POSSIBILITIES FOR PROLONGING THE TOKAPOLE II DISCHARGE

J.C. Sprott

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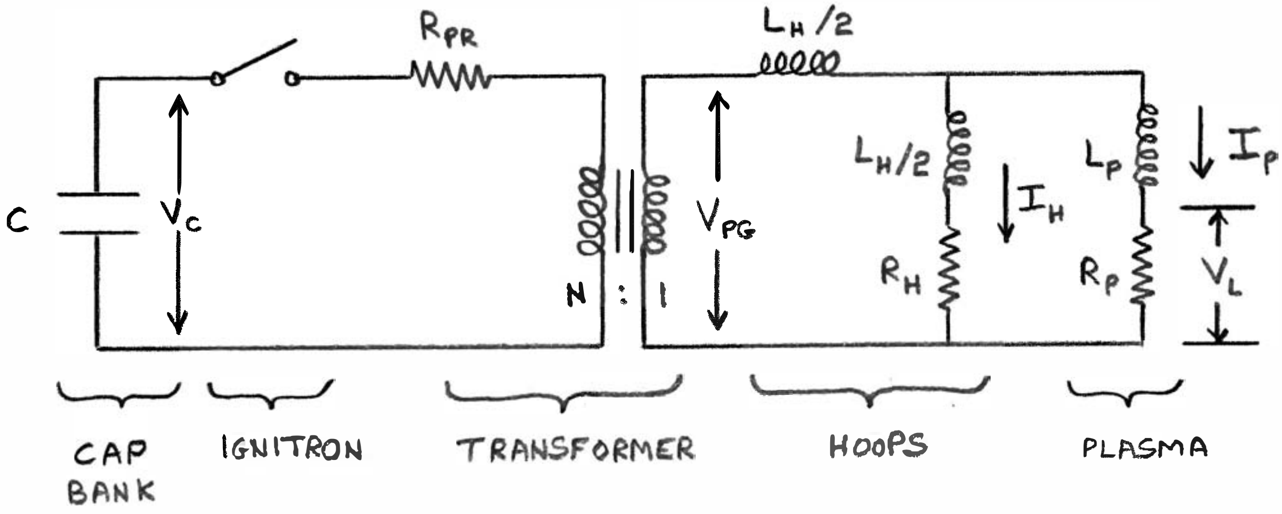
It is desirable to extend the duration of the Tokapole II discharge current pulse for a number of reasons: (1) to facilitate the study of slowly growing instabilities, (2) to insure that equilibrium is achieved with wall refluxed neutrals, and (3) to achieve higher temperatures and poloidal beta. In this note, an extension of the circuit model of PLP 756 will be used to determine what changes to the external poloidal field circuitry would be required to prolong the plasma current pulse in Tokapole II.

The circuit model used is as follows



The values assumed for the circuit components are:

Capacitance of B_P bank: $C = 0.0072$ F

Primary resistance: $R_{PR} = 0.0485\ \Omega$

Turns ratio: $N = 40$

Hoop inductance: $L_H = 0.219 + 0.06834\,(1 - e^{-396t})\ \mu H$

Hoop resistance: $R_H = 7.8 + 16.9\, e^{-396t}$ μΩ

Plasma inductance: $L_P = 0.8 - 0.5\, L_H$ μH

Plasma resistance: $R_P = 12.26\, Z_{eff}/a^2\, T_e^{3/2}$ ohms

The primary resistance and hoop inductance at t = 0 were taken from PLP 744. The hoop resistance and inductance correction are approximations to the calculation of Spencer (PLP 771 and subsequent private communications), where t is the time in seconds after the ignition fires. The plasma inductance was measured by Miller (PLP 756). The plasma resistance comes from Spitzer, where Z_{eff} is taken equal to one, a is the distance in cm from the geometric axis to the field nulls, and T_e is the plasma electron temperature in eV. The quantity a is determined from the ratio of plasma current I_P to hoop current I_H by

$$a = 21\, (I_P/I_H)^{1/4} \text{ cm}$$

The electrical circuit equations which relate the variables, V_C, I_H, and I_P are:

$$C \frac{dV_C}{dt} = - \frac{I_H + I_P}{N}$$

$$\frac{L_H}{2} \frac{d(I_H + I_P)}{dt} + \frac{L_H}{2} \frac{dI_H}{dt} + R_H I_H = V_{PG}$$

$$\frac{L_H}{2} \frac{d(I_H + I_P)}{dt} + L_P \frac{dI_P}{dt} + R_P I_P = V_{PG}$$

where

$$V_{PG} = \frac{V_C}{N} - \frac{R_{PR}\,(I_H + I_P)}{N^2}$$

With a bit of algebra, these equations can be put in a more convenient form:

$$\frac{dV_C}{dt} = - \frac{I_H + I_P}{NC} \tag{1}$$

$$\frac{dI_H}{dt} = \frac{2V_{PG}\,L_P/L_H - R_H\,(2L_P/L_H + 1)\,I_H + R_P\,I_P}{2L_P + \frac{1}{2}\,L_H} \tag{2}$$

$$\frac{dI_P}{dt} = \frac{V_{PG} + R_H\,I_H - 2R_P\,I_P}{2L_P + \frac{1}{2}\,L_H} \tag{3}$$

The set of non-linear differential equations (1)-(3) can in principle be solved provided $T_e(t)$ is known. Since $T_e(t)$ is not known a priori, it is better to calculate it self-consistently from the ohmic heating input power and some reasonable model of the energy loss rate:

$$\frac{d}{dt}\,(1.5\text{ ne } T_e\,\upsilon) = I_P{}^2\,R_P - 1.5\text{ ne } T_e\,\upsilon/\tau_E \tag{4}$$

In equation (4) the density n is taken as a constant equal to 1×10^{13} cm^{-3}, and the volume υ is taken as 6×10^5 cm^3. The energy confinement time τ_E is taken as an adjustable parameter whose value is time-independent. These assumptions are reasonable because tokamak densities tend to be determined only by the toroidal field strength, and energy confinement times are determined almost entirely by the density (see PLP 753). Late gas puffing may be required to keep the density constant throughout the discharge. Note also that the loss term in equation (4) contains the quantity n/τ_E which presumably remains approximately constant even if n varies somewhat.

