GENERAL FORMULAS FOR FLAT-TOPPED WAVEFORMS

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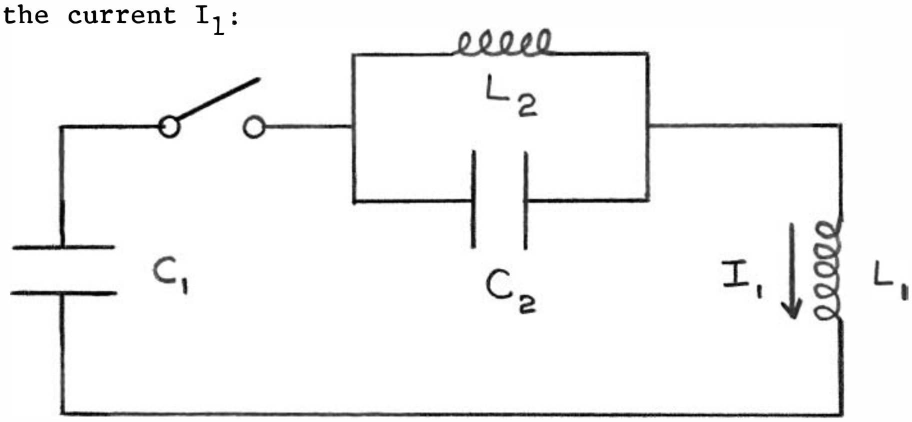
In PLP 849, Rose and Kerst discussed circuitry for producing a flat top on the Levitated Octupole magnetic field waveform. This note summarizes their results and treats several cases not considered by them for a general capacitive discharge into an inductive load.

The first case considered by Rose and Kerst as shown below has a series parasitic LC circuit whose purpose is to add a third-harmonic component to the current I_1:



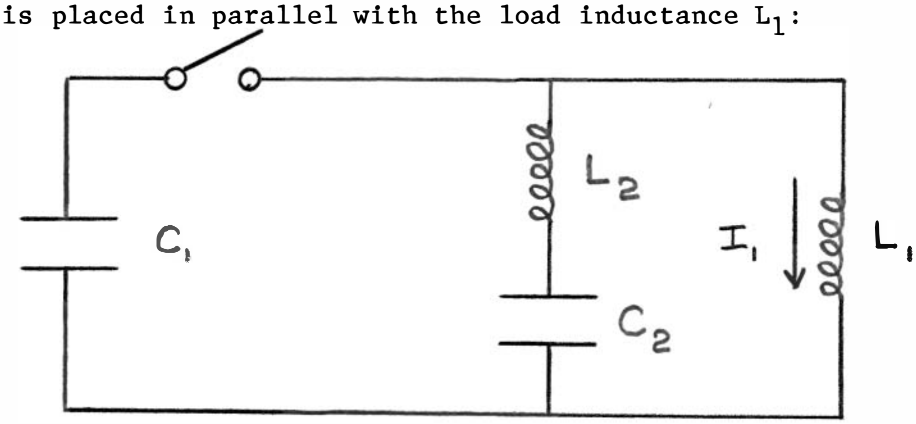
CIRCUIT A

Solutions are sought which are of the form

$$I_1 = I_0 \left(\sin \omega t + \frac{1}{S} \sin 3\,\omega t\right)$$

where $t = 0$ is taken as the time the switch closes. The quantity S determines the amount of third harmonic. As can easily be shown, $S = 9$ represents the condition for which $\ddot{I}(t) = 0$ at the time at which $\dot{I}(t) = 0$ and thus represents a reasonable criterion for best flattening. For $S > 9$ the waveform has a single peak at $\omega t = \pi/2$, and for $S < 9$ it has two peaks on either side of a dip at $\omega t = \pi/2$. The results of Rose and Kerst for $S = 9$ and C_2 initially discharged are summarized in Table I.

