MODEL OF PLASMA CURRENT IN THE TOKAPOLE

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The magnitude of the toroidal plasma current in the Tokapole discharge has in the past been inferred from the poloidal gap voltage, V_{PG}, and primary current, I_{Pr}, via a circuit model of the machine (Ref. 1). It is assumed that the plasma current is localized about the minor axis. The justification and limitation of this model are given here.

If the transformer core has infinite permeability, Ampere's circuital law (applied along a path through the core) states that the core links zero total current or

$$nI_{Pr} + I_H + I_{Pl} = 0 \tag{1}$$

where n is the number of primary turns and I_{Pr}, I_H, and I_{Pl} are the primary current, total hoop current and plasma current, respectively. If the hoops are perfectly conducting, the time varying magnetic flux linking them must sum to zero, i.e.,

$$\dot{\phi}_C + \dot{\phi}_H + \dot{\phi}_{HP} = 0 \tag{2}$$

where ϕ_C, ϕ_H and ϕ_{HP} are the core flux, flux created by the hoop and flux created by the plasma that links the hoop, respectively. We may write $\dot{\phi}_C = V_{PG}$ and $\dot{\phi}_H = L_H\dot{I}_H$ where L_H is the total hoop inductance determined with plasma image current in hoops included but hoop image current in plasma set to zero. The flux linkage between the hoops and plasma can be described by a mutual inductance, M, such that $\phi_{HP} = MI_{Pl}$ and $\phi_{PH} = MI_H$ where ϕ_{PH} is the hoop flux that links the plasma. It is known from computer generated vacuum flux plots that the total flux in the machine, without plasma, is about evenly divided between private and common flux. Therefore, half of the hoop-generated flux links the plasma filament, or $\phi_{PH} = \frac{L_H I_H}{2}$ and $M = L_H/2$. We may now rewrite Eq. (2) as

$$V_{PG} - L_H \dot{I}_H - \frac{L_H \dot{I}_{Pl}}{2} = 0 \tag{3}$$

Eliminating I_H using Eq. (1) yields

$$V_{PG} = L_H n \dot{I}_{Pr} - \frac{L_H}{2} \dot{I}_{Pl} \tag{4}$$

Therefore, from the measurable quantities V_{PG} and I_{Pr} one can infer I_{Pl}. Eq. (4) is consistent with the circuit of Fig. (1), given by Sprott (Ref.1). For a non-filamentary current profile the plasma-hoop coupling changes, and the above model is inaccurate. It is pointed out in Ref. (2) that for 1/R current profile Eq. (4) still agrees with the results of computer flux calculations to within $\sim$ 15%.

However, if the hoop current is measured, Eq. (1) alone yields the plasma current <u>independent</u> of its spatial profile. Also, it is clear from Eq. (3) that, for a fixed V_{PG}, as the plasma current changes by δI_{Pl}, the hoop current changes by $-\frac{\delta I_{Pl}}{2}$. A measurement of the distribution of this image current among the four hoops may yield information about the plasma position. To these ends, Rogowski loops have been installed around the hoops to measure the hoop current.

Pursuing the localized plasma model further, we may relate the plasma loop voltage, V_{Pl}, to the flux linking the loop as

$$V_{Pl} = I_{pl}R_{Pl} = V_{PG} - L_{Pl}\dot{I}_{Pl} - \frac{L_H}{2}\dot{I}_H \tag{5}$$

Where R_{Pl} and L_{Pl} are the plasma resistance and inductance (evaluated with hoop image current in plasma included, but plasma image current in hoops absent) respectively. To represent Eqs. (1), (4) and (5) by one circuit requires extending the circuit of Fig. 1 to that of Fig. 2. At this point, perhaps the circuit loses some intuitive appeal because of the presence of the inductor, $-L_H/2$, in series with the plasma elements. This is not too surprising since the plasma filament case involves two transformer secondaries (hoops and plasma) mutually coupled to each other. Reduction of a circuit is not straightforward. The last term in Eq. (5) is $-\frac{L_H}{2}\dot{I}_H = \frac{L_H}{2}(\dot{I}_{pr} - \dot{I}_{Pl})$. Therefore in Fig. 2 the $-\frac{L_H}{2}$ inductance represents part of the mutual coupling.

Recently, the plasma loop voltage, V_{Pl}, was measured experimentally. Using this measurement in conjunction with the above model, one can infer the value of L_{Pl}. V_{Pl} is measured with a loop of wire inserted to the minor axis, as shown in Fig. 3. Invoking axisymmetry, the probe measures the flux, ϕ_{Pr}, between the minor axis and outside wall. The plasma loop links the flux contained within the minor axis or $(\phi_C - \phi_{probe})$ since the outside wall links only the core flux. Then in terms of voltage

$$V_{Pl} = V_{PG} - V_{probe} \tag{6}$$

Where $V_{probe} = \dot{\phi}_{probe}$. Furthermore, the hoop current may be eliminated from Eqs. (3) and (5) to yield

$$V_{Pl} - \frac{V_{PG}}{2} = (-L_{Pl} + \frac{L_H}{4})\dot{I}_{Pl} \tag{7}$$

Therefore, after measuring V_{P1} through Eq. (6), L_{P1} may be obtained from Eq. (7) if I_{P1} is measured. It is seen from Eq. (7) that V_{P1}, the voltage around the minor axis, deviates from its vacuum value, $\frac{V_{PG}}{2}$, from two effects--(1) The plasma self-flux or back EMF ($L_{P1}\dot{I}_{P1}$) and (2) the deviation of the coupling term $\dot{\phi}_{PH}$ by $\delta\dot{\phi}_{PH} = \frac{L_H}{2}\delta\dot{I}_H = \frac{L_H}{4}\dot{I}_{P1}$ due to plasma current imaging on the hoops.