The set of equations (1)-(4) has been solved numerically using program TOKII listed in Appendix A. The initial conditions are normally taken as

$V_C = 2000$ volts

$I_H = 0$

$I_P = 0$

$T_e = 5$ eV

The resulting plasma current as a function of time for various values of τ_E is shown in figure 1. Also shown is the result of a typical experimental pulse with the divertor baffle inserted to reduce the current which flows in the common flux region. The experiment is in reasonable agreement with the model provided the energy confinement time is the order of a few hundred μsec. The discrepancy may result from some combination of the following: (1) an increase in the plasma inductance as a function of time, (2) a decrease in energy confinement time as a function of time, (3) a rise in plasma density as a function of time, (4) a rise in Z_{eff} as a function of time, or (5) a shift in the current distribution toward the rings as a function of time. Note that the current waveform is a sensitive function of confinement time for τ_E in the range below about 400 μsec, but for high τ_E, the plasma inductance dominates, and the plasma and hoops form an inductive current divider so that the plasma current follows the hoop current according to

$$I_P = L_H I_H/2L_P \cong 0.20 I_H$$

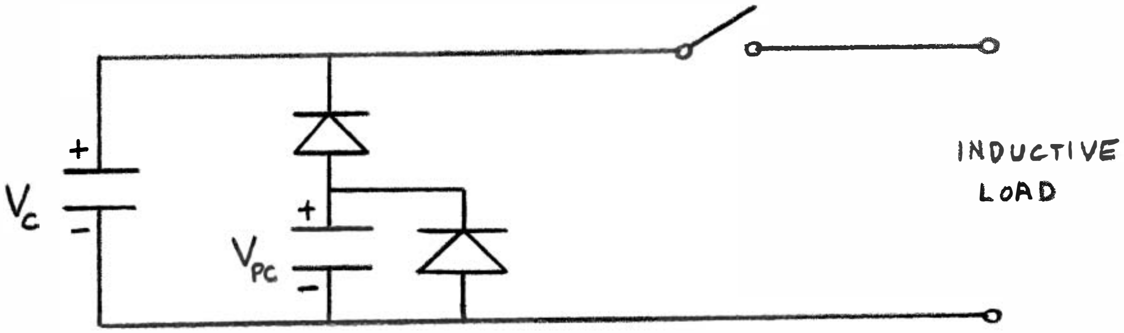
This result is further exhibited in figure 2 where the amp-seconds ($\int I_P dt$) are plotted as a function of τ_E.

The amp-seconds is apparently a good figure of merit for low τ_E, but not for high τ_E. As a further point of interest, the electron temperature as a function of time is plotted in figure 3 for various values of assumed energy confinement time. Note that the electron temperature is volume-averaged over the entire machine. The central current channel temperature is undoubtedly higher, but then so probably is Z_{eff}, and so the current waveforms are probably reasonable.

In order to extend the discharge time, several solutions come to mind. Perhaps the simplest is to increase the primary resistance and raise the capacitor voltage to compensate for the additional IR drop. Figure 4 shows such a case with $V_C = 5000$ volts and $R_{PR} = 0.5\ \Omega$. Although the high initial gap voltage ($V_{PG} = 125$ volts) does help the discharge get started, especially at low τ_E, the decay of the plasma current is disappointingly rapid. The $\tau_E = 1600$ μsec case still only has 130 amp-seconds and a peak electron temperature of 98 eV.

A second possible solution is to passively crowbar the poloidal field so that the capacitor voltage is clamped at zero rather than allowed to swing negative. The result is shown in figure 5. Little improvement over the uncrowbarred case (Fig. 1) is noted. The $\tau_E = 1600$ μsec case has 139 amp-seconds and a peak electron temperature of 89 eV.

The next level of complexity would be a power crowbar which will hold the capacitor bank voltage constant at a prescribed voltage (V_{PC}) whenever it attempts to drop below that voltage. In practice this is done by switching a large bank of capacitors charged to V_{PC} in parallel with the initial capacitor bank at the prescribed time. Actually a diode would in principle suffice as shown below:



A second diode is also shown in parallel with the power crowbar bank to provide a passive crowbar when the voltage eventually tries to reverse. This extra diode is also desired to prevent the power crowbar bank voltage from reversing in the event that electrolytic capacitors are used. We begin by examining the case of an ideal power crowbar (that is, one with an infinite capacitance) and return later to the question of how large a bank is actually required. Figure 6 shows the result of such a case with V_{PC} = 450 volts. This value of voltage was chosen because it is the highest voltage in which electrolytic capacitors are commonly available. It is clear that a spectacular improvement is achieved for $\tau_E \gtrsim 1000$ μsec. The 1600 μsec case has 288 amp-seconds (ignoring times later than 10 msec) and a peak electron temperature of 94 eV. At 10 msec the loop voltage V_L and the poloidal gap voltage V_{PG} are about 2 volts and the core flux is about 0.1 volt-sec which is about the limit without cocking.

Another case was examined in which the power crowbar voltage was 900 volts (requiring a series parallel combination of electrolytics), and the result is shown in figure 7. This is a case of overkill. The 1600 μsec case has 491 amp-seconds (up to 10 msec) and a peak electron temperature of 174 eV. At 10 msec the loop voltage V_L is 2 volts and the poloidal gap voltage V_{PG} is 7 volts. The core flux is 0.15 volt-sec which might barely be achievable with an elaborate core cocking ciruit.

Now we return to the question of how much capacitance is required in a reasonable power crowbar bank. We take as a standard the τ_E = 800 μsec case in figure 6 and vary the capacitance C_{PC}. The result is shown in figure 8. To take full advantage of the 10 msec pulse apparently requires $C_{PC} \gtrsim 0.5$ farads.

The variation of $\int I_P dt$ vs τ_E for the crowbarred case with V_{PC} = 450 volts and C_{PC} = 0.4 farads is shown in figure 9. Comparison with figure 2 shows that the amp-seconds is a reasonable figure of merit in the presence of a suitable power crowbar, although it becomes less sensitive as τ_E increases.

In summary, it has been shown that in the absence of a power crowbar, the plasma current waveform can never exceed in duration that of the hoop current, or in amplitude a value of about 20% of the hoop current. With a power crowbar, the amplitude is still limited to about 20% of the hoop current, but the duration is limited only by saturation of the iron core. A power crowbar of ~ 0.5 F and 450 volts should be about right for extending the current pulse to ~ 10 msec without saturating the core, assuming an energy confinement time of ~ 1 msec.