Rose and Kerst also considered the case below in which the parasitic LC is placed in parallel with the load inductance L_1:

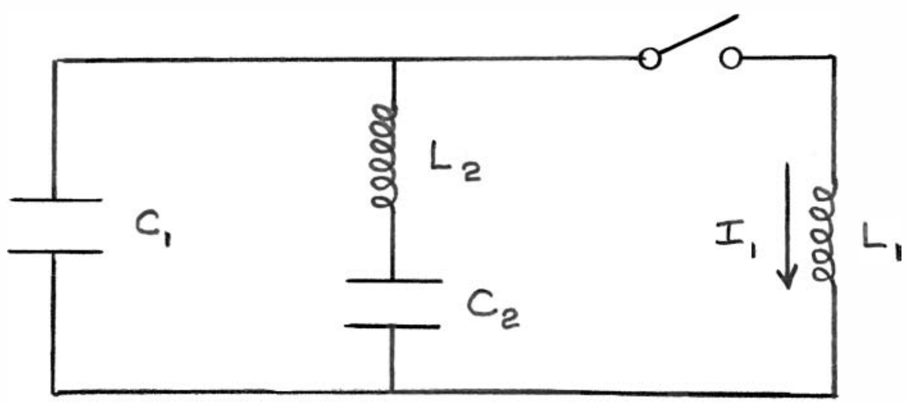


CIRCUIT B

This circuit requires less capacitance in the parasite and doesn't require one to float C_2 or L_1, but it does require a larger L_2 and a higher voltage capacitor for C_2 as shown in Table I.

A problem with both circuit A and circuit B is that the parasite robs energy from L_1 and thus lowers the peak field. Rose and Kerst propose charging C_2 with an initial voltage (0.75 $V_{C1}(0)$ for circuit A and 2.4 $V_{C1}(0)$ for circuit B) to achieve the same peak current in 1, as would have existed in the absence of the parasite. This would require a separate switch in series with C_2, however. Furthermore, for both cases, the voltage rating for the parasitic capacitor is different from that of the main capacitor bank.

An alternative solution, not discussed in PLP 849 is to place the series parasite of circuit B on the other side of the switch and charge it to the same voltage as the main bank as shown below:



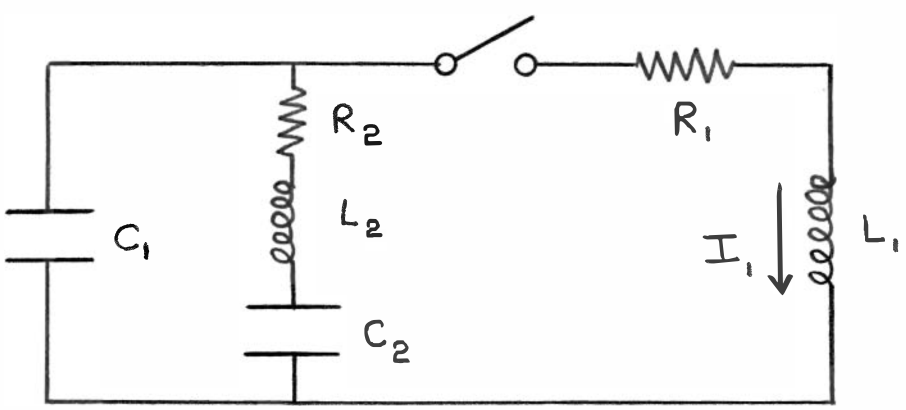
CIRCUIT C

This circuit has the following desirable features:

1) Only one switch is required.
2) A separate charging circuit for C_2 is not required.
3) Both capacitors and L_1 have a common ground.
4) All capacitors can have the same voltage rating.
5) One can change from the flattest field to the highest field by shorting out L_2.

Circuit C can be implemented by taking a portion of an existing bank and adding an appropriate series inductor. The required values are listed in Table I. The effect of varying L_2 with the other values fixed is illustrated in figures 1 and 2 respectively. Note that for values away from the optimum, the symmetry of the waveform is destroyed. Rose and Kerst point out that this may be useful for countering resistive losses in the circuit, emulating a power crowbar. One should be cautious, however, because deviations from the ideal value in either direction generally causes in excess of 100% voltage reversal of the parasitic capacitor in the absence of resistive losses.

