TOKAPOLE II

EXPECTED PARAMETERS

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With upwards of twenty tokamaks operating around the world, the empirical scaling laws for tokamak plasmas are now fairly well known. In this note, these scaling laws will be used to predict the plasma parameters that are expected in Tokapole II at a nominal 3.5 kG toroidal field and at the full 10 kG for which the machine was designed. The presence of the internal rings are assumed to neither enhance nor degrade the performance of the device.

The symbols used and their units are given below:

B_T = toroidal field strength on minor axis (kG)

R = major radius of toroid (cm)

a = minor radius of plasma (cm)

n = volume-averaged electron density (cm^{-3})

T_e = peak electron temperatures (eV)

T_i = peak ion temperature (eV)

I = toroidal plasma current (kA)

V = single turn loop voltage (volts)

τ_E = energy confinement time (msec)

β = plasma pressure/total magnetic field pressure.

Two widely accepted empirical scaling laws for tokamaks are the following:

$$n \propto B_T/R \tag{1}$$

and

$$\tau_E \propto na^2 \quad . \tag{2}$$

Electron energy balance requires that the input ohmic heating power equal the electron energy loss rate:

$$IV \propto \frac{nT_e a^2 R}{\tau_E} \quad . \tag{3}$$

Classical (Spitzer) resistivity requires $(Z_{eff} = \text{const.})$:

$$V/I \propto R/a^2 T_e^{3/2} \tag{4}$$

Stability against kink modes requires that the Kruskal-Shafranov limit be satisfied (q = const.):

$$I/a^2 \propto B_T/R \quad . \tag{5}$$

Finally, ion energy balance (assuming $T_i << T_e$ and $\tau_{Ei} \simeq \tau_{Ee}$) requires

$$n/T_e^{1/2} \propto T_i/\tau_E \tag{6}$$

With a bit of algebra, equations 1-6 can be written in a form in which the plasma quantities appear on the left and the machine quantities appear on the right:

$$n \propto B_T/R \tag{7}$$

$$\tau_E \propto B_T a^2/R \tag{8}$$

$$T_e \propto \left(B_T a/R\right)^{4/5} \tag{9}$$

$$T_i \propto \left(B_T a/R\right)^{8/5} \tag{10}$$

$$I \propto B_T a^2/R \tag{11}$$

$$V \propto B_T^{-1/5} (R/a)^{6/5} \tag{12}$$

Note that in every case the plasma parameters are improved by increasing the toroidal field and decreasing the aspect ratio (R/a). Note also that energy confinement time is directly proportional to plasma current, and that the loop voltage is essentially constant for all tokamaks of similar aspect ratio (typically 1-5 volts).

In order to make predictions, it is necessary to have constants of proportionality for equations 7-12. To do this, the parameters of eight tokamaks described in WASH-1295 were plotted in figures 1-5, and fitted to straight lines of 45° slope. The various machines and their respective parameters are listed below:

	DEVICE	B_T	I	R	a	n	T_e	T_i	τ_E
1.	S T	40 kG	70 kA	109 cm	13 cm	4×10^{13} cm^{-3}	2500 eV	600 eV	10 msec
2.	Ormak	18	120	80	23	3×10^{13}	700	300	11
3.	A T C	15	60	90	17	1.5×10^{13}	1000	250	5
4.	Doublet II	8	130	59	12×15	1.3×10^{13}	550	250	2
5.	T F R	40	200	98	20	2×10^{13}	2500	-	20
6.	T M - 3	40	70	40	8	7×10^{13}	500	350	3-4
7.	T - 4	40	120	100	17	4×10^{13}	1500	700	16
8.	T - 6	10	60	70	25	1×10^{13}	300	200	1

The best fit straight lines give the following equalities, valid to about a factor of two:

$$n = 10^{14}\, B_T/R \tag{13}$$

$$\tau_E = 0.1\, B_T a^2/R \tag{14}$$

$$T_e = 300(B_T a/R)^{0.8} \tag{15}$$

$$T_i = 30(B_T a/R)^{1.6} \tag{16}$$

$$I = B_T a^2/R \quad . \tag{17}$$

Using these equations, we can make the following predictions for Tokapole II:

B_T	3.5 kG	10 kG
n	7×10^{12} cm^{-3}	2×10^{13} cm^{-3}
τ_E	1.3 msec	3.6 msec
T_e	220 eV	660 eV
T_i	16 eV	150 eV
I	13 kA	36 kA

In the above equations, we have taken $R = 50$ cm and $a = 13.5$ cm (the radius of a circle with the same area as a 24×24 cm square). The above predictions ignore any advantages (or disadvantages) of having internal rings, and do not consider the possibility of improving performance by gas puffing, auxiliary heating, feedback control of the equilibrium, or any number of other tricks which might be developed in the future. The regions of projected operation of Tokapole II are indicated by the boxes in figures 1-5.