APPENDIX A

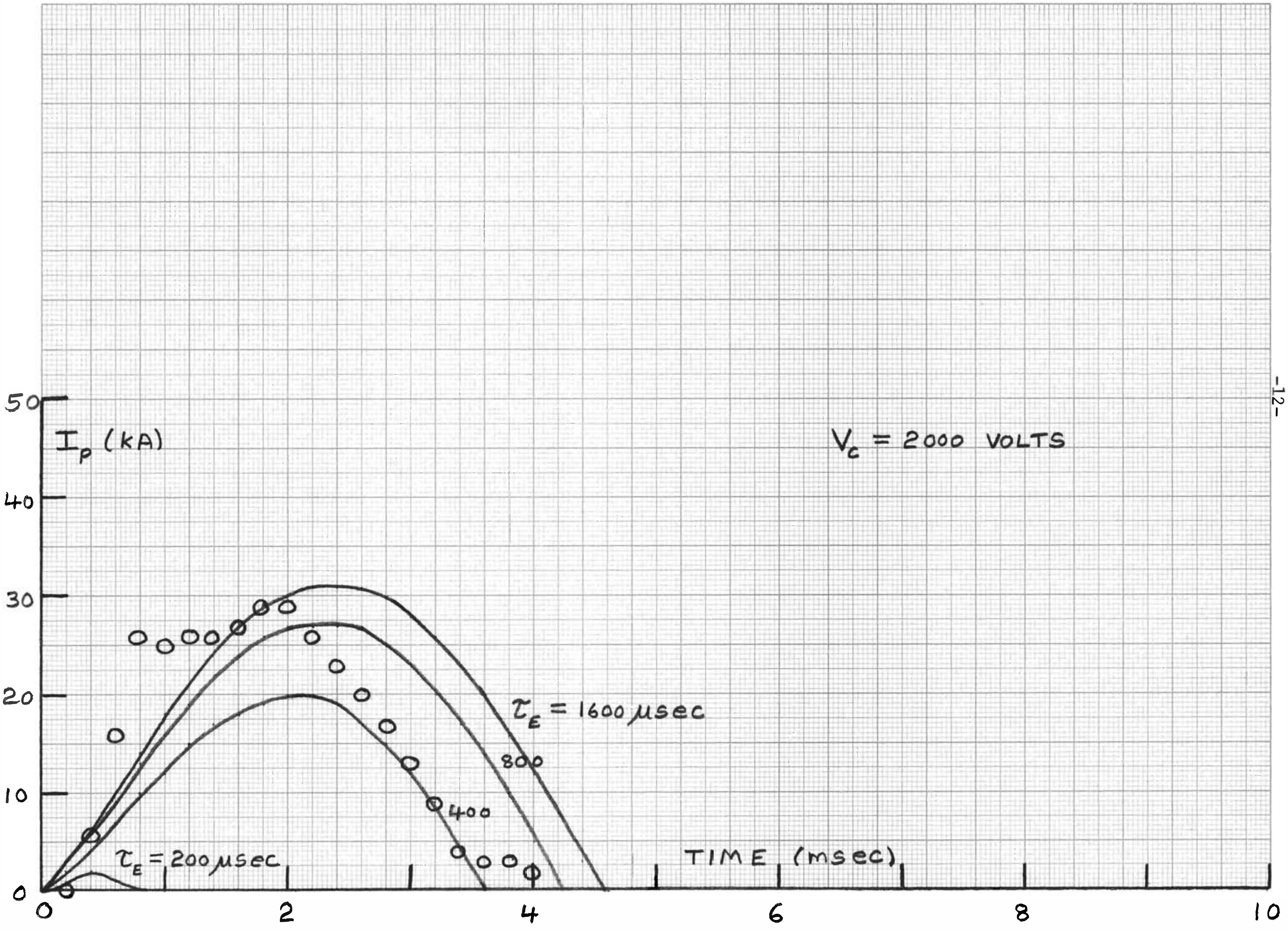
```
2980,4126810219
KII
1.17S-01/05/79-14:36:44                TOKII
     C      PROGRAM TOKII  -  J. C. SPROTT  -  JULY 2, 1978
            DIMENSION YI(4),YF(5),SAVE(52,6)
            COMMON C,RPR,N,ALP,DENS,ZEFF,TAUE
            EXTERNAL DERIVS
            DO 900 ICASE=1,9
            C=0.0072
            RPR=0.0485
            N=40
            VC=2000.0
            ZEFF=1.0
            TAUE=1.0E-4*1.4142136**(ICASE-1)
            AIH=0.1
            AIP=0.1
            DENS=1.0E13
            TE=5.0
            AMPSEC=0.0
            VSEC=0.0
            YI(1)=VC
            YI(2)=AIH
            YI(3)=AIP
            YI(4)=1.44E-13*DENS*TE
            WRITE(6,300)
      300   FORMAT(1H1,'     TIME        IP       VPG    IHOOP    VLOOP     VCAP      P
           20H        TE')
            CALL DEPC(4,0.0,YI,0.010,YF,DERIVS,1.0E-3,0.0,1.0E-7,1.0E-3,1.0E-4
           2,2,2.0E-4,2H..,SAVE,52,6,NPOINT,NOTIFY,$400)
      400   CONTINUE
            DO 600 I=1,NPOINT
            IT=200*(I-1)
            IP=1.0E-3*SAVE(I,3)
            IVPG=SAVE(I,5)
            IH=1.0E-3*SAVE(I,2)
            IVL=SAVE(I,4)
            IVC=SAVE(I,1)
            IPOH=1.0E-3*SAVE(I,3)*SAVE(I,4)
            ITE=SAVE(I,6)
            WRITE(6,500) IT,IP,IVPG,IH,IVL,IVC,IPOH,ITE
      500   FORMAT(1H ,8I8)
            AMPSEC=AMPSEC+2.0E-4*SAVE(I,3)
            VSEC=VSEC+2.0E-4*AMAX1(SAVE(I,5),0.0)
      600   CONTINUE
            WRITE(6,700) AMPSEC
      700   FORMAT(1H0,'AMP SECONDS=',F11.6)
            WRITE(6,720) VSEC
      720   FORMAT(1H0,'VOLT SECONDS',F10.6)
      900   CONTINUE
            END

 COMPILATION:          NO  DIAGNOSTICS.
IVS
```

```
1.17S-01/05/79-14:37:02                    DERIVS
          SUBROUTINE DERIVS(T,Y,DY,STORE,ITEST)
          DIMENSION Y(1),DY(1),STORE(1)
          COMMON C,RPR,N,ALP,DENS,ZEFF,TAUE
          VC=Y(1)
          AIH=Y(2)
          AIP=Y(3)
          CO=C
          VCROW=450.0
          CCROW=0.5
          IF(VC.LE.VCROW) CO=C+1.0E4*C*(VCROW-VC)
          IF(CO.GT.CCROW) CO=CCROW
          IF(VC.LE.50.0) CO=CCROW+1.0E4*CCROW*(50.0-VC)
          RH=7.8E-6+16.9E-6*EXP(-396.0*T)
          ALH=0.319E-6+0.06834E-6*(1.0-EXP(-396.0*T))
          ALP=0.8E-6-0.5*ALH
          TE=Y(4)/DENS/1.44E-13
          A=AMAX1(3.0,AMIN1(31.11,21.0*(ABS(AIP/AIH))**0.25))
          RP=12.26*ZEFF/A/A/(AMAX1(TE,0.025))**1.5
          VL=AIP*RP
          VPG=VC/FLOAT(N)-RPR*(AIH+AIP)/FLOAT(N*N)
          DY(1)=-(AIH+AIP)/CO/FLOAT(N)
          DY(2)=(2.0*VPG*ALP/ALH-RH*(2.0*ALP/ALH+1.0)*AIH+RP*AIP)
         2/(2.0*ALP+0.5*ALH)
          DY(3)=(VPG+RH*AIH-2.0*RP*AIP)/(2.0*ALP+0.5*ALH)
          DY(4)=AIP*VL-1.44E-13*DENS*TE/TAUE
          DO 300 I=1,3
      300 STORE(I)=Y(I)
          STORE(4)=VL
          STORE(5)=VPG
          STORE(6)=TE
          RETURN
          END

 COMPILATION:          NO  DIAGNOSTICS.

5 01/05-14:37:10
```

```
   TIME        IP      VPG    IHOOP    VLOOP     VCAP      POH       TE
      0         0       49        0        0     2000        0        5
    200         3       48       42       15     1983       52        7
    400         6       45       81       12     1937       73       13
    600         9       42      115        9     1863       90       20
    800        13       39      146        8     1764      104       28
   1000        16       35      173        6     1642      114       35
   1200        19       30      196        6     1501      122       42
   1400        22       26      215        5     1343      128       48
   1600        24       21      230        5     1172      131       53
   1800        26       16      241        5      990      132       58
   2000        27       11      249        4      800      131       61
   2200        27        6      252        4      607      128       63
   2400        27        2      253        4      446      123       65
   2600        27        2      252        4      443      119       65
   2800        27        2      252        4      440      117       65
   3000        26        2      252        4      437      115       65
   3200        26        2      251        4      435      113       65
   3400        25        2      251        4      432      112       64
   3600        25        2      251        4      429      110       63
   3800        25        2      250        4      426      109       63
   4000        24        2      250        4      424      107       62
   4200        23        2      250        4      421      106       61
   4400        23        2      250        4      418      104       61
   4600        22        2      250        4      415      102       60
   4800        22        2      250        4      413      100       59
   5000        21        2      250        4      410       98       58
   5200        21        1      250        4      407       96       57
   5400        20        1      250        4      404       94       56
   5600        19        1      250        4      402       92       55
   5800        18        1      250        4      399       90       54
   6000        18        1      250        4      396       87       53
   6200        17        1      250        4      394       85       52
   6400        16        1      250        4      391       82       50
   6600        15        1      250        5      388       79       49
   6800        15        1      250        5      386       76       48
   7000        14        1      250        5      383       73       46
   7200        13        1      250        5      380       69       45
   7400        12        1      250        5      378       65       43
   7600        11        1      250        5      375       61       41
   7800        10        1      250        5      373       57       39
   8000         9        1      250        5      370       52       37
   8200         8        1      250        5      367       47       35
   8400         6        1      250        6      365       41       33
   8600         5        1      250        6      362       35       30
   8800         4        1      250        6      360       28       27
   9000         3        1      250        6      357       20       24
   9200         1        1      250        6      355       11       21
   9400         0        1      250        5      352        3       17
   9600         0        1      250        3      350        0       13
   9800         0        1      249        1      347        0       10
  10000         0        1      248        1      345        0        8

AMP SECONDS= 162.497196

VOLT SECONDS    .089051
```

```
    RUNID:  CW1142   PROJECT: 02980             USER: 4126810219

   ITEM                                    AMOUNT       COST

 CPU TIME                         00:00:14.630          $0.43
 FILE I/O REQUESTS                         363          $0.13
 FILE I/O WORDS                         302008          $0.04
 SWAP REQUESTS                               1          $0.00
 SWAP WORDS                               3584          $0.00
 MEMORY USAGE                            0.148          $0.10
 CARDS IN                                   85          $0.08
 PAGES PRINTED                              11          $0.27
 SHARED FILES                                1          $0.10
 SOFTWARE SUPPORT                           16          $0.23
 JOB CHARGE                                  1          $0.05

 TOTAL COST                                             $1.43

    THE ABOVE DOLLAR AMOUNTS ARE APPROXIMATE AND ARE BASED ON RATES FOR WHILE
 USER BALANCE                                         $432.29

    INITIATION  TIME:   14:36:39 JAN  5,1979

    TERMINATION TIME:   14:41:54 JAN  5,1979

   PREVIOUS RUN TIME:   14:51:32 DEC 31,1978




 ***EOF***      12 PAGES COST      $0.28    USER BALANCE      $432.28     CW1142
```