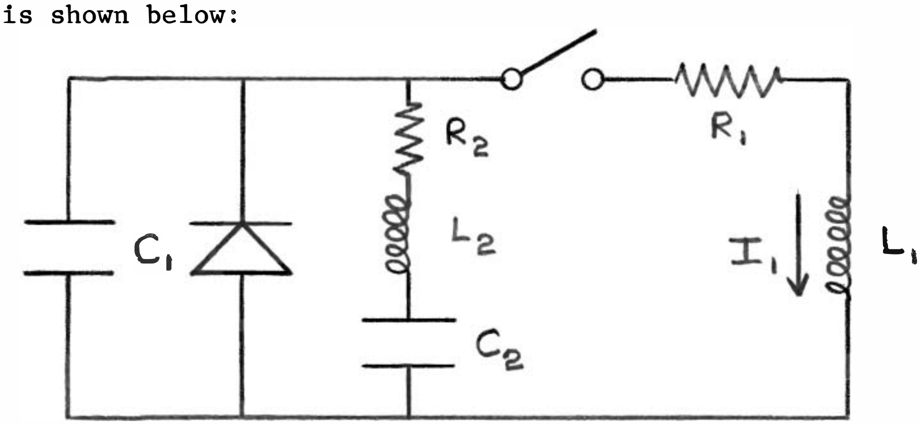
A real circuit contains resistive losses in both inductors and can be represented as follows:



CIRCUIT D

Rather than attempt an analytic solution, the optimum values of L_2 and C_2 were determined by numerical solution of I(t) with $R_2/L_2 = R_1/L_1$ for $Q = \omega L_1/R_1 = 1$ and Q = 2, using the qualitative behavior of figures 1 and 2 for guidance (see Table II). The optimum values were then fit to functions of the form $1 + \alpha/Q^\beta$. The results of this procedure are given in Table I. Figure 3 shows the waveforms for (1) no parasite, (2) parasite calculated neglecting losses, and (3) optimized parasite.

Often it is desirable to clamp the capacitor voltage at zero when it attempts to reverse. In the absence of resistive losses, such a crowbar would hold the current I_1 constant forever, and no parasite would be required. With resistance, a crowbar may still be desirable to prolong the current and prevent voltage reversal on the capacitor bank. Such a circuit is shown below:



CIRCUIT E

The addition of the diode across C_1 has no effect on the preceeding calculations except to reduce the voltage reversal on the parasitic capacitor (see Table I). Voltage reversal of C_2 can be eliminated entirely by providing it with its own crowbar diode, and there will be no effect on I_1 since the voltage across C_2 reverses after the voltage across C_1 has been clamped to zero. The resulting waveforms are shown as dashed curves on figure 3.

As a design example, we will calculate the parasite that would be required to flatten the toroidal field waveform on Tokapole II. For a charging voltage of 3000 volts, the present circuit gives a toroidal field as shown in figure 4, where $t = 0$ is 6.0 msec after the pulse begins. From the toroidal field, one can calculate the winding current from $I_1 = 2.5 \times 10^6$ B/N where B is the field in teslas and N = 96 is the number of turns. Over the range of interest ($0 < t < 20$ msec), the current is adequately modeled by a crowbarred RLC with $C_1 = 0.031$ F, $L_1 = 1.3$ mH and $R_1 = 80$ mΩ. From circuit E of Table I, one can calculate $\omega = 91$ sec^{-1}, $C_2 = 0.060$ F, $L_2 = 700$ μH, $R_2 = 43$ mΩ, and voltage reversal = 19%. The cost of optimally wound copper coils is estimated to be \$28/msec/meter of length (installed). Thus for a one-meter-long coil, the cost would be ~\$500. In practice, we would probably want an inductor with various taps so that one could optimize the flattening (at ~ 8kG for $L_2 = 700$ μH and 5 kV charge) or increase the field (to ~ 10 kG for $L_2 < 100$ μH and 5 kV charge). Figure 5 shows the resulting waveforms for various values of L_2 (for constant R_2/L_2).