Of particular interest is the value of toroidal beta, given by

$$\beta = \frac{1.21}{RB_T}\left(\frac{B_T a}{R}\right)^{0.8} \quad . \tag{18}$$

For Tokapole II we predict β = 0.66% at 3.5 kG and 0.54% at 10 kG. We might hope to test β-limits (currently believed to be $\sim$10%) by auxiliary ICRH. Assuming τ_E to remain constant in the presence of ICRH, the ion temperature that we might hope to obtain as a function of the ICRH power P (in watts) is given by

$$T_i = 0.317\ P/R\ . \tag{19}$$

With P = 1 MW, T_i is 6340 eV, independent of B_T, and the beta is 14.6% at 3.5 kG and 5.1% at 10 kG. In addition to testing beta limits, the proposed experiment offers the possibility of a definitive test of energy confinement vs. ion temperature since T_i can be varied by more than a factor of a hundred instead of the factor of two that is typical of previous and other proposed ICRH experiments.

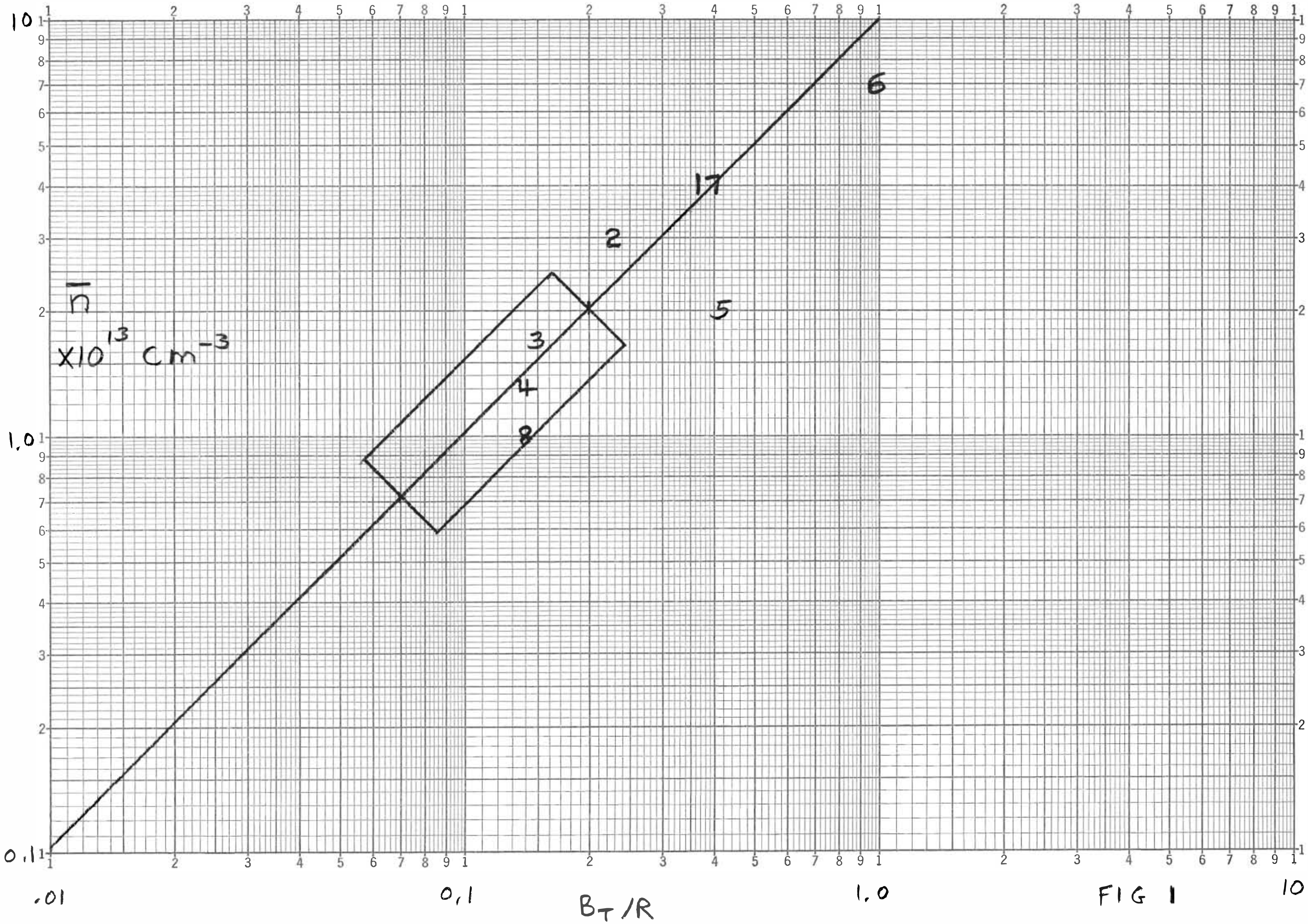

FIG 1

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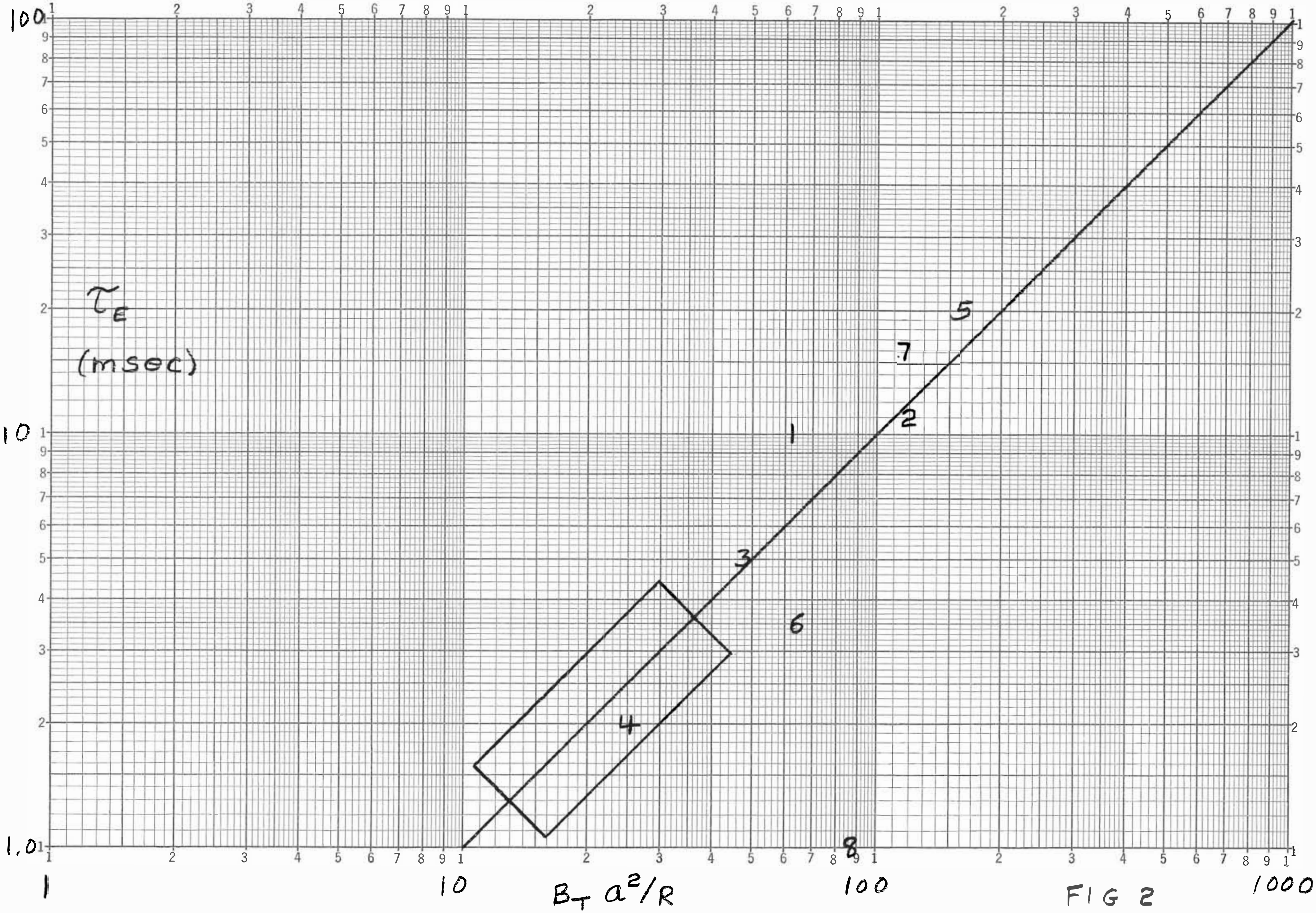