FIGURE 1



50
40
30
20
10
0
I_p (kA)
V_c = 2000 VOLTS
τ_E = 1600 μsec
800
400
τ_E = 200 μsec
TIME (msec)
0
2
4
6
8
10

FIGURE 2

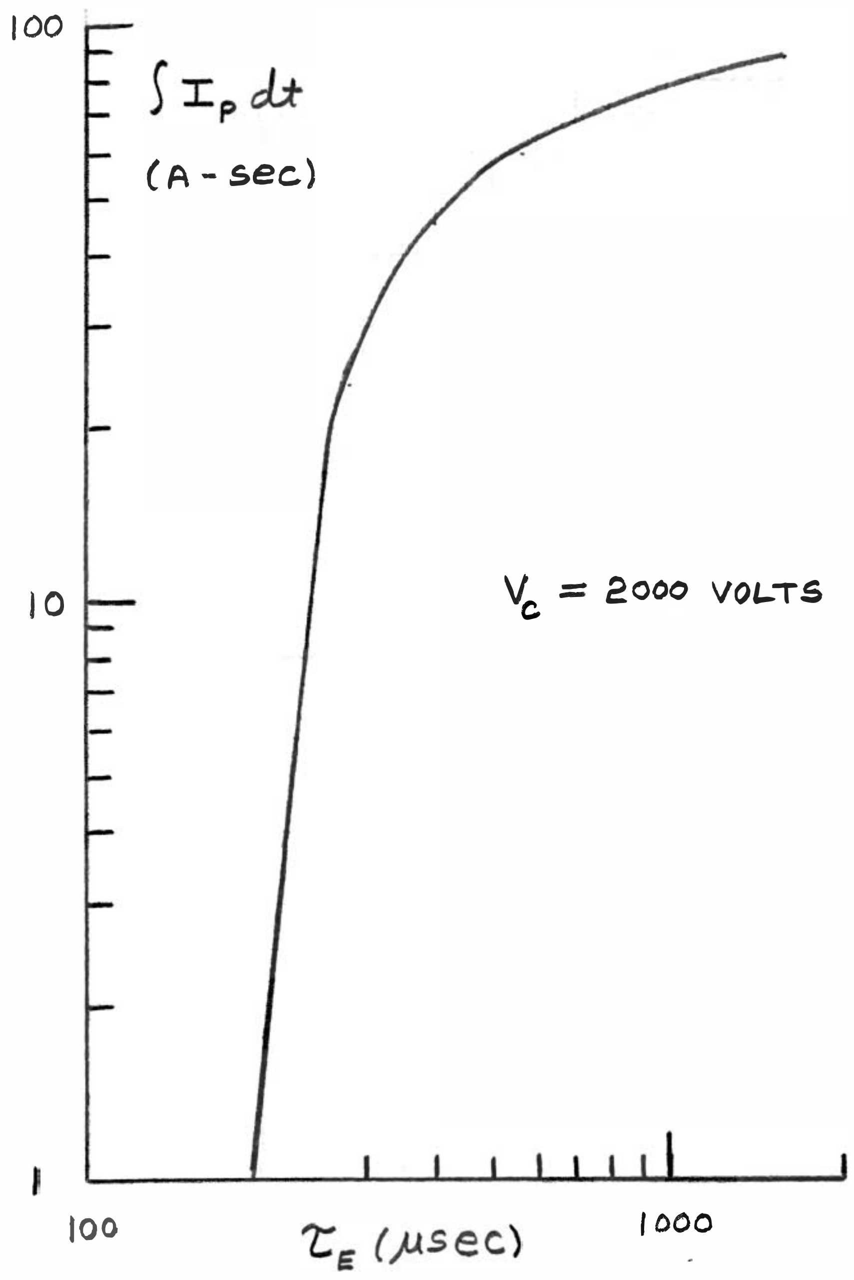


FIGURE 3

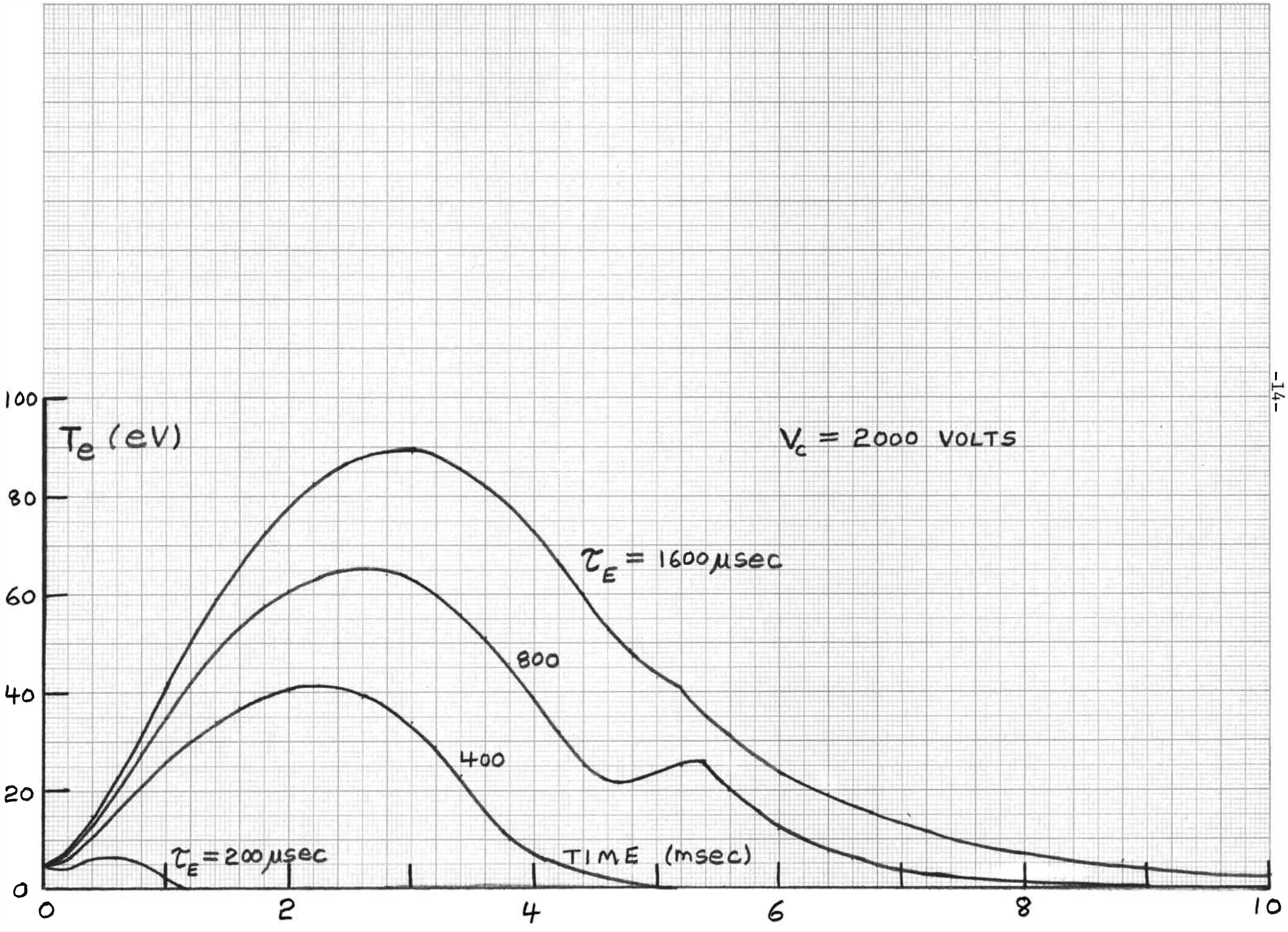

100
80
60
40
20
0
T_e (eV)
