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J.C. Sprott

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Most toroidal fusion reactor concepts require hundreds of MW of auxiliary heating power to reach ingnition temperatures. Until recently, little information has been available on the effects of very high power heating on plasma confinement. The reason for this is that most tokamaks have relied primarily on ohmic heating. In such devices the energy confinement time typically follows an empirical scaling law, first observed on Alcator A[1,2]

$$\tau_E = 6 \times 10^{-21}\ \bar{n}a^2 \qquad (1)$$

where $\bar{n}$ is the line-averaged electron density, a is the minor radius, and all units are MKS. The energy losses are dominated by anomalous electron heat transport. Furthermore, the density in such devices tends to follow the empirical scaling first noted by Murakami, et al.:[3]

$$\bar{n} = 10^{19}\ B_T/R_o \qquad (2)$$

where B_T is the toroidal field on axis and R_o is the major radius. As a point of local interest, these scaling laws predict $\bar{n} = 10^{13}$ cm^{-3} and τ_E = 860 μsec for Tokapole II with B_T = 5 kG, R_o = 50 cm, and a = 12 cm. These values are quite consistent with observations. The explanation of the observed scalings is still conjectural, and many recent experiments have documented departures from these simple scalings. For example, at very high densities, the empirical scaling predicts very long confinement times that exceed the neoclassical prediction for which τ_E varies inversely with $\bar{n}$ and the energy losses are dominated by ion heat transport.

The scaling of τ_E with $\bar{n}a^2$ implies that ~~this~~ there is something fundamental about the role of the plasma density in determining the energy losses. Such is not necessarily the case, however, since an ohmically-heated tokamak (with $T_i \cong T_e$) is subject to other constraints such as the steady-state power balance:

$$\frac{3\bar{n}kT}{\tau_E} = \eta j_T^2 = \eta \left(\frac{2B_T}{\mu_o R_o q}\right)^2 \tag{3}$$

where j_T is the toroidal current density, q is the safety factor,

$$q = \frac{2B_T}{\mu_o j_T R_o} \tag{4}$$

and η is the resistivity, as given by Spitzer and Harm[4]

$$\eta = 7.8 \times 10^{-4} \frac{Z_{eff}}{(kT_e/e)^{1.5}} \tag{5}$$

Combining Eqs. (1)-(5) allows one to calculate the temperature for an ohmically-heated tokamak:

$$\frac{kT}{e} = 906\left[\frac{a^2 Z_{eff} B_T^{\,2}}{R_o^2 q^2}\right]^{0.4} \qquad (6)$$

Equation (6) illustrates the origin of the cliche that "you can't heat a tokamak to ignition with ohmic heating alone." In a hydrogenic plasma (Z_{eff}=1), operated at the Kruskal-Shafranov limit (q=1) with an aspect ratio (R_o/a) of 3 at a field of 100 kG, the temperature only reaches ~2.4 keV, whereas ignition requires at least 4.5 keV.[5] For Tokapole II with B_T = 5kG, R_o/a = 4.2 and q = 2, the expected temperature at Z_{eff} = 1 is 95 eV, in rough agreement with observations.

By substituting B_T from Eq. (2) into Eq. (6), one concludes that the temperature and density are very closely related in an ohmically-heated tokamak:

$$\frac{kT}{e} = 5.7 \times 10^{-13} \left[\frac{a^2 Z_{eff}}{q^2}\right]^{0.4} \bar{n}^{0.8} \qquad (7)$$

Thus any scaling law, such as Eq. (1), that depends on a single plasma parameter, such as $\bar{n}$, can also be written in terms of other parameters, such as T:

$$\tau_E = 1.2 \times 10^{-5} \frac{aq}{\sqrt{Z_{eff}}} \left[\frac{kT}{e}\right]^{1.25} \qquad (8)$$

Equation (8) implies that temperature is the relevant quantity which determines the energy confinement in a tokamak, rather than density as suggested by Eq. (1). This argument illustrates the importance of auxiliary heating, not only for reaching ignition, but also for understanding the energy losses in the present genre of devices.