FIG 2

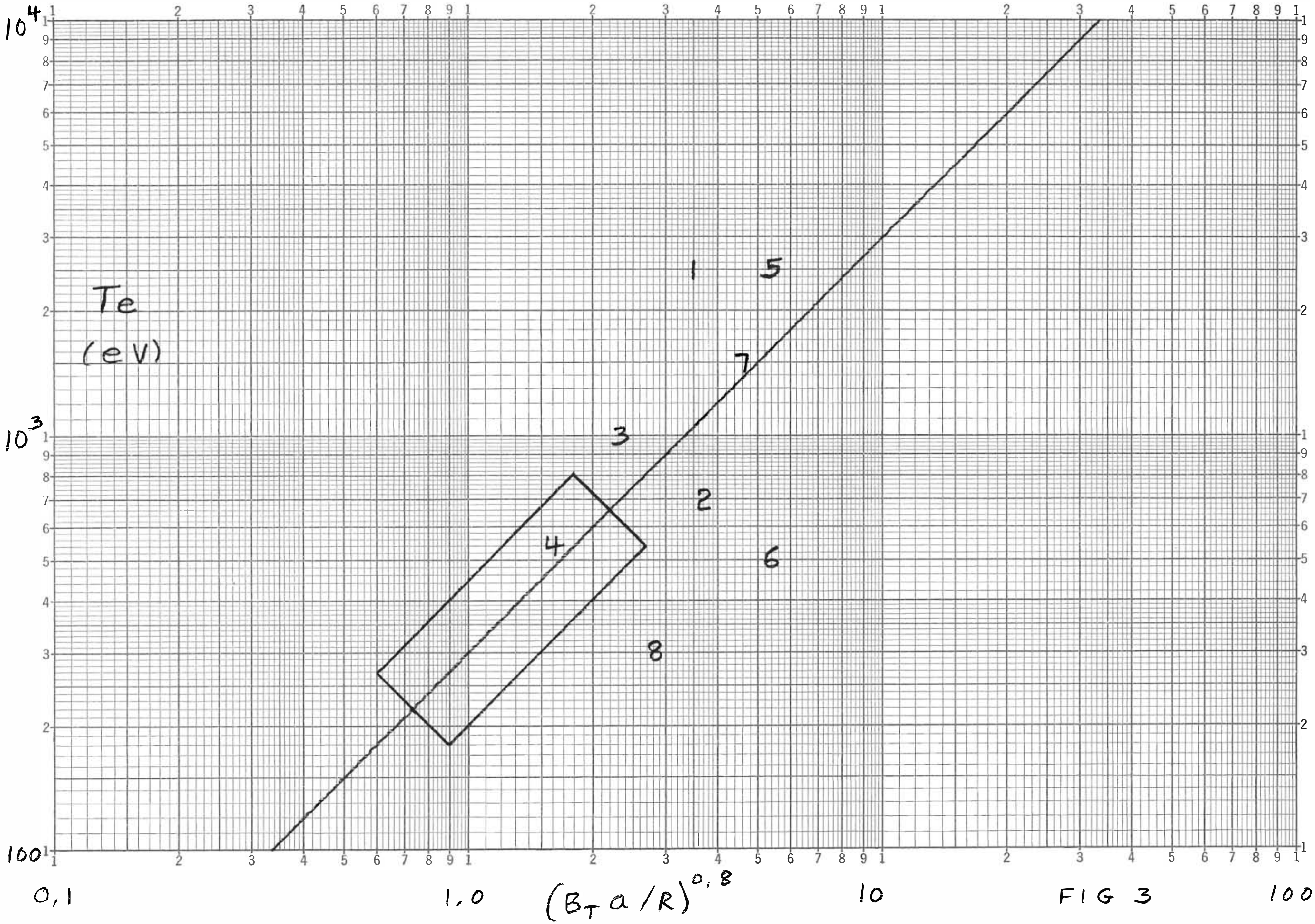