V_c = 2000 VOLTS
τ_E = 1600 μsec
800
400
τ_E = 200 μsec
TIME (msec)
0
2
4
6
8
10

FIGURE 4

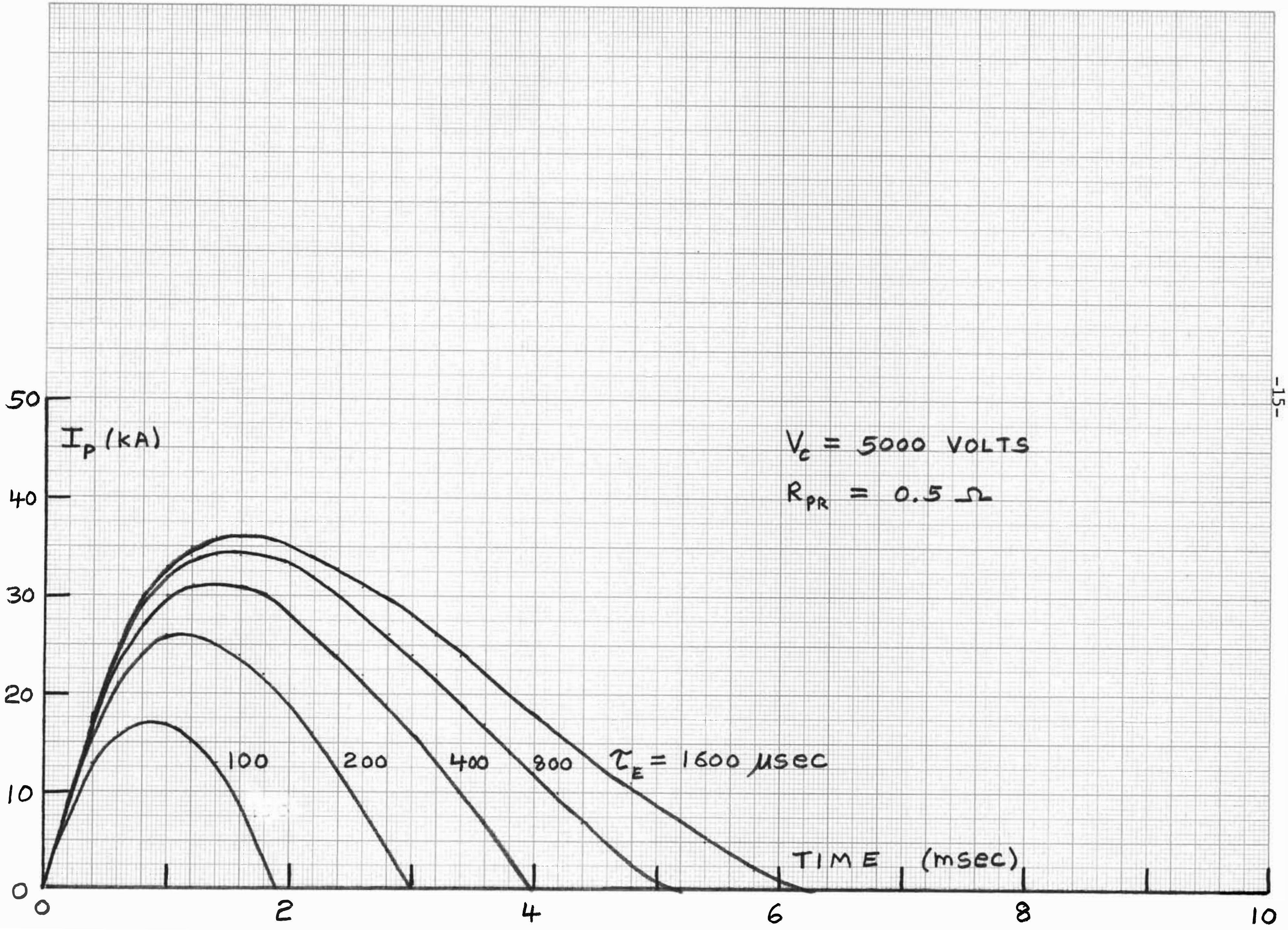

50
40
30
20
10
0
I_P (kA)
V_c = 5000 VOLTS
R_{PR} = 0.5 Ω
100
200
400
800
τ_E = 1600 μsec
TIME (msec)
0
2
4
6
8
10