Neutral beam and rf heating provide the means for varying the temperature independently of the density in a tokamak. One is thus able to distinguish among possibilities such as Eqs. (1) and (8) to decide which is most fundamental. To accomplish this, the confinement time is usually measured by dividing the stored energy by the input power:

$$\tau_E = \frac{3\bar{n}kT}{P}(2\pi^2 a^2 R_o) \qquad (9)$$

Note that if the confinement time is determined by neither $\bar{n}$, nor T, nor P, but rather by some "hidden variable" such as the wall conditions, then this method of measuring τ_E will yield a strong linear correlation with either $\bar{n}$ or T. It is thus easy to draw an erroneous conclusion concerning the transport mechanism by this method.

Nevertheless, auxiliary heating gives one the capability of deducing an improved empirical scaling law that is valid at higher temperatures than would be accessible with ohmic heating alone. For this purpose, one often plots $\bar{n}kT$, or, equivalently

$$\beta = \frac{4\mu_o\bar{n}kT}{B_T^2} \qquad (10)$$

vs the input power P, in which case the slope is proportional to the confinement time, and one can see at a glance whether the confinement improves or degrades as reactor conditions (high betas) are approached.

With ohmic heating alone, the beta is essentially fixed by the constraints of tokamak operation at a value of

$$\beta = 7.3 \times 10^{-3}\left[\frac{a^4 Z_{eff}^2}{R_o^9 B_T q^4}\right]^{0.2} \tag{11}$$

If one wants to maintain a constant q, the only free parameter for varying beta in a given machine is thus the toroidal field, and its influence on beta is remarkably small. One could, however, attempt to measure beta versus ohmic heating power at constant q by varying B_T, in which case the expected result is $\beta \propto P^{-0.25}$. The inverse dependence is amusing since it says that the highest beta is achieved at the smallest power, but useless because the scaling laws are not sufficiently accurate over the very wide range of B_T necessary to do the experiment.

With auxiliary heating, the beta can be varied over a wide range now that the auxiliary heating power levels considerably exceed the ohmic power levels in many experiments. Perhaps the most detailed work has been done on ISX-B[6], where it was noted that $\beta \propto P^{1/3}/q^{3/2}B_T^{1/2}$ over a range of $0.3 < P < 2.4$ MW, $7.5 < B_T < 15$ kG, and $2.2 < q < 5.7$. The $P^{1/3}$ dependence has often been referred to as a "soft beta limit," since it implies a decrease in energy confinement time with increasing power. This result can be interpreted as a beam-driven enhanced loss, as a manifestation of a beta-related instability, or as a dependence of τ_E on temperature or density. Only further experimentation will tell.

In order to further address this question, data taken by Biddle[7] several years ago on an ion-cyclotron-heated plasma in Tokapole II were re-examined. Biddle plotted the ion energy confinement time versus the average ion energy for four values of plasma current ($23<I_p<36$ kA) at a constant B_T of 4 kG and essentially constant density ($\bar{n}\sim 6\times 10^{12}$ cm^{-3}). Ion energy was varied ($36<\bar{E}_i<110$ eV) by varying the ICRH power ($P\lesssim 130$ kW), and the plasma current was varied by varying the OH voltage (and hence q). From Biddle's data, one can plot the quantity $\beta q^{3/2} B_T{}^{1/2}$ vs ICRH power, and the result is shown in Fig. 1. One observes the same $P^{1/3}$ scaling as observed on ISX-B, despite the fact that the beta is very much smaller. The highest beta in Fig. 1 is only ~0.1%, well below any ballooning or other finite beta stability limits. (The ISX-B data ranged from $\beta\sim 0.3\%$ to 2.5%.) One is thus led to conclude that the "soft beta limit" has little to do with beta, per se.

In the Tokapole II experiment, an explanation for the "soft beta limit" is readily available. Since the device has a relatively small minor radius and weak magnetic field, the energetic ions produced by ICRH are poorly confined due to their large drift orbits. If one takes as a criterion that the maximum ion energy that can be confined is such that the ion poloidal gyroradius is equal to the minor radius of the plasma, the limiting ion energy is thus

$$E_i < \frac{e^2 a^4 B_T{}^2}{32 M R_o^2 q^2} \qquad (12)$$

where M is the ion mass ($M=1.67\times10^{-27}$ kg for protons). For Tokapole II with a = 12 cm, B_T = 4 kG, and q = 2, the limiting ion energy is ~100 eV, in good agreement with observations of Biddle.[7] To test this limit, Biddle plotted the maximum ion energy that could be observed on the charge exchange analyzer versus toroidal plasma current I_p and found that E_{iMAX} was proportional to I_p^2 as predicted by Eq. (12) above.[8] The corresponding limit for ISX-B at B_T = 10 kG and q = 2 is ~4600 eV.

Equation (12) also predicts a beta limit if we take $kT_e = E_i$ and a density from Eq. (2):

$$\beta < \frac{48a^4B_T}{R_o^3q^2} \qquad (13)$$

For Tokapole II with a = 12 cm, B_T = 5 kG and q = 2, this limit is ~0.1% in good agreement with the observations of the ICRH plasmas. By contrast, the corresponding limit for ISX-B with a = 27 cm, B_T = 14 kG, R_o = 93 cm, and q = 1 is ~11%. This difference illustrates the difficulty of doing high beta exeriments in low-field tokamaks with large aspect ratios.

One can go one step further and calculate the expected variation of beta with heating power in the limit where the energy losses are dominated by unconfined ion orbits. The energy confinement time is assumed equal to 1/5 the time for an ion to ∇B drift in the toroidal field over a distance equal to the minor radius of the plasma:

$$\tau_D = \frac{eB_TR_oa}{kT} \qquad (14)$$

For Tokapole II at 100 eV with B_T = 5 kG, this time is 300 μsec. The electrons are assumed to somehow also escape on this time scale in order to preserve charge neutrality. Under such conditions, the plasma is expected to acquire a strong (~kT/e) negative ambipolar potential and to rotate by ($\vec{E}\times\vec{B}$ drifts) accordingly. Combining Eq. (14) with Eqs. (2), (9), and (10), we obtain

$$\beta = 3.7 \times 10^{-7}\left[\frac{P}{B_T^2 R_o a}\right]^{1/2} \qquad (15)$$

The $\beta \propto P^{1/2}$ dependence is slightly different from the $P^{1/3}$ observed experimentally, but probably within experimental error. Equation (15) predicts a beta of ~0.13% for Tokapole II with 130 kW of auxiliary heating at B_T = 4 kG, in good agreement with the observation. The corresponding prediction for ISX-B at B_T = 10 kG and P = 2.6 MW is also ~0.12% implying that ISX-B is almost certainly not limited by this mechanism as expected from its containment limit of 4600 eV at 10 kG and q = 2.

Much local interest has recently centered around studies of beta limits in multipoles. The reason is, that without the strong toroidal field necessary for stability in a tokamak, it is relatively easy to achieve high-beta plasmas in a multipole. Furthermore, since the plasma production and confinement are completely decoupled, there are no constraints on the plasma parameters as in a tokamak. In particular, almost arbitrarily large values of beta can be achieved in a multipole by simply reducing the magnetic field strength while keeping the density and temperature relatively constant. An ultimate limit is reached only when the field becomes so weak that the ion gyro-orbits are no longer confined in the machine. Thus the easiest high-beta plasmas to produce in a multipole have high density, low temperature, and large gyroradius and thus tend to be collisional and subject to FLR effects. The 44% beta plasmas previously reported in the Levitated Octupole[9] are of this type, although β = 11% cases also have been studied[10] in which the collision frequency is on the order of the bounce frequency, and 5 ion gyroradii exist across a pressure-gradient scale-length.

The availability of high power ICRH on the Levitated Octupole (1.5 MW at present and 4 MW in a few months) raises the exciting possibility of producing plasmas well into the collisionless regime with a sufficient number of ion gyroradii that FLR effects will not preclude an observation of the ballooning mode stability limit, if it exists. As a first step in this direction, the beta was measured vs ICRH power by Strait, Fortgang, and Kellman to see if anything resembling the tokamak "soft-beta limit" exists. The result is shown in Fig. 2. For these data the temperature was relatively constant ($12<T_e<15$ eV, $40<T_i<50$ eV), and gas puffing was used to vary the density over the range $2 \times 10^{12} < n < 1 \times 10^{13}$ cm^{-3}. The beta is observed to increase as roughly the 3/4 power of the ICRH power and to be within a factor of 2 to 3 of the value expected for ion heat transport:

$$\beta = 4 \times 10^{-7} \frac{P^{1/2}}{B}\left(\frac{kT_i}{e}\right)^{3/4} \qquad (16)$$

Thus there is no evidence for a "soft beta limit" in the Octupole experiments, and thus presumably no barrier to the production of plasmas which should exhibit the ballooning-mode instability.

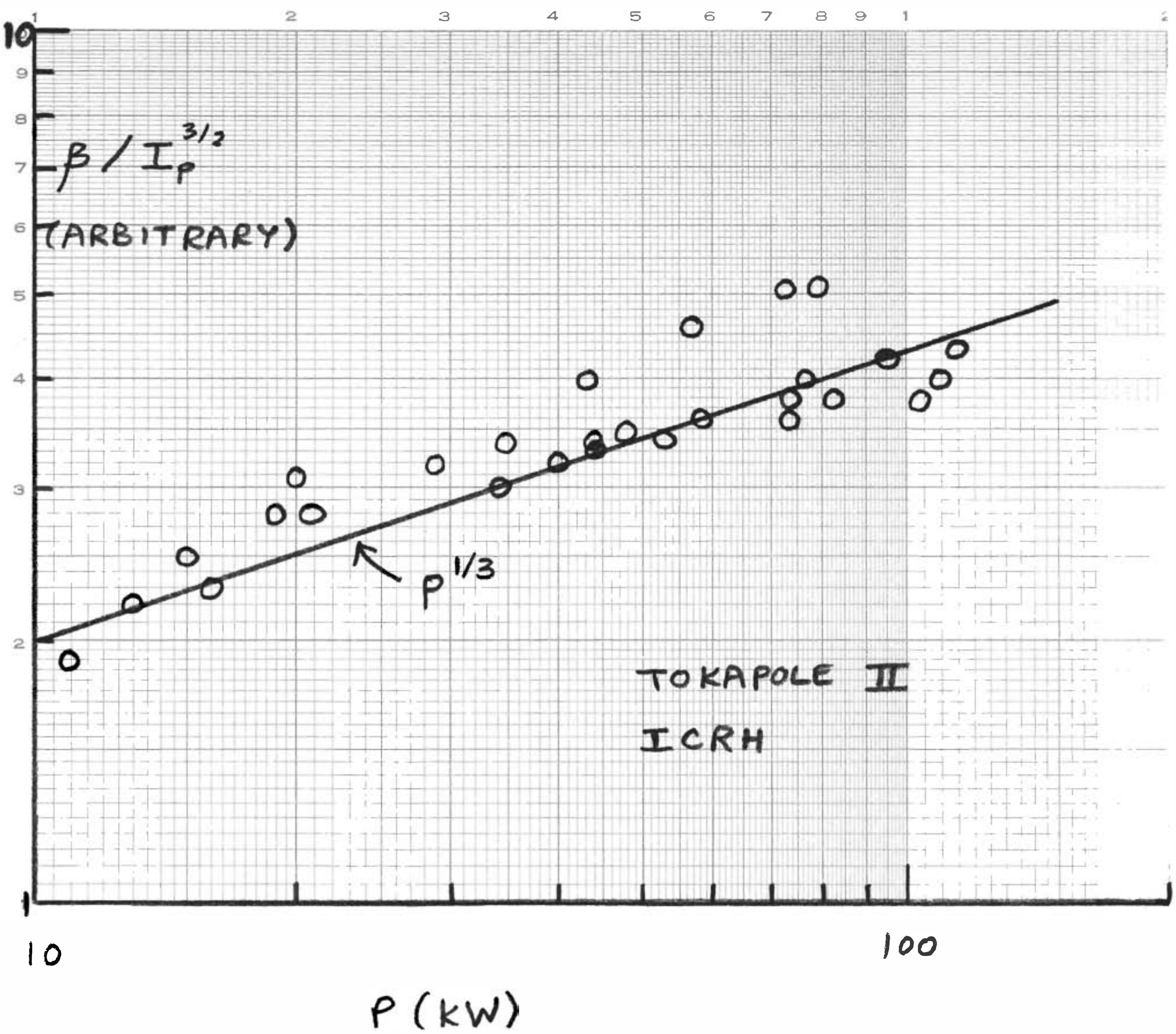

Figure 1: Beta normalized to the 3/2 power of the plasma current vs ICRH power in Tokapole II showing the "soft beta limit" ($\beta \propto P^{1/3}$).

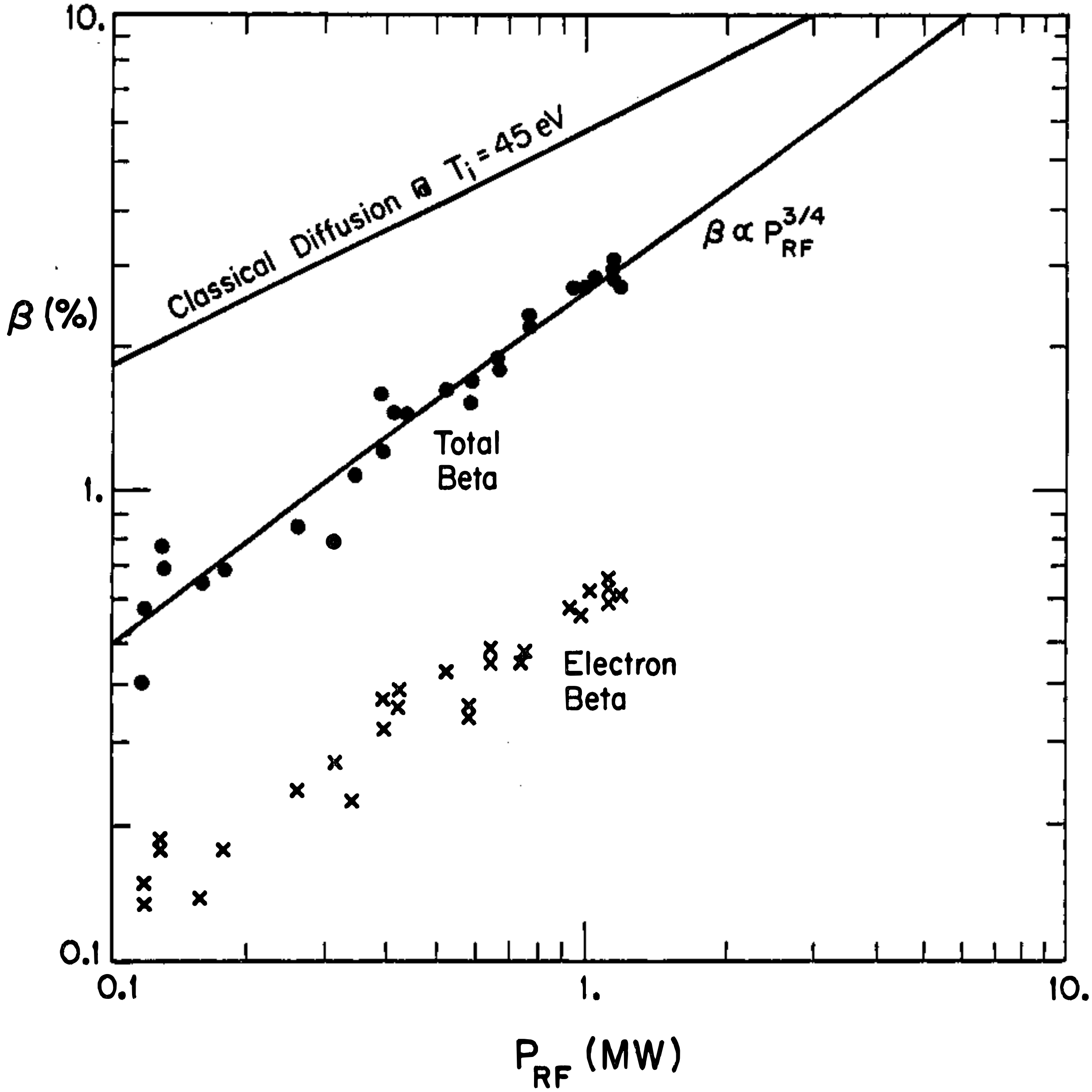

Figure 2: Beta vs ICRH power in the Levitated Octupole showing a nearly linear increase. Also shown is the classical ion heat transport limit for a constant ion temperature of 45 eV.

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