FIG 3

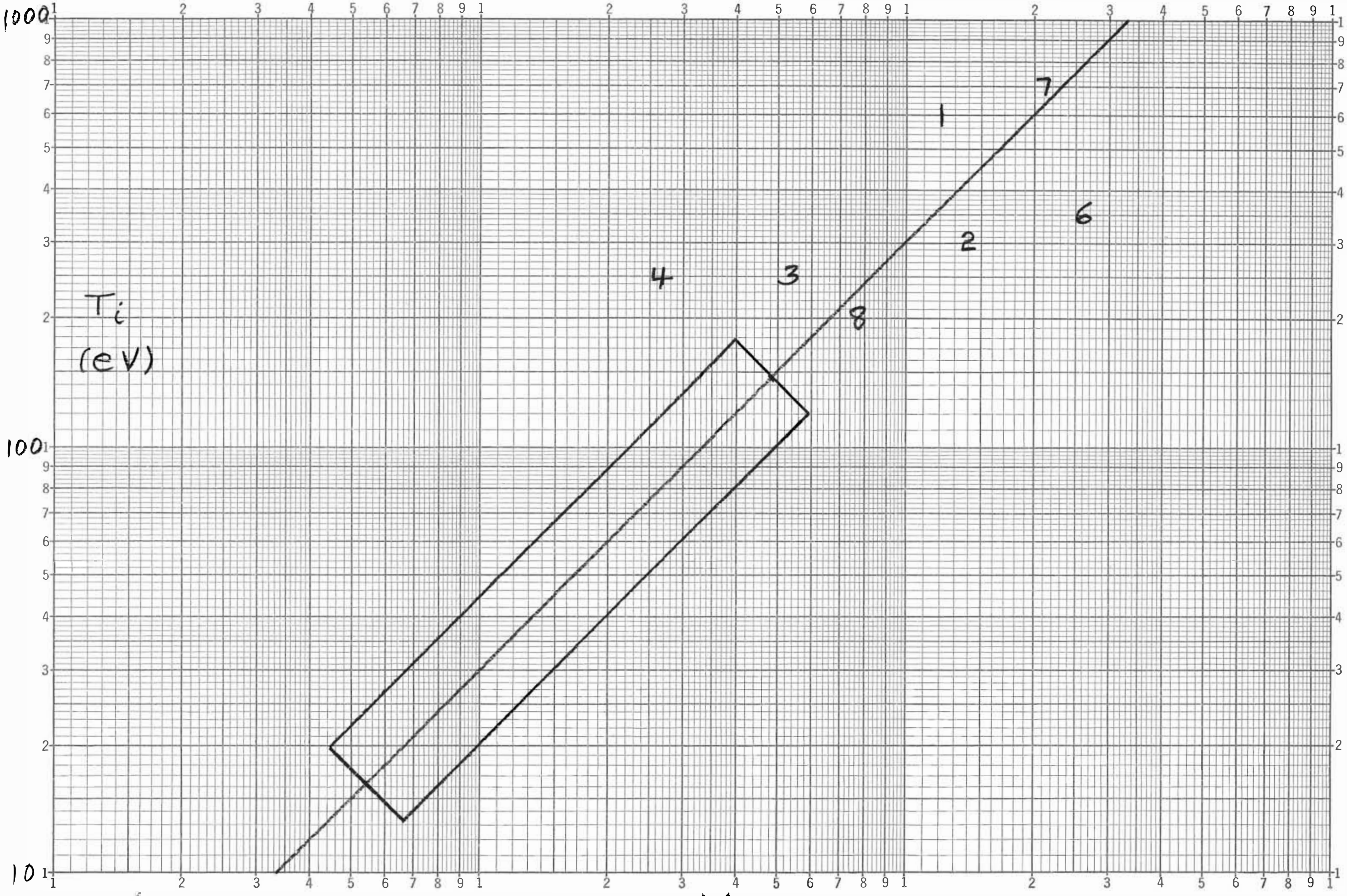