FIGURE 5

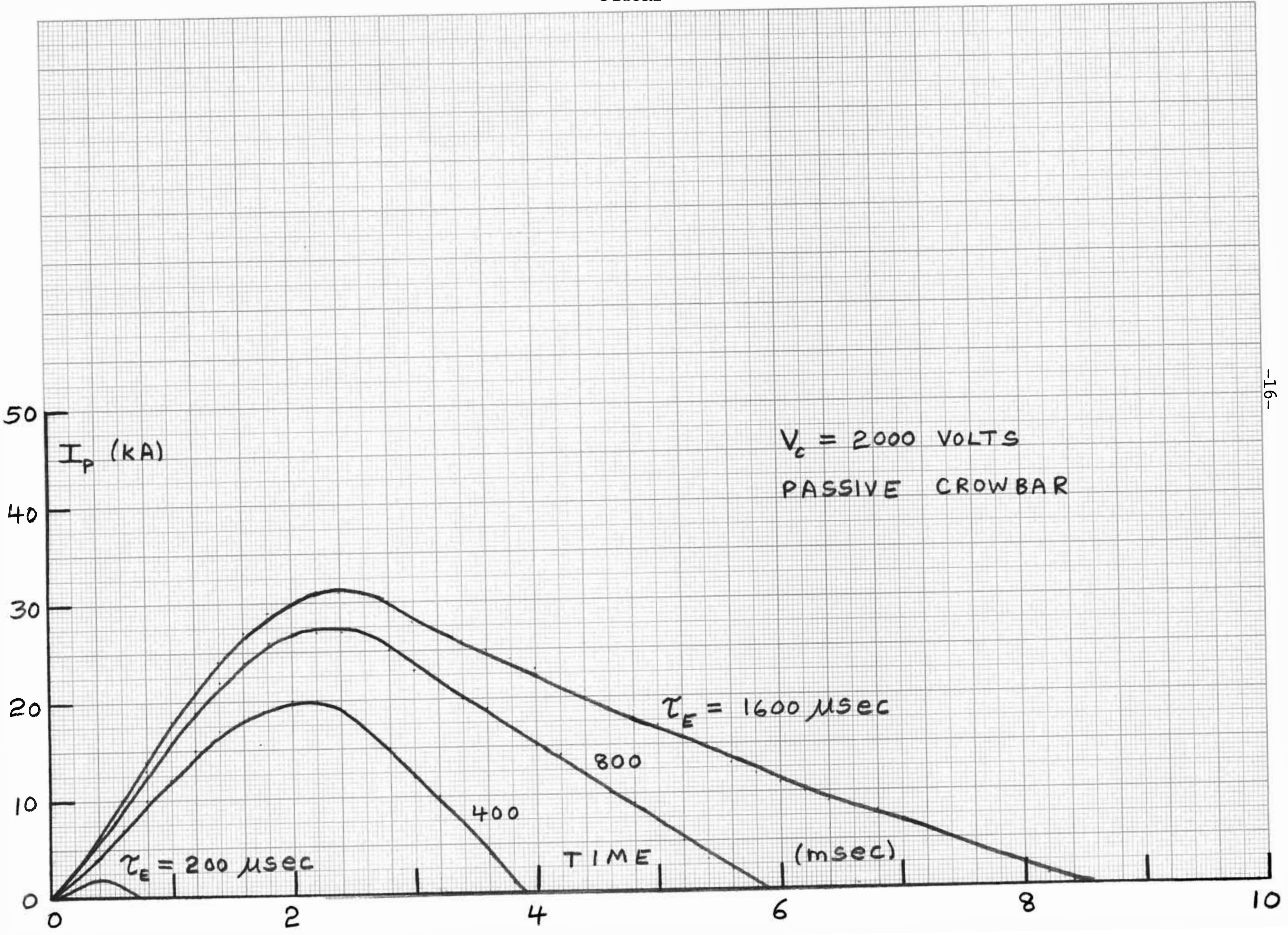
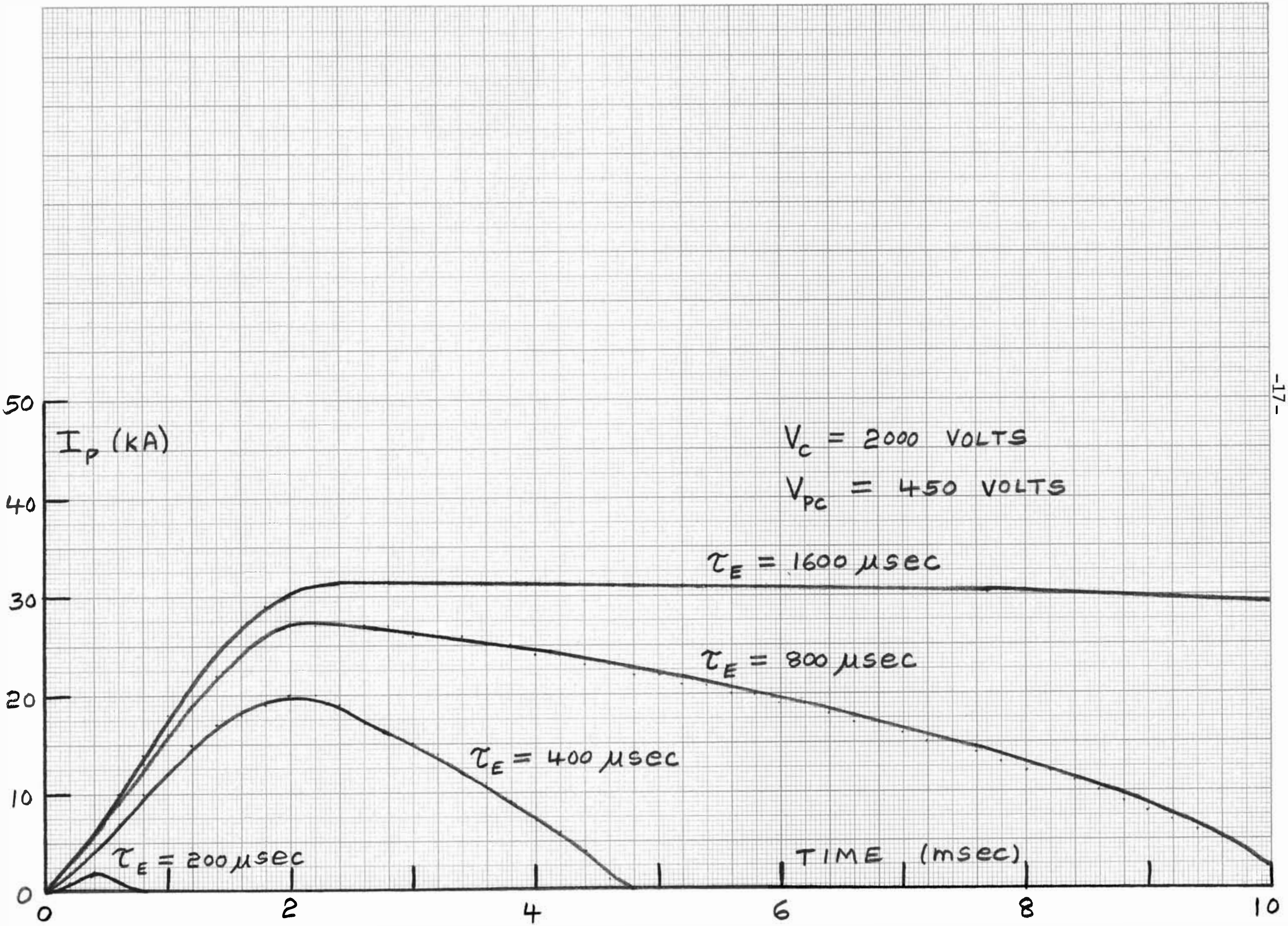

V_c = 2000 VOLTS
PASSIVE CROWBAR
I_p (kA)
50
40
30
20
10
0
τ_E = 1600 μsec
800
400
τ_E = 200 μsec
TIME (msec)
0
2
4
6
8
10

FIGURE 6



I_p (kA)
V_c = 2000 VOLTS
V_{pc} = 450 VOLTS
τ_E = 1600 μsec
τ_E = 800 μsec
τ_E = 400 μsec
τ_E = 200 μsec
TIME (msec)
50
40
30
20
10
0
0
2
4
6
8
10

FIGURE 7

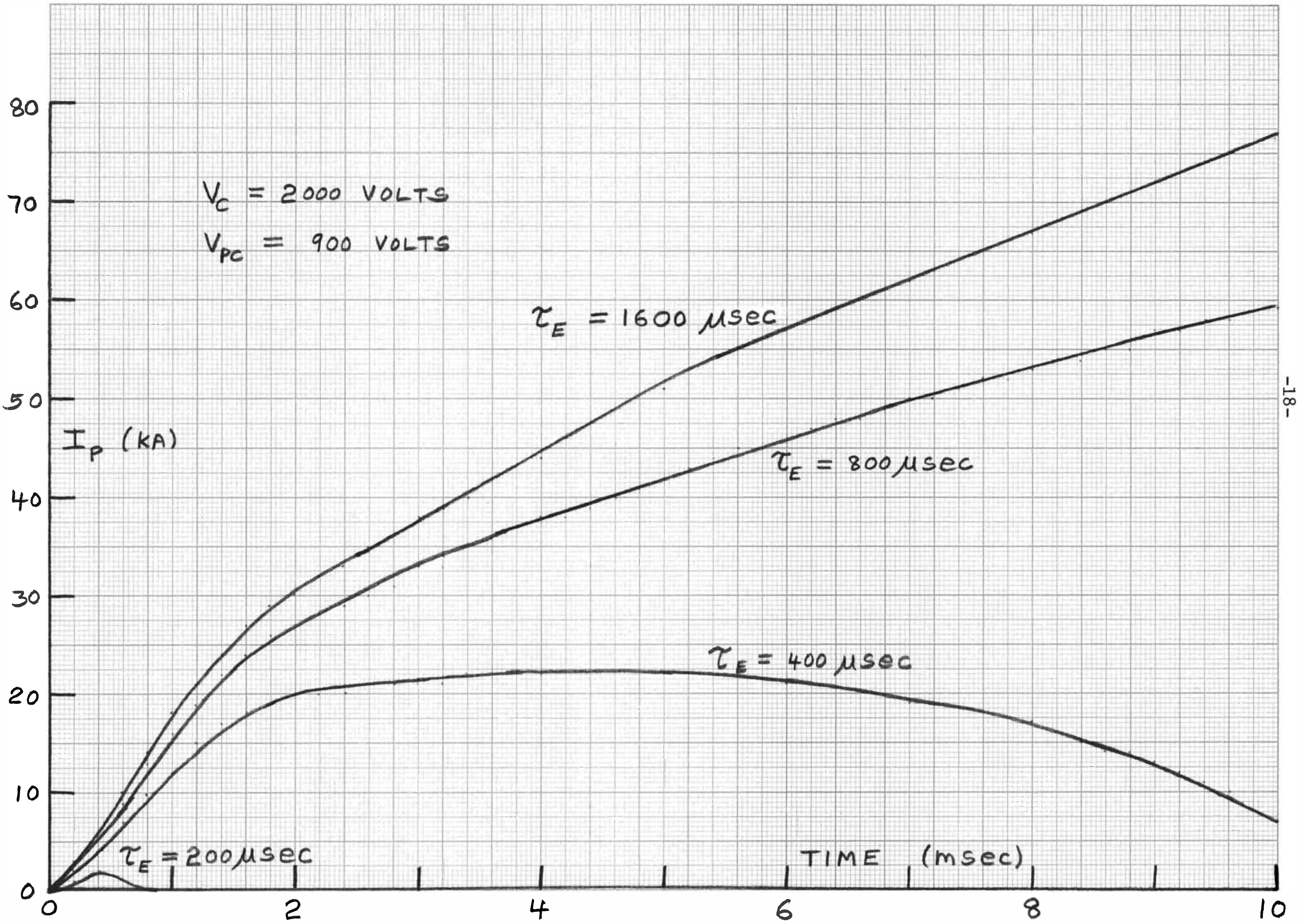

80
70
60
50
40
30
20
10
0
V_C = 2000 VOLTS
V_{PC} = 900 VOLTS
I_P (KA)
τ_E = 1600 μsec
τ_E = 800 μsec
τ_E = 400 μsec
τ_E = 200 μsec
TIME (msec)
0
2
4
6
8
10