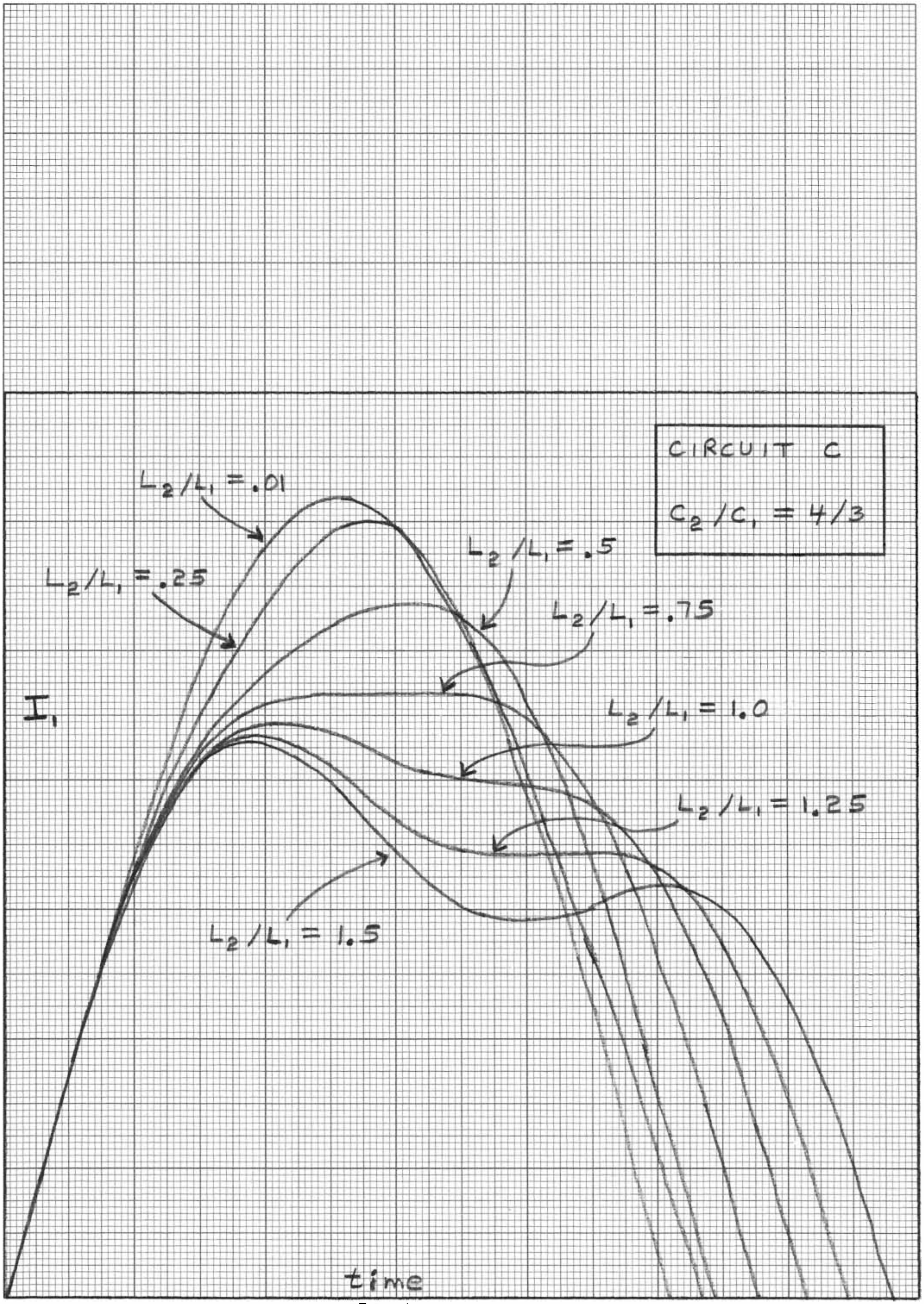
TABLE I
CIRCUIT PARAMETERS FOR FLATTEST WAVEFORM

CIRCUIT	A	B	C	D	E
ω	$\left(\frac{L_1+L_2}{3L_1L_2C_2}\right)^{1/2}$	$\left(\frac{L_1+L_2}{3L_1L_2C_1}\right)^{1/2}$	$\left(\frac{1}{3L_1C_1}\right)^{1/2}$	$\left(\frac{1}{3L_1C_1}\right)^{1/2}$	$\left(\frac{1}{3L_1C_1}\right)^{1/2}$
C_2/C_1	0.75	0.244898	1.333333	$\frac{4}{3} + 0.934 \left(\frac{R_1}{\omega L_1}\right)^{1.08}$	$\frac{4}{3} + 0.934 \left(\frac{R_1}{\omega L_1}\right)^{1.08}$
L_2/L_1	0.244898	0.75	0.75	$\frac{3}{4} - 0.284 \left(\frac{R_1}{\omega L_1}\right)^{0.793}$	$\frac{3}{4} - 0.284 \left(\frac{R_1}{\omega L_1}\right)^{0.793}$
% REVERSAL	32.9914%	134.7151%	100%	$100-89\left(\frac{R_1}{\omega L_1}\right)^{0.745}$%	$100-91.5 \left(1-\frac{R_1}{\omega L_1}\right)^{0.302}$%

