finally a test was made to see how sensitive the admittance probe is to changes in floating potential and to measure its response time. A 100 volt square pulse was applied to the hoops for 10 μsec. Experiments have shown that when this is done, the plasma potential at the center of the machine changes by about 50 volts. With a positive pulse, the admittance increases about a factor of 3 at 1 MHz and returns to equilibrium in about 10 μsec. Negative pulses gave similar results.

We now summarize the results of the above experiments:

1. The parallel RC circuit is a good representation of the plasma sheath, at least up to frequencies ten times the ion plasma frequency.
2. The sheath admittance appears to be about a factor of 4 smaller than predicted theoretically, but this may be due in part to errors in determining the plasma density.
3. The sheath resistance is very nearly proportional to the reciprocal of the density as measured by Langmuir probes, at least in the 10^8 - 10^9 cm^{-3} range.
4. The admittance probe is as good a diagnostic tool as the single Langmuir probe, its major advantage being the fact that it will operate over a very wide range of plasma potentials, provided the potential changes slowly with time.

Octupole Hoop Displacement

An interesting application of the admittance probe is the measurement of the hoop displacement in the octupole when the magnetic field is pulsed. If the admittance probe is placed very near a grounded hoop and the field is pulsed without plasma in the machine, the hoop-to-probe capacitance will change as a function of time if the hoop moves relative to the probe. This varying capacitance will produce a modulated RF signal which can be detected in the usual way.

A more convenient method, if high sensitivity is not required, is to apply the RF signal to the hoop, eliminating the need for a capacitance bridge. The modulated RF signal is then detected and fed into the a.c. input of a high gain scope preamplifier.

If a probe with a large blunt end is brought near the hoop, the geometry is approximately planar and the capacitance is expressed as $C = \frac{\varepsilon_o A}{\ell}$ where A is the area of the probe end and ℓ is hoop/probe separation. If ℓ changes with time, the capacitance changes according to

$$\frac{dC}{dt} = -\frac{\varepsilon_o A}{\ell^2}\frac{d\ell}{dt} .$$

Integrating with respect to time gives

$$\delta C = -\frac{\varepsilon_o A}{\ell^2}\delta\ell = -C\frac{\delta\ell}{\ell} .$$

Since $C \ll 200$ pF, the output of the admittance probe is proportional to C or

$$\delta V = -V\frac{\delta\ell}{\ell} .$$

This relation is true, of course, only for $\delta\ell \ll 1$.

Figure 9 shows the output signal of an admittance probe located about 1 mm above the top outer hoop at the rod port, before and after detection. A careful analysis of the undetected signal shows that $\delta V/V \approx 5\%$. Hence the amplitude of the hoop displacement is apparently about 0.05 mm at the rod port for 2KV on the multipole capacitor bank. The detected signal shows the detailed structure of the displacement $\ell(t)$ which resembles a damped sine wave of period slightly greater than 10μsec, and decay time of about 25 μsec. The lower trace is the time derivative of the multipole field $\frac{dB}{dt}$, obtained from the single turn link around the transformer core. The third picture shows the hoop displacement with 1 KV on the multipole bank. With the present 100 Hz field period, the hoops are driven very near their natural resonant frequency. When the admittance probe is used in this way, two words of caution are in order. The cables from the probe to the scope tend to produce piezoelectric signals when they are shaken. Hence they must be placed in such a way that vibration from the multipole does not affect them. The probe itself is also subject to vibration. At the rod port, this effect is not too serious, but in cases where a probe extends all the way across the vacuum tank, some effort must be made to steady it.