Measurement indicates (using Eq. (6)) that V_{P1} can differ significantly from its vacuum value ($V_{PG}/2$). It is seen experimentally that V_{P1} is equal to its vacuum value when the plasma current within the hoops, as measured by a Rogowski loop, peaks (i.e. $\dot{I}_{P1} = 0$). This agrees with Eq. (7). The plasma current flowing outside of hoops has a low inductance and therefore contributes little to the argument. L_{P1} is found to be ~ 0.8 μH. The hoop inductance L_H in the absence of plasma is 0.242 μH.

In summary, the above simplified description facilitates qualitative interpretations of experimental results. The discussion is approximate (except Eq. (2)) since it is assumed that the plasma current is well localized. Transformer losses and time varying inductances and resistances could be incorporated into the model.

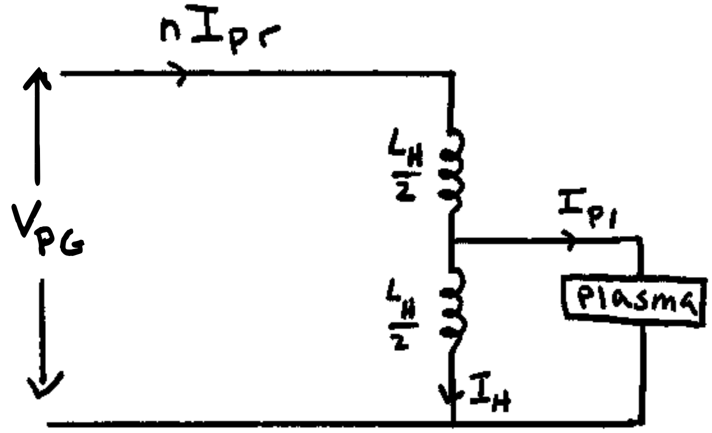

Fig. 1: Circuit describing Eq. (4) and given in Ref. 1.

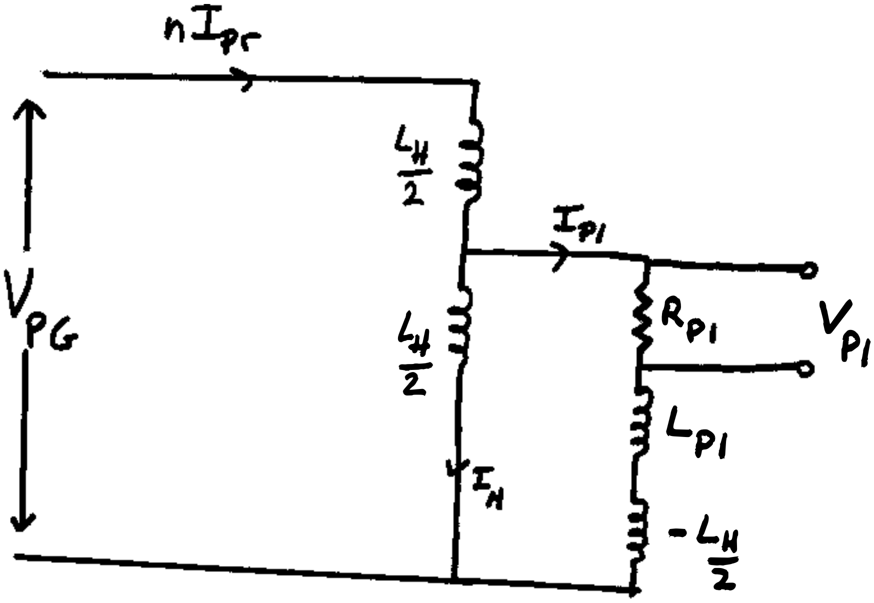

Fig. 2: Circuit describing Eqs. (4) and (5).

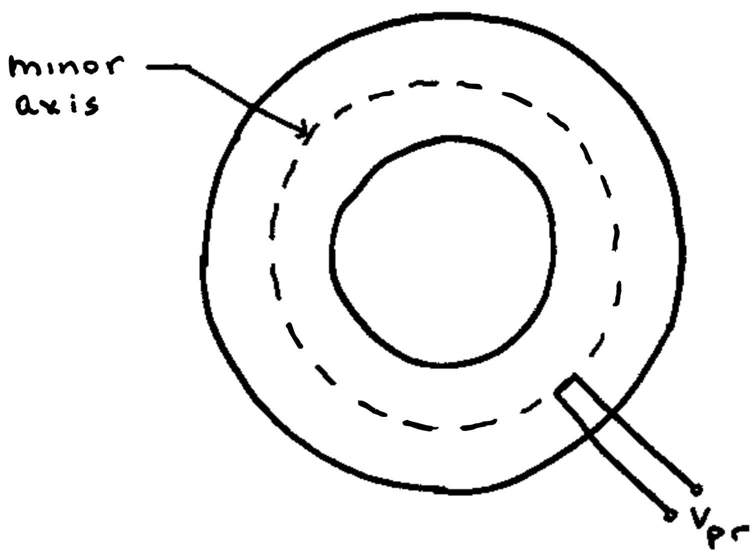

Fig. 3: Loop voltage probe viewed from top of machine.

REFERENCES

1. J.C. Sprott, PLP 712.

2. J.C. Sprott and T. Lovell, PLP 744.