FIGURE 8

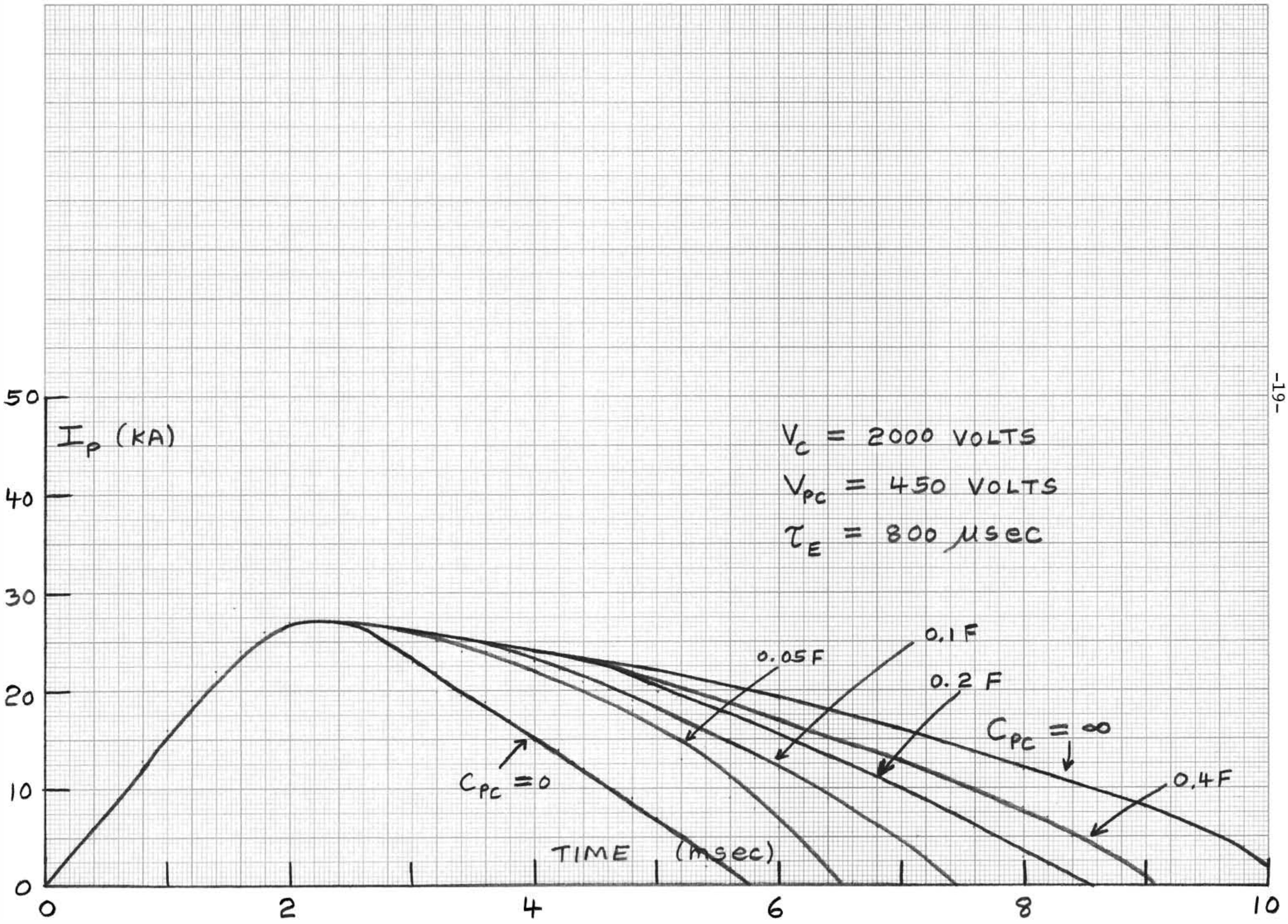
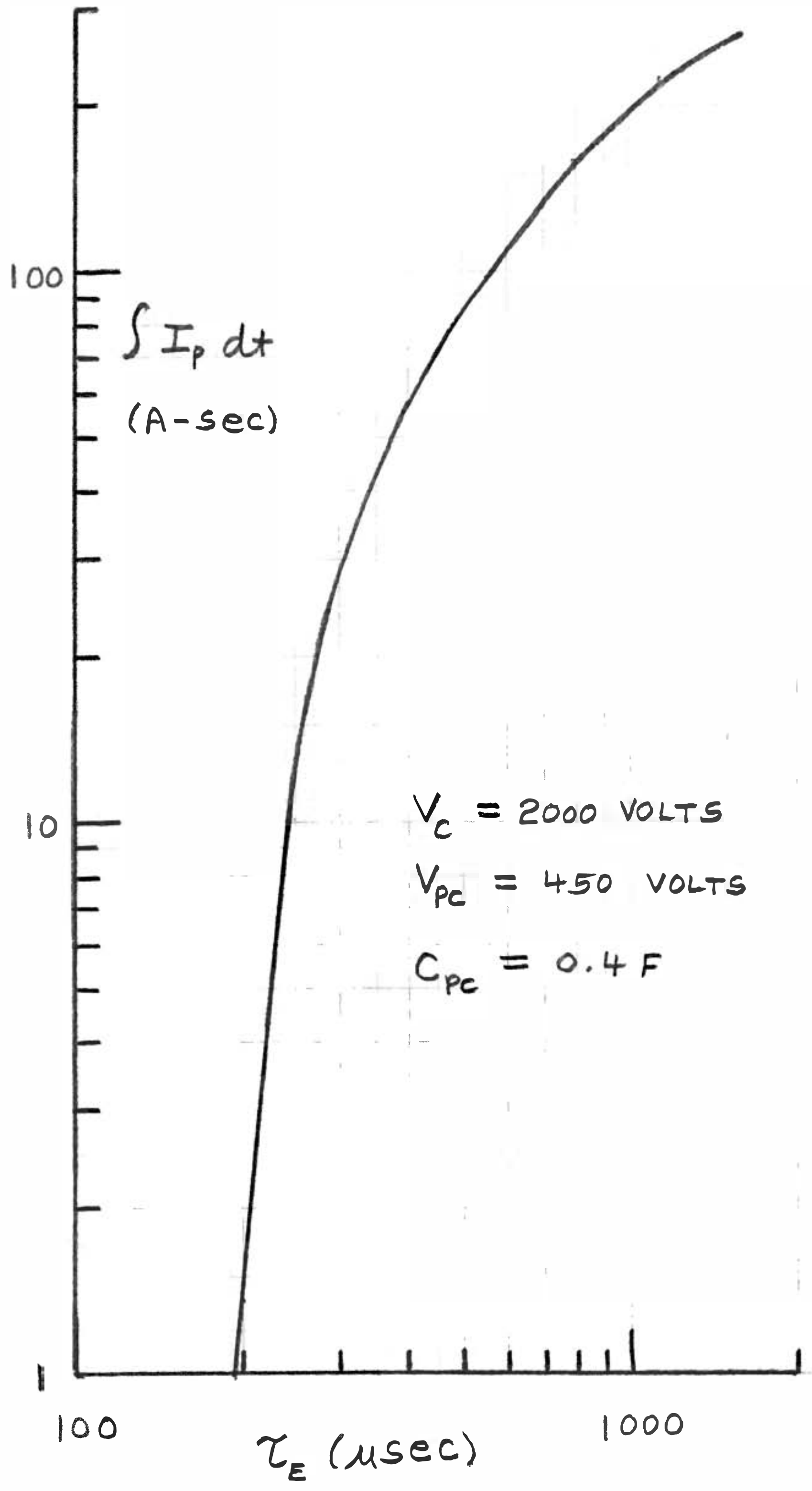

I_p (KA)
50
40
30
20
10
0
0
2
4
6
8
10
TIME (msec)
V_C = 2000 VOLTS
V_{PC} = 450 VOLTS
τ_E = 800 μsec
$C_{PC} = 0$
0.05F
0.1F
0.2 F
$C_{PC} = \infty$
0.4F

FIGURE 9


100
10
1
$\int I_p\,dt$
(A-sec)
V_C = 2000 VOLTS
V_{PC} = 450 VOLTS
C_{PC} = 0.4 F
100
1000
τ_E (μsec)