TABLE II

ALGORITHM FOR OPTIMIZING WAVEFORM

$$\text{DIP?}\begin{cases}\text{NO:} & \text{RAISE } L_2 \text{ OR } C_2\\ \text{YES:} & \begin{cases}1^{\text{st}} \text{ PEAK HIGHER? LOWER } L_2\\ 2^{\text{nd}} \text{ PEAK HIGHER? LOWER } C_2\end{cases}\end{cases}$$


CIRCUIT C
C2/C1 = 4/3
L2/L1 = .01
L2/L1 = .25
L2/L1 = .5
L2/L1 = .75
L2/L1 = 1.0
L2/L1 = 1.25
L2/L1 = 1.5
I1
time
FIG. 1

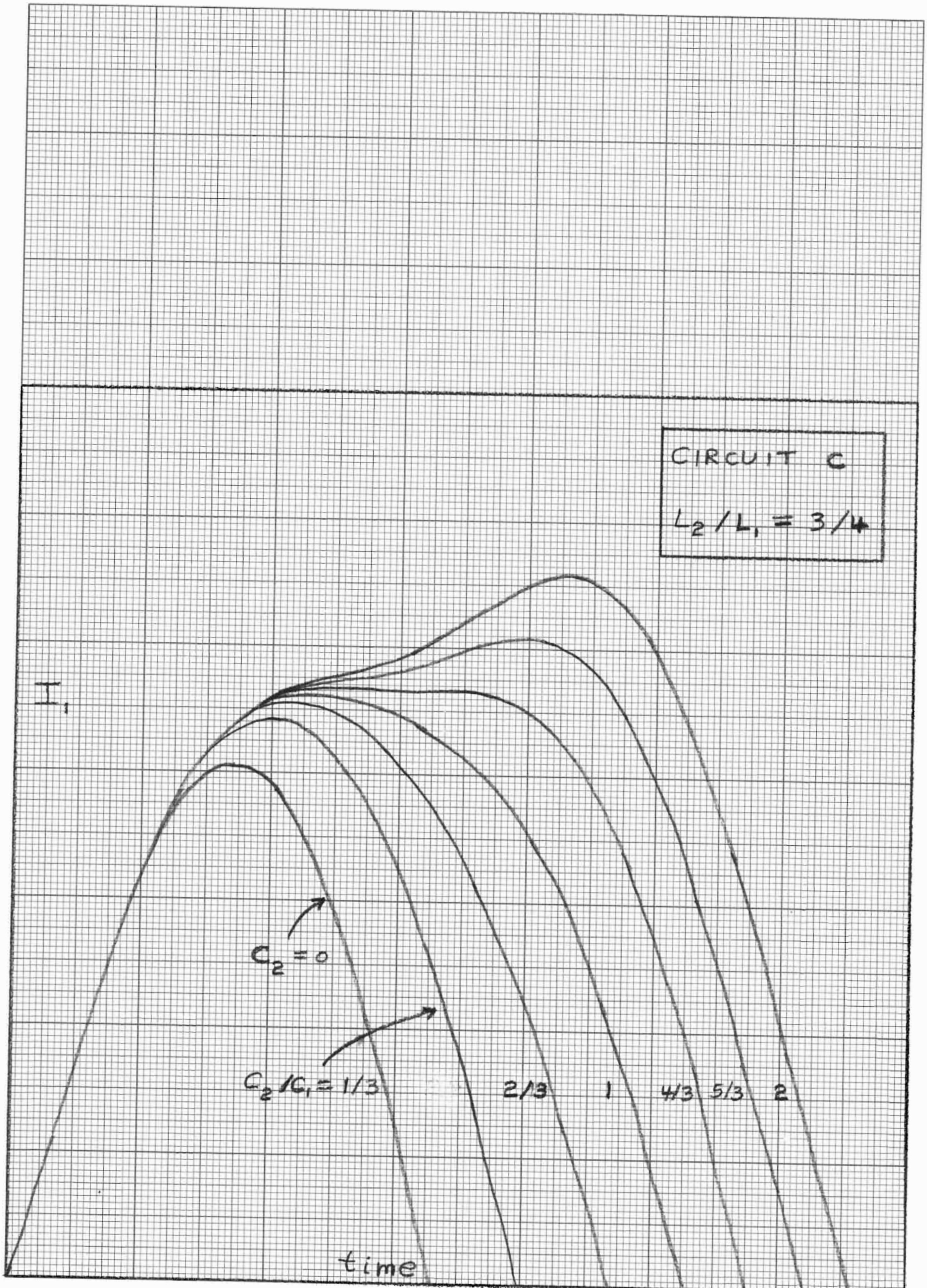


FIG. 2

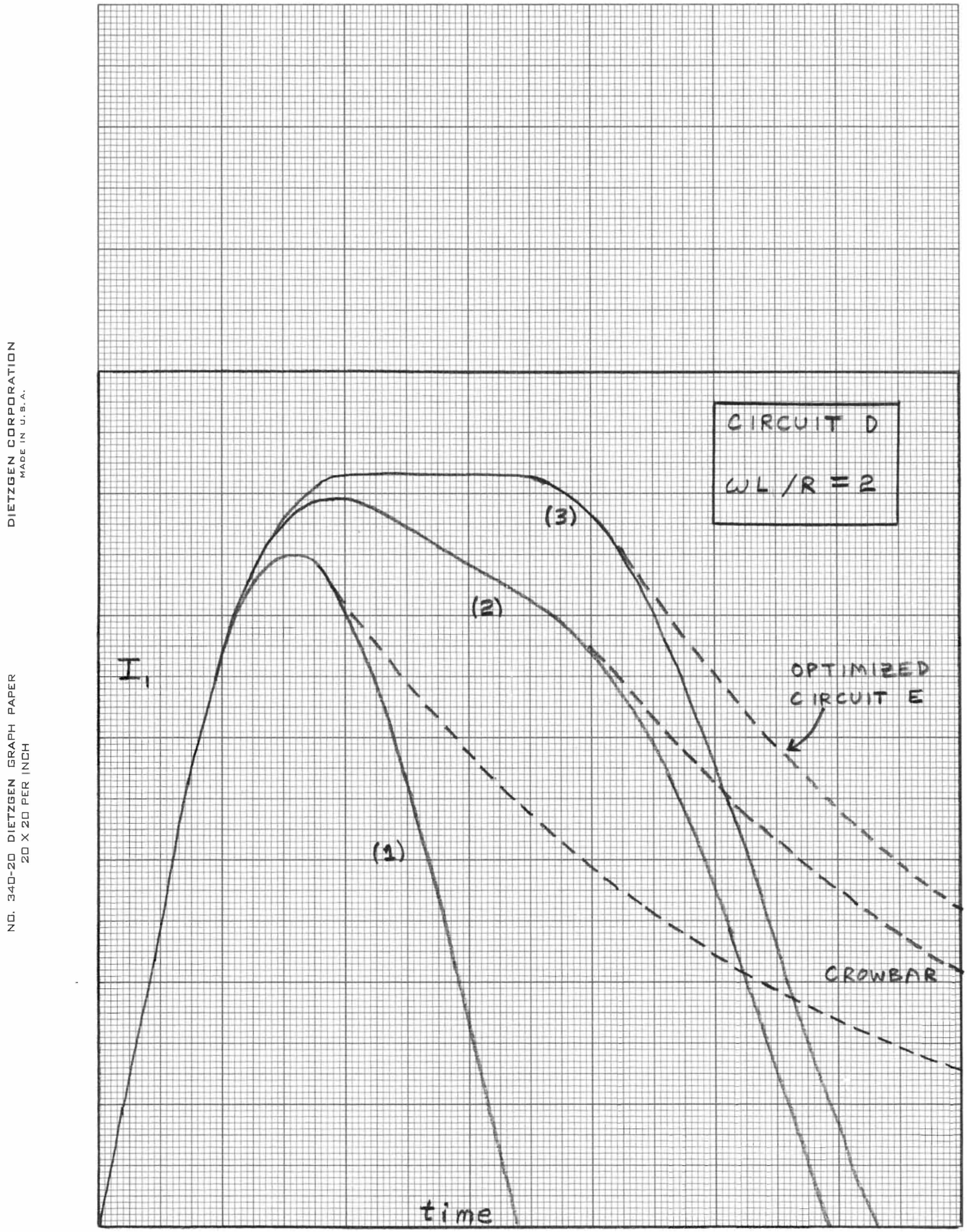

CIