RELAXATION TIMES IN OCTUPOLE

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This paper has been written to clear up some confusion about the various relaxation times in the octupole and to provide a convenient reference for formulas and constants taken from several sources. The ion and electron distributions are assumed Maxwellian with $E_i = 40$ eV (determined from electrostatic analyzer measurements), and $E_e = 10$ eV (determined from Langmuir probe measurements).

Consider first the case of a fully ionized plasma of hydrogen ions (protons) and electrons. Three types of collisions are possible: (1) ion-ion, (2) electron-electron, and (3) ion-electron. Energy transfer is most efficient between particles of identical mass, so ions are thermalized by the first type collision while electrons are thermalized by the second. In the third type, very little energy is transferred between ions and electrons, but the electrons experience large momentum changes.

(1) Ion-ion Collisions:

The time required for an ion to be deflected through 90° by collisions with other ions is a factor of 1.14 longer than the time required for an originally non-Maxwellian ion distribution to thermalize. According to Spitzer[1], this time is given by

$$\tau_{ii} = \frac{11.4\ T_i^{3/2}}{n\ \ell n\ \Lambda} \qquad (1)$$

where T is in °K, n in cm^{-3}, and τ is seconds. $\ln \Lambda$ is a slowly varying function of temperature and density and is plotted in Figure 1 from values given in Spitzer[2]. For the octupole, $\ln \Lambda \approx 18$. Some useful relations between temperature T in °K and energy E in eV are:

$$\left.\begin{aligned} E &= \frac{kT}{e} \\ T(°K) &= 11{,}600\ E(eV) \\ E(eV) &= 8.62 \times 10^{-5} T(°K) \end{aligned}\right\} \qquad (2)$$

Hence, in the octupole, $T_i \approx 4 \times 10^5$ °K and $T_e \approx 10^5$ °K. Using these values, equation (1) becomes

$$\tau_{ii} = \frac{1.8 \times 10^8}{n} \quad . \qquad (3)$$

Equation (3) is plotted in Figure 2. For the typical dens

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nearly one second, which is much longer than the several millisecond duration of the experiment. Hence, we are justified in calling the plasma "collisionless". The collision frequency $\nu = 1/\tau$ is also plotted in Figure 2.

(2) <u>Electron-electron Collisions:</u>

The electron-electron collision time as given by Spitzer[1] is

$$\tau_{ee} = \frac{0.266\ T_e^{3/2}}{n \ln \Lambda} \quad . \qquad (4)$$

Substituting the numerical values appropriate to the octupole gives:

$$\tau_{ee} = \frac{4.6 \times 10^5}{n} \qquad (5)$$

The electron-electron collision time in the octupole is about 1 millisecond. Hence we would expect some change in the electron distribution function if it were originally non-Maxwellian.

(3) Ion-electron Collisions:

When an ion collides with an electron, the ion is deflected very little and experiences almost no energy change compared to ion-ion collisions. The electrons, on the other hand, may experience appreciable momentum changes. Detailed calculations[3] show that unless the ion energy greatly exceeds the electron energy, the relaxation time for electron-ion collisions is $2\sqrt{2}$ times as long as for electron-electron collisions. Hence to our degree of accuracy, the deflection time for electrons is given very nearly by the electron-electron collision time.

Ion-electron collisions, however, are the only means by which ions and electrons can reach the same temperature. This equipartition time τ_{eq} is geven by Spitzer[4] as

$$\tau_{eq} = \frac{5.87}{1836\ n\ \ell n \Lambda} (T_i + 1836T_e)^{3/2} \qquad (6)$$

For the octupole, equation (6) reduces to

$$\tau_{eq} = \frac{4.4 \times 10^8}{n} \qquad (7)$$

τ_{eq} is plotted in Fugure 2. In the octupole, there is almost no tendency for ions and electrons to reach the same energy.

Now consider the case of a plasma in the presence of a background gas at room temperature. For simplicity, the background gas will be assumed to consist of nitrogen (N_2) and hydrogen (H_2) with partial pressures p_N and p_H. The partial pressure of a gas is related to the number density of molecules by the expression

$$p = nkT. \qquad (8)$$

At room temperature, pressure and density are related by

$$\left.\begin{aligned} p(\text{Torr}) &= 3.4 \times 10^{-18}\ n(\text{cm}^{-3}) \\ n(\text{cm}^{-3}) &= 3.2 \times 10^{17}\ p(\text{Torr}) \end{aligned}\right. \qquad (9)$$

Pressure is plotted along the top axis of Figure 2 corresponding to the densities on the bottom axis. The base pressure in the octupole is about 3×10^{-7} Torr corresponding to a neutral particle density of 10''cm^{-3}.

The collision frequency ν is defined in terms of the reaction cross section σ by the relation

$$\nu = 1/\tau = n\sigma v \qquad (10)$$

where n is the density and v is the charged particle velocity. For a Maxwellian distribution, the rms particle velocity is given in terms of the temperature by

$$v = \sqrt{\frac{3kT}{m}}\ . \qquad (11)$$

Using the ion and electron masses, the following useful relations are derived:

$$\left.\begin{aligned} v_i\ (\text{cm/sec}) &= 1.7 \times 10^{6}\ \sqrt{E_i(\text{eV})} \\ v_e\ (\text{cm/sec}) &= 7.3 \times 10^{7}\ \sqrt{E_e(\text{eV})} \end{aligned}\right\} \qquad (12)$$

For plasma in the octupole, $v_i = 1.1 \times 10^7$ cm/sec and $v_e = 2.3 \times 10^8$ cm/sec.

Cross sections for the various reactions of interest are scattered throughout the literature; however, a useful collection is included in a report by ORNL entitled "Atomic and Molecular Collision Cross Sections of Interest in Controlled Thermonuclear Research" (ORNL 3113, revised, UC-20-Controlled Thermonuclear Processes, TlO-4500, 30th ed.). Page numbers in the following discussion refer to this work. The most interesting reactions are elastic scattering of ions and electrons off molecular hydrogen, and charge exchange between hot ions and thermal hydrogen molecules.

(4) Ion-H_2 Collisions:

From ORNL 3113, Page 191, the elastic scattering cross section σ_{iH_2} for 40 eV protons is about $7 \times 10^{-16} cm^2$ and decreases with increasing energy. Substituting into equation (9) leads to the result:

$$\tau_{iH_2} = \frac{1.3 \times 10^8}{n} \quad . \tag{13}$$

This result is plotted in Figure 2. At 10^{-6} Torr, the ion-H_2 collision time is about 400 μsec.

(5) Electron-H_2 Collisions:

The elastic scattering cross section for electrons on molecular hydrogen is given on page 187 as $\sigma_{eH_2} = 6 \times 10^{-17}$ for E_e = 10 eV. σ_{eH_2} decreases with increasing electron energy. The corresponding collision time is

$$\tau_{eH_2} = \frac{7.2 \times 10^7}{n} \quad . \tag{14}$$

The result is plotted in Figure 2. τ_{eH_2} is about half as large as τ_{iH_2}.

(6) Charge Exchange:

Finally, we consider collisions in which a hot ion gains an electron from a cold molecule and leaves a hot neutral and a cold ion. This reaction is called charge exchange or electron capture and is a serious loss mechanism at high energies. The charge exchange cross section is given on page 65 and increases very sharply with energy. It is not plotted below 50 eV, but it appears that $\sigma_{ch\ ex}$ is less than $10^{-17}cm^2$ at 40 eV. Using this value of $\sigma_{ch\ ex}$ gives a loss time of

$$\tau_{ch\ ex} = \frac{1 \times 10^{10}}{n} \quad . \tag{15}$$

The charge exchange time is plotted in Figure 2. It is larger than any other times previously discussed, but this is only because of the low energies in the octupole.