References

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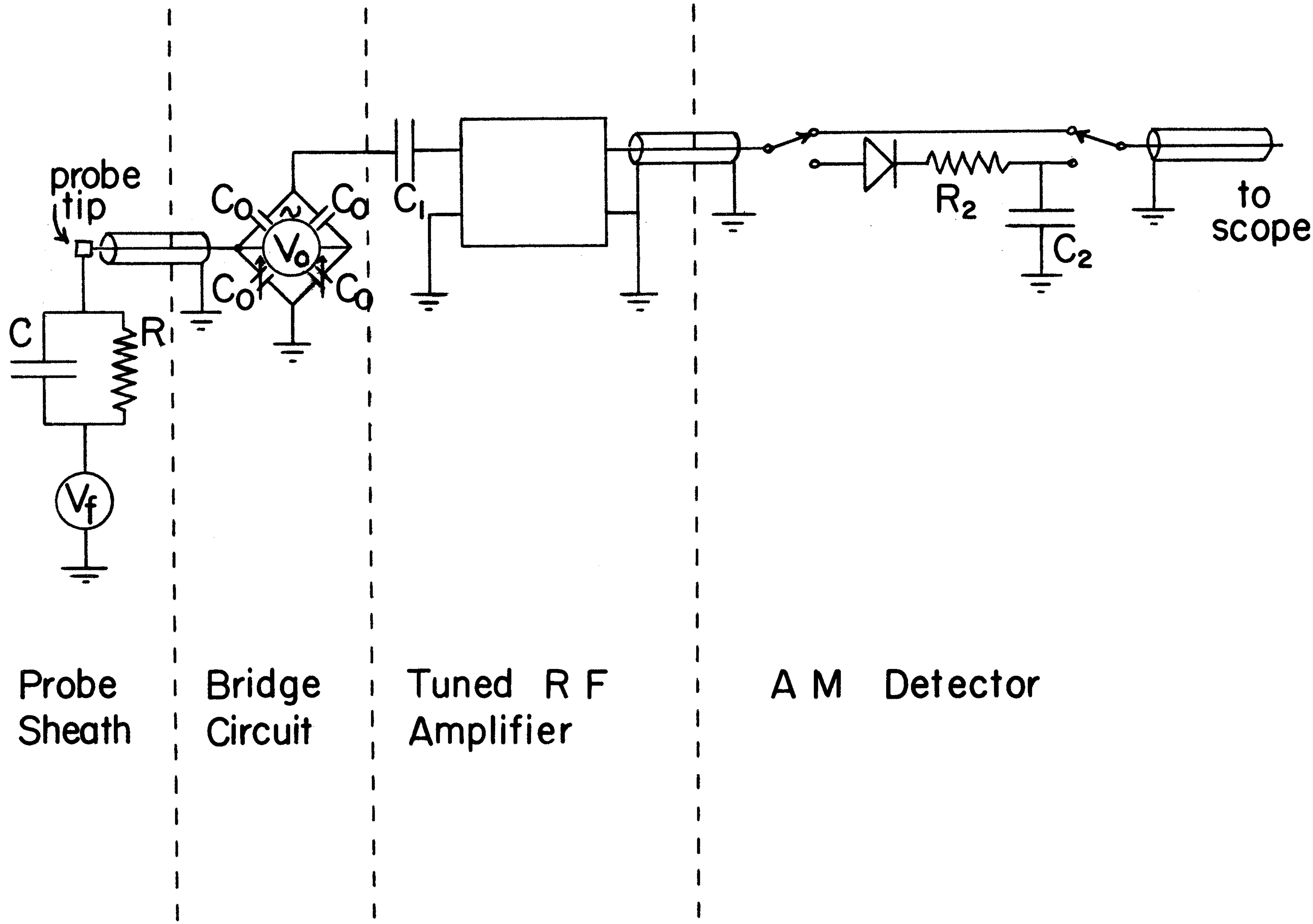
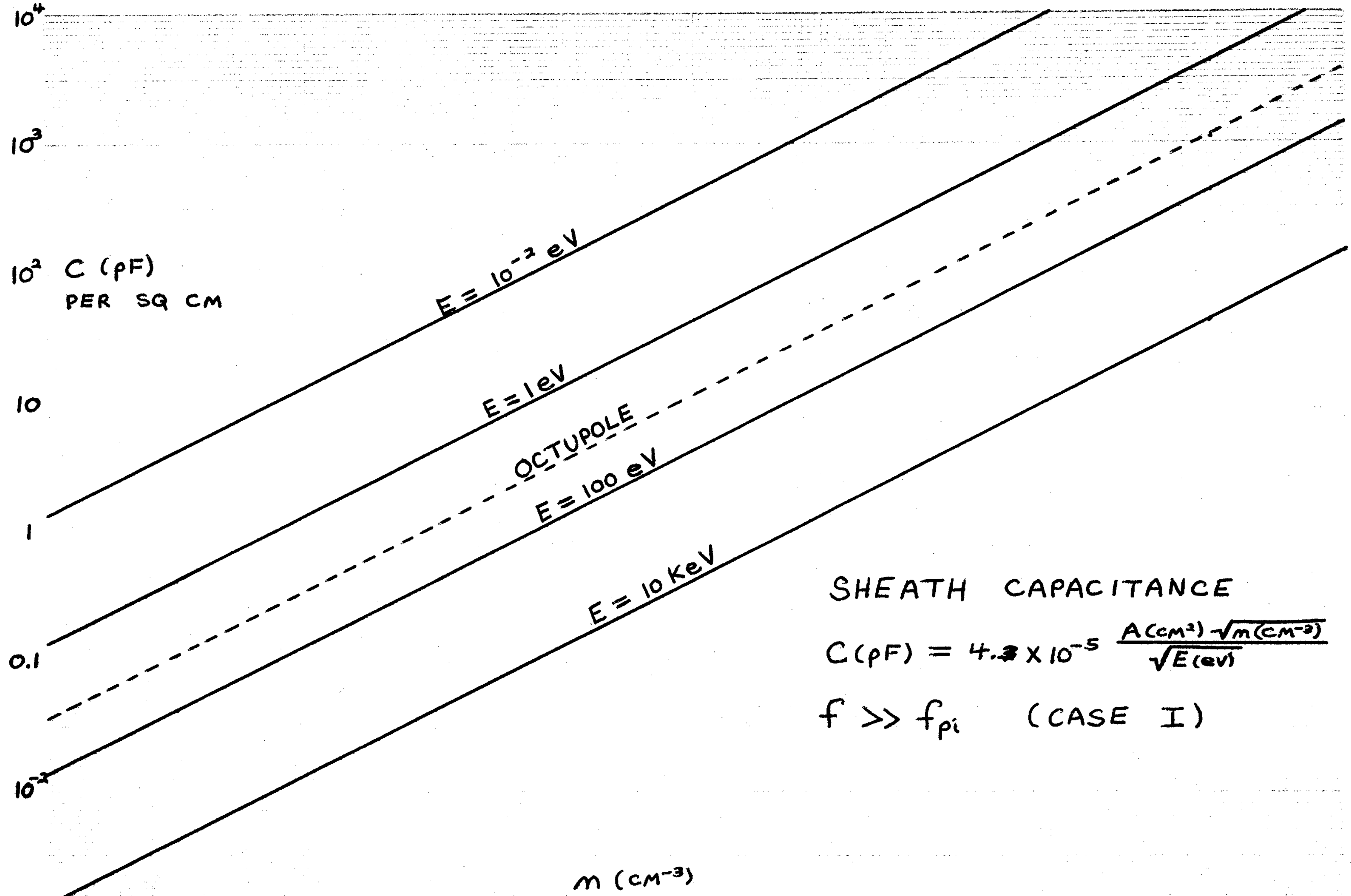
3. F.F. Chen in Plasma Diagnostic Techniques, R.D. Huddlestone and S.L. Leonard, Eds. (Academic Press Inc.; New York, 1965), Chapter 3.

Figures



ADMITTANCE PROBE CIRCUIT

FIG 1



10^4
10^3
10^2 C (pF)
PER SQ CM
10
1
0.1
10^{-2}
10^{-3}
$E = 10^{-2}$ eV
E = 1eV
OCTUPOLE
E = 100 eV
E = 10 keV
SHEATH CAPACITANCE
$C(pF) = 4.3 \times 10^{-5} \frac{A(cm^2)\sqrt{n(cm^{-3})}}{\sqrt{E(eV)}}$
$f \gg f_{pi}$ (CASE I)
n (cm^{-3})
10^7
10^8
10^9
10^{10}
10^{11}
10^{12}
10^{13}
10^{14}
10^{15}
10^{16}
10^{17}

FIG 2

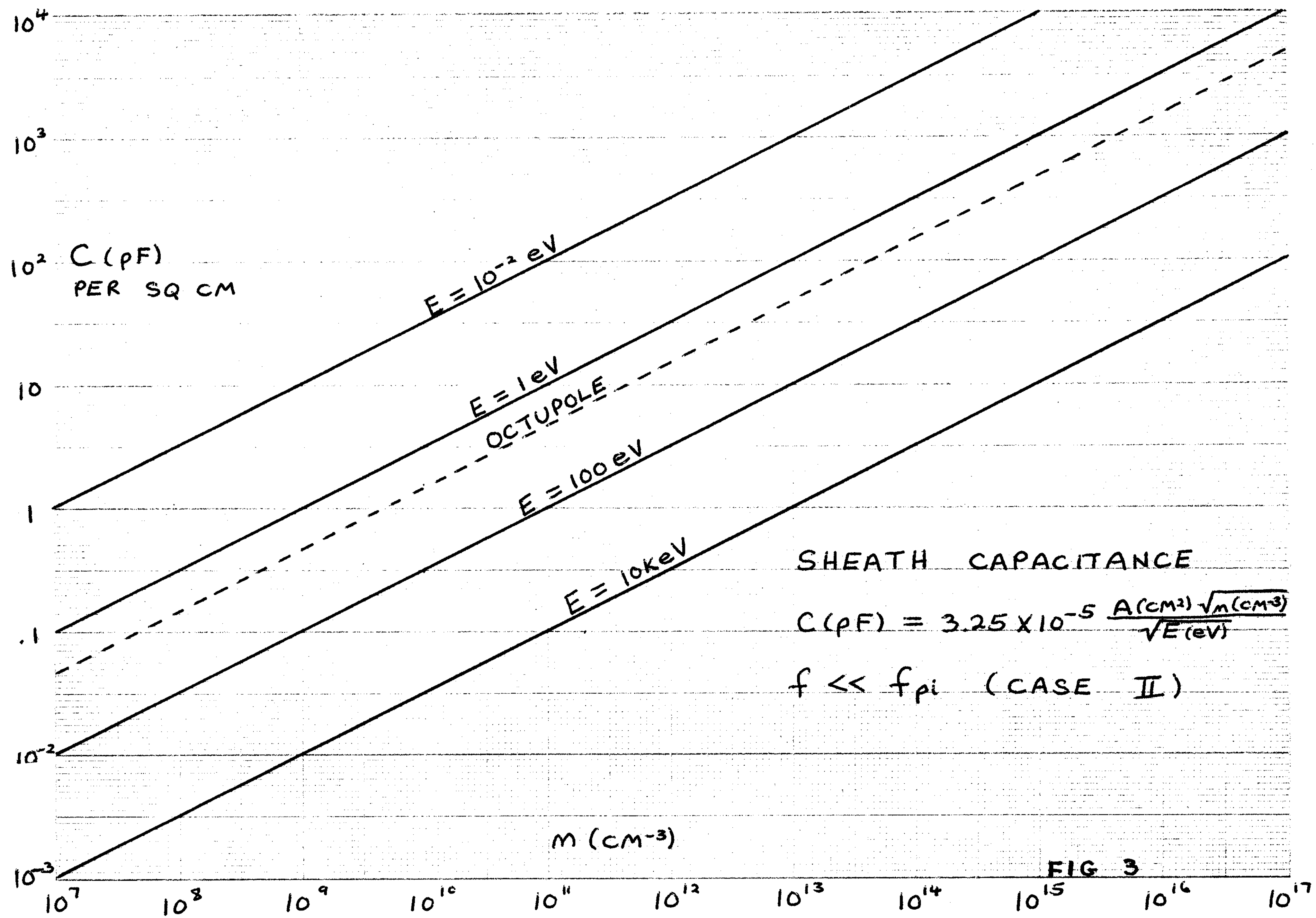
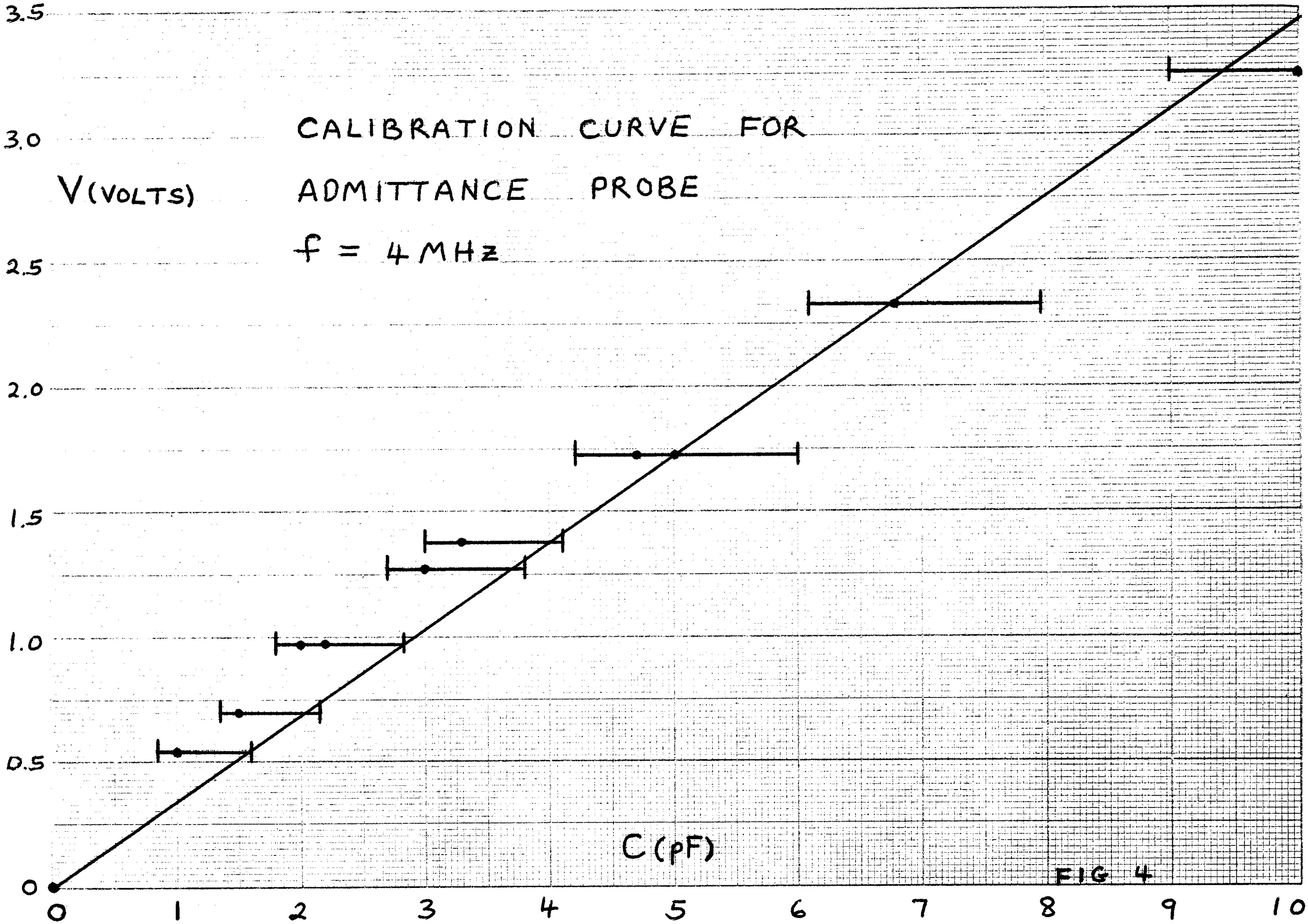
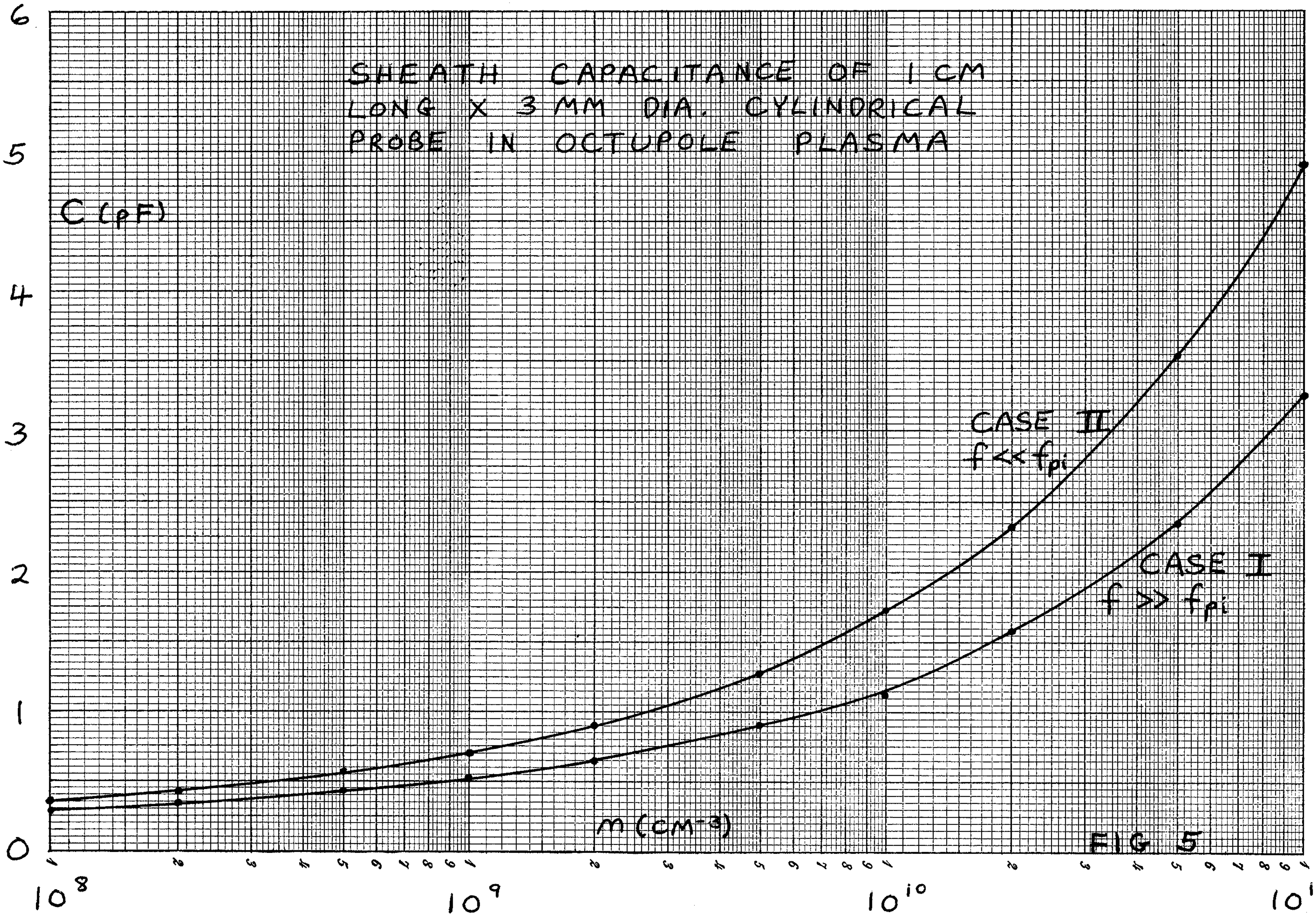

C (pF)
PER SQ CM
10^4
10^3
10^2
10
1
.1
10^{-2}
10^{-3}
E = 10^{-2} eV
E = 1 eV
OCTUPOLE
E = 100 eV
E = 10 keV
SHEATH CAPACITANCE
$C(pF) = 3.25 \times 10^{-5} \frac{A(cm^2)\sqrt{n(cm^{-3})}}{\sqrt{E(eV)}}$
$f \ll f_{pi}$ (CASE II)
n (cm^{-3})
10^7
10^8
10^9
10^{10}
10^{11}
10^{12}
10^{13}
10^{14}
10^{15}
10^{16}
10^{17}

FIG 3



CALIBRATION CURVE FOR
ADMITTANCE PROBE
f = 4 MHz
V (VOLTS)
C (pF)
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
0
1
2
3
4
5
6
7
8
9
10

FIG 4


SHEATH CAPACITANCE OF 1 CM
LONG X 3 MM DIA. CYLINDRICAL
PROBE IN OCTUPOLE PLASMA
C (pF)
n (CM⁻³)
CASE II
f << fpi
CASE I
f >> fpi
0
1
2
3
4
5
6
10⁸
10⁹
10¹⁰
10¹¹
FIG 5

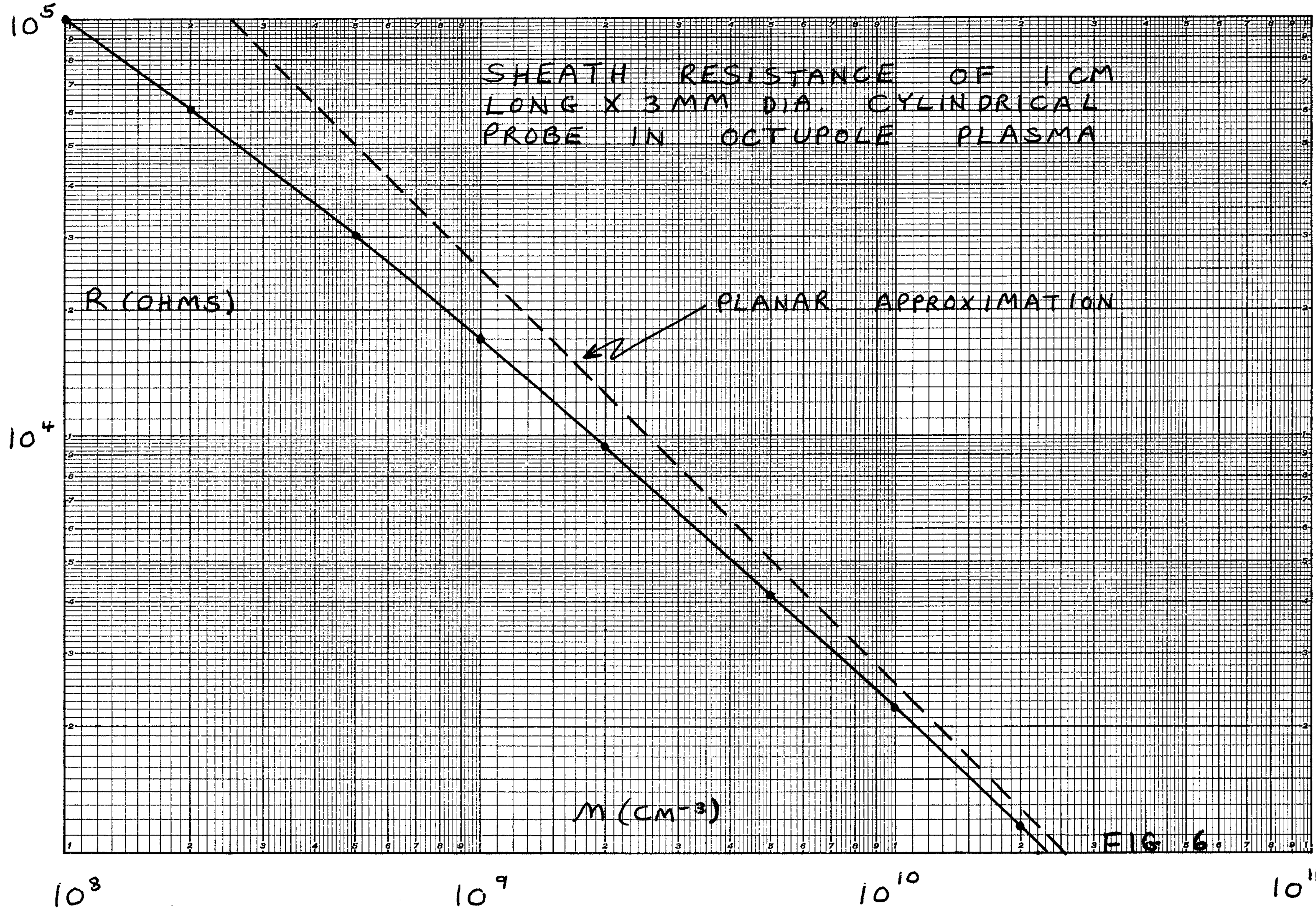

SHEATH RESISTANCE OF 1 CM
LONG X 3MM DIA. CYLINDRICAL
PROBE IN OCTUPOLE PLASMA
PLANAR APPROXIMATION
R (OHMS)
n (CM⁻³)
10⁵
10⁴
10⁸
10⁹
10¹⁰
10¹¹

FIG 6

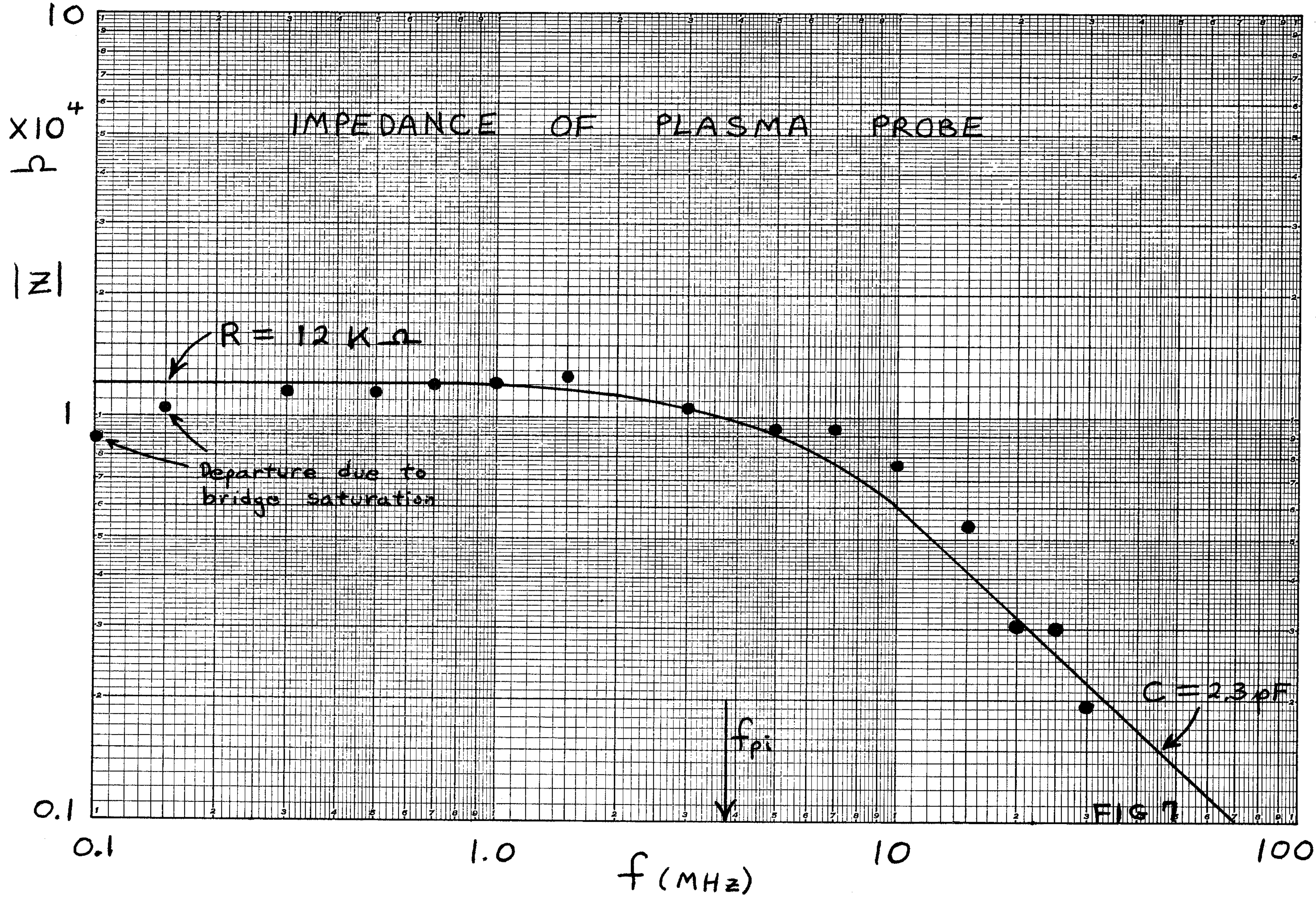

IMPEDANCE OF PLASMA PROBE
10
X10^4 Ω
|Z|
1
0.1
R = 12 K Ω
Departure due to bridge saturation
C = 2.3 pF
f_pi
0.1
1.0
10
100
f (MHz)

FIG 7

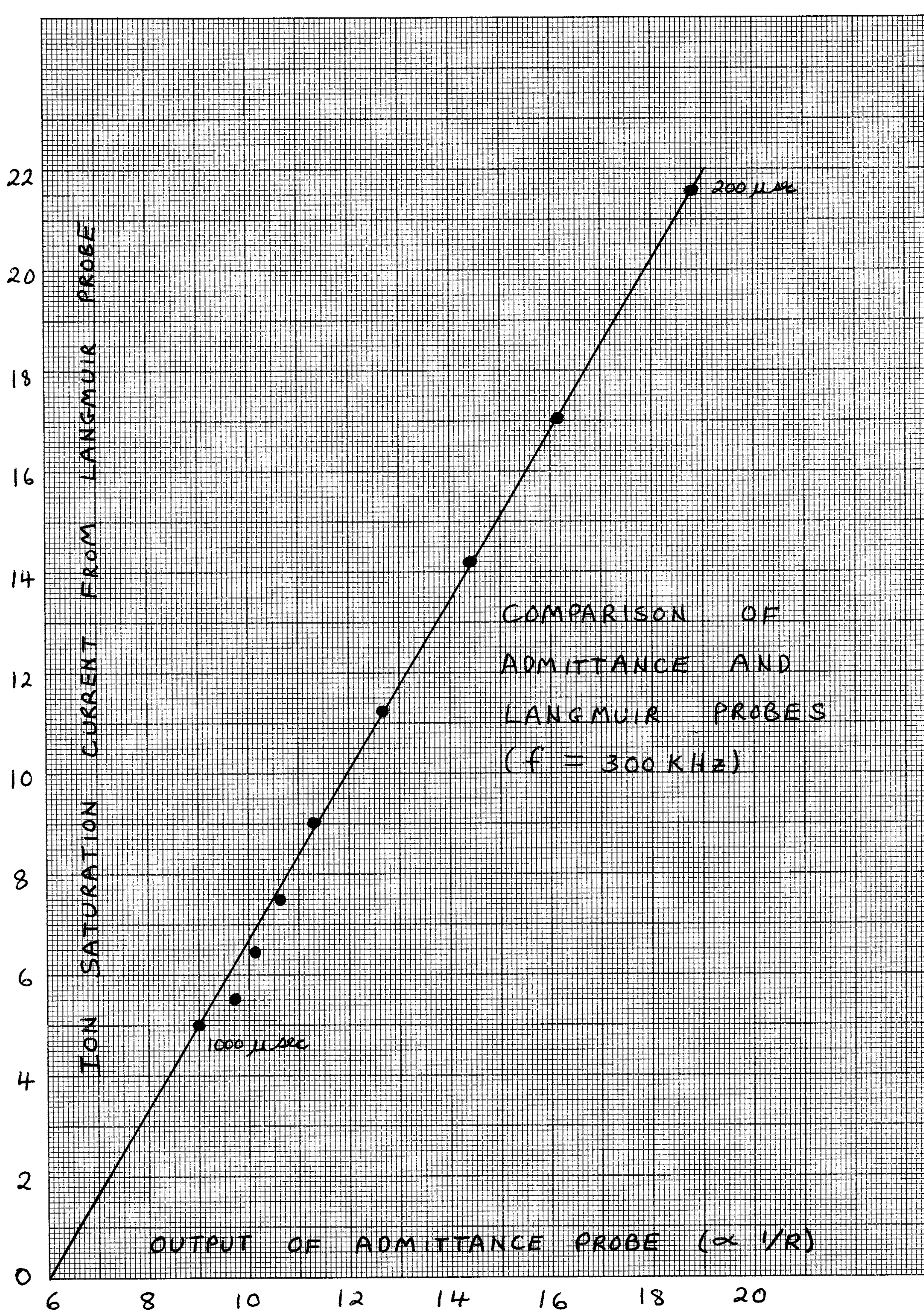
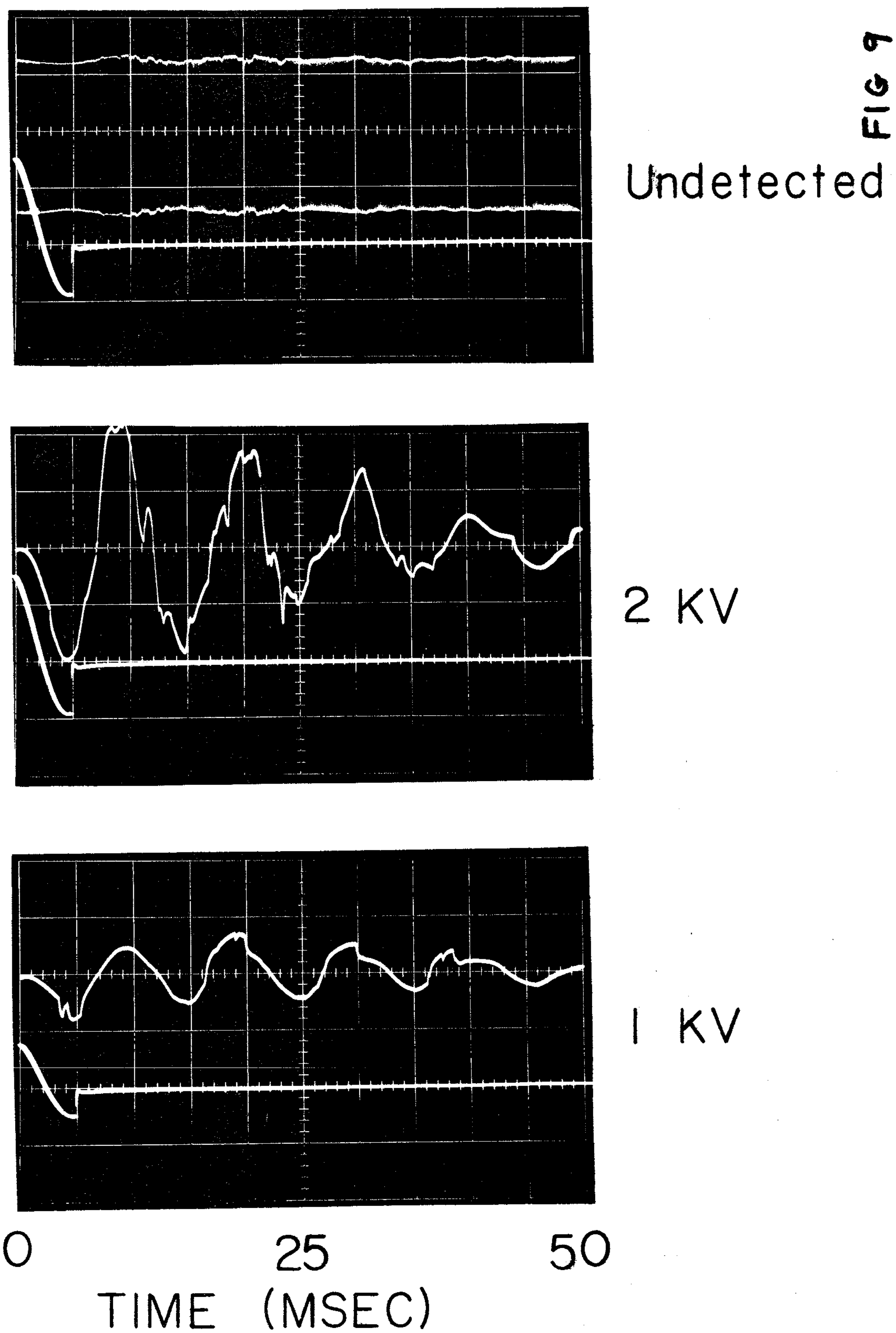

COMPARISON OF
ADMITTANCE AND
LANGMUIR PROBES
(f = 300 KHz)
ION SATURATION CURRENT FROM LANGMUIR PROBE
OUTPUT OF ADMITTANCE PROBE (∝ 1/R)
200 μsec
1000 μsec
0
2
4
6
8
10
12
14
16
18
20
22
6
8
10
12
14
16
18
20

FIG 8



MULTIPOLE HOOP DISPLACEMENT