FIG 4

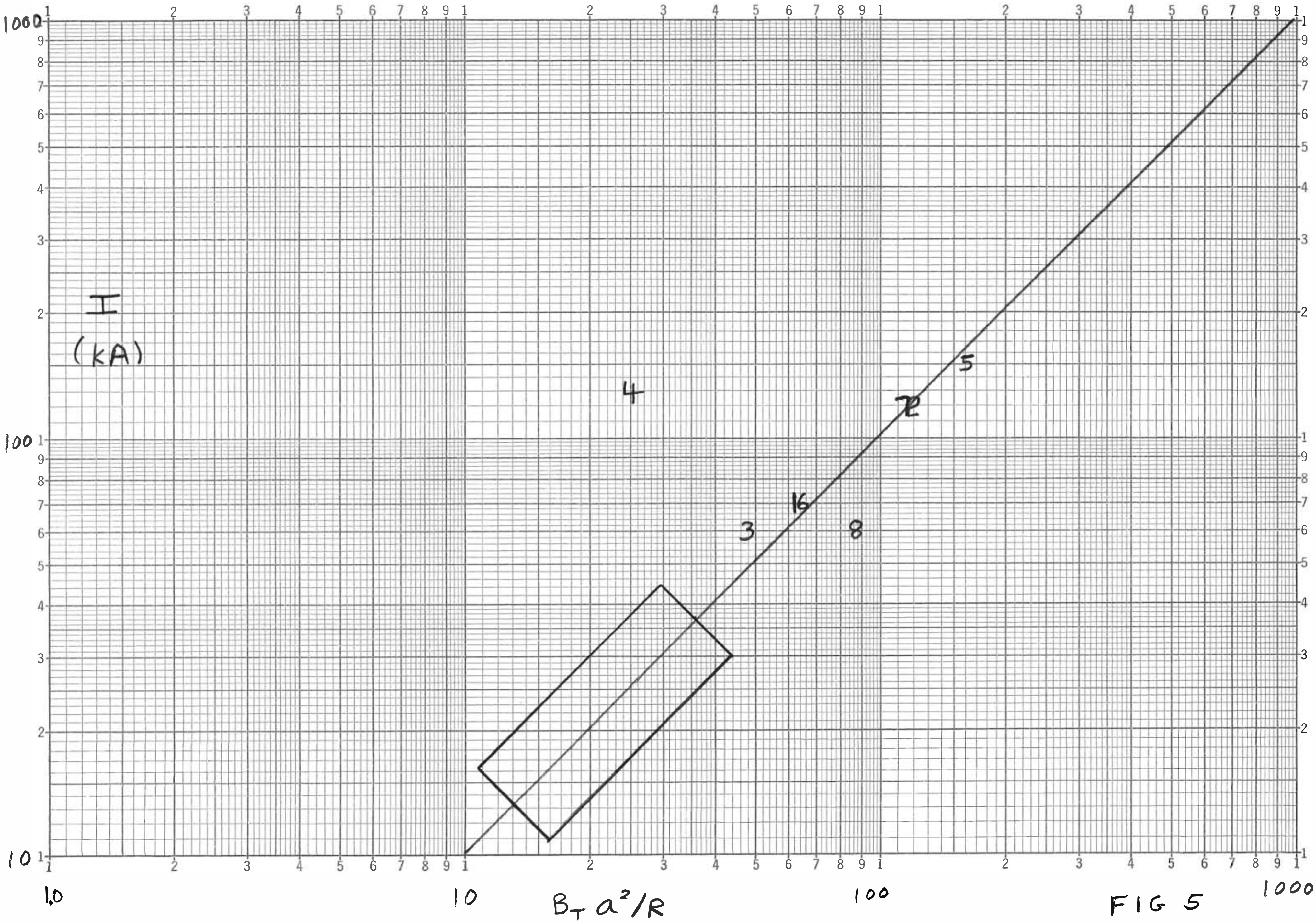

FIG 5