Note that since collision frequencies add directly,

$$\nu = \sum_i \nu_i \quad ,$$

the collision time for any group of reactions is found from

$$\frac{1}{\tau} = \sum_i \frac{1}{\tau_i} \quad . \tag{16}$$

REFERENCES

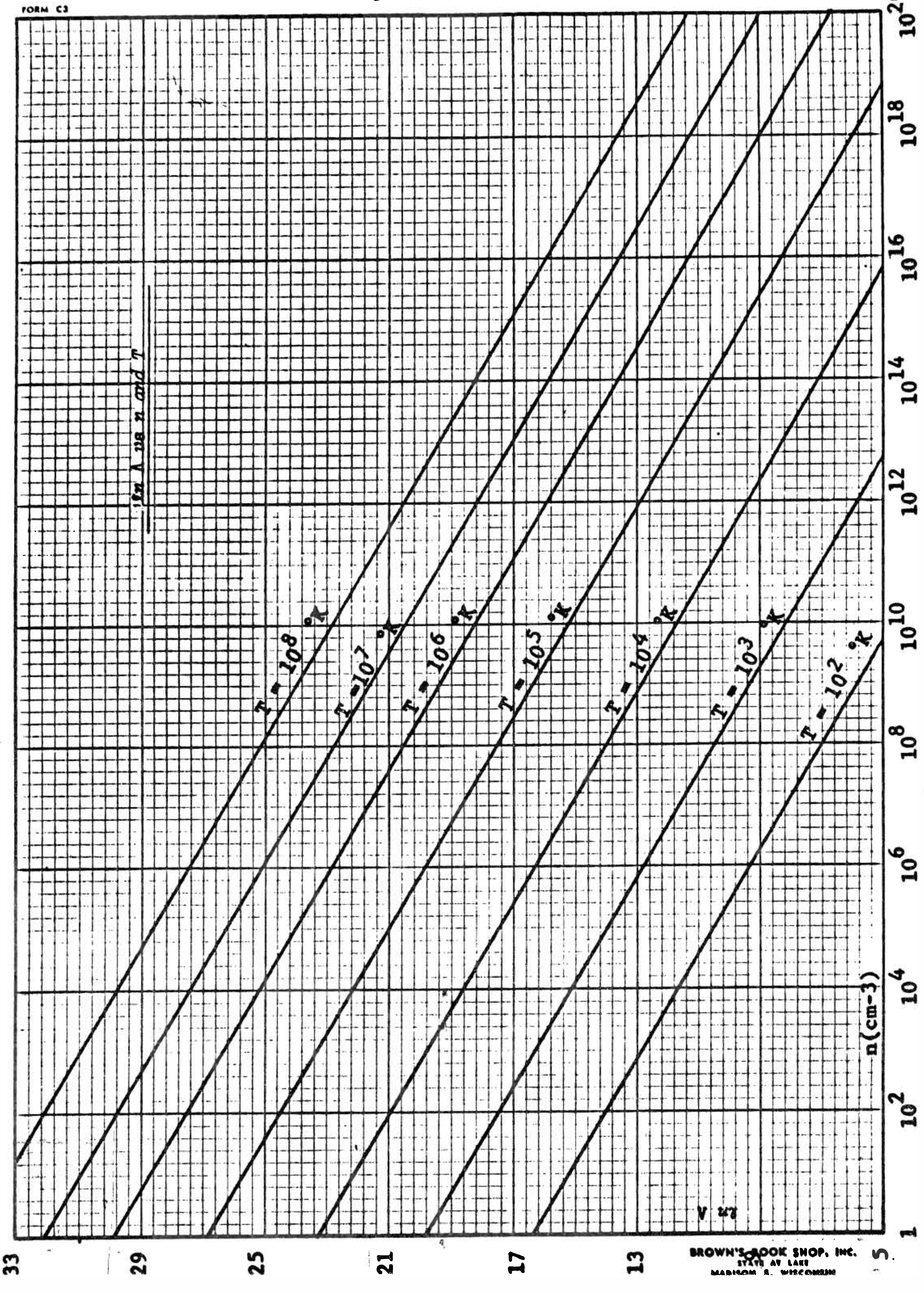
1. L. Spitzer, Physics of Fully Ionized Gases, second edition, page 133.

2. L. Spitzer, page 128.

3. D. J. Rose and M. Clark, Plasmas and Controlled Fusion, page 167.

4. L. Spitzer, page 135.

Figure 1



FORM C3
ℓn Λ vs n and T
T = 10^8 °K
T = 10^7 °K
T = 10^6 °K
T = 10^5 °K
T = 10^4 °K
T = 10^3 °K
T = 10^2 °K
n(cm-3)
ℓn Λ
1
10^2
10^4
10^6
10^8
10^10
10^12
10^14
10^16
10^18
10^20
33
29
25
21
17
13
5
BROWN'S BOOK SHOP, INC.
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MADISON 5, WISCONSIN

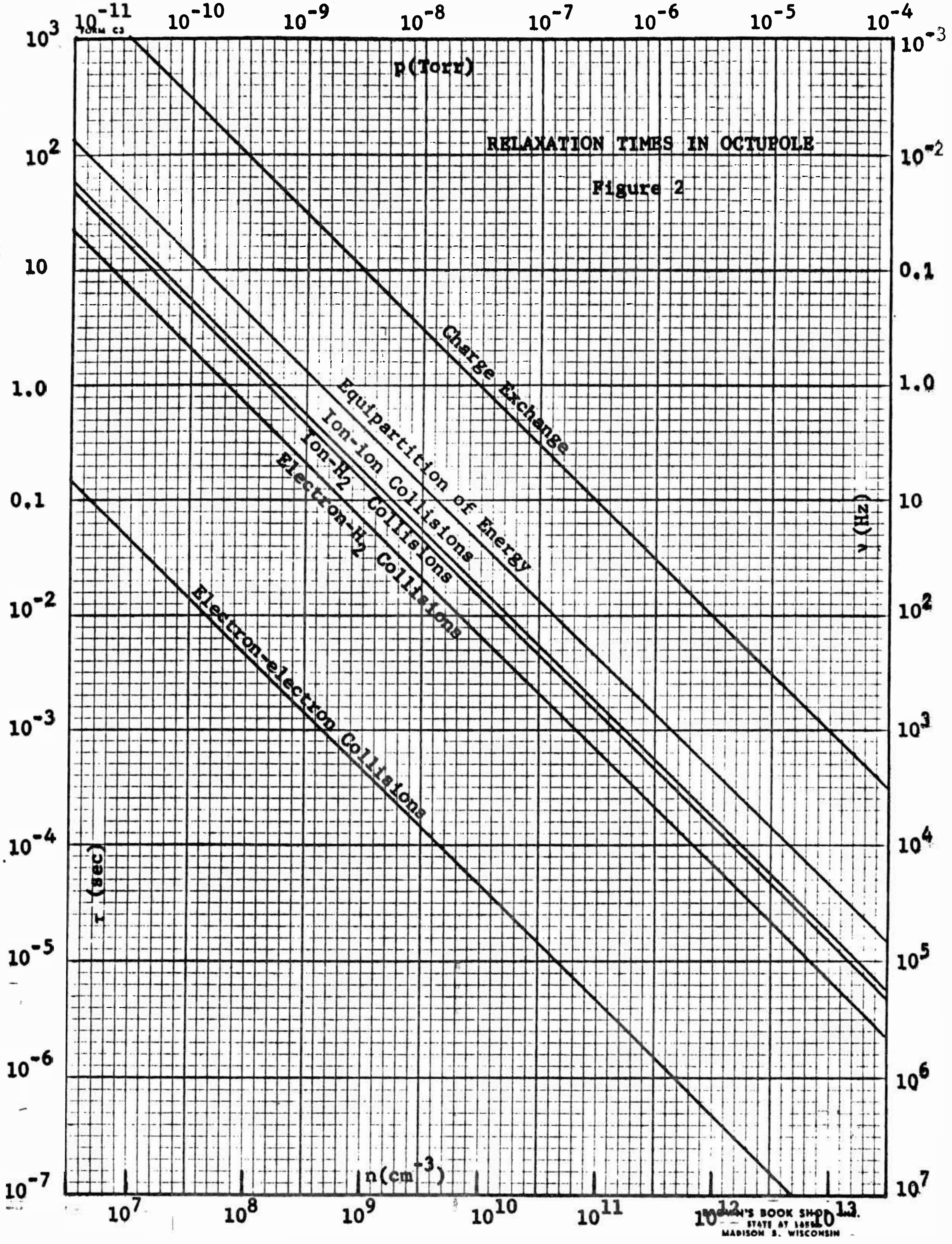

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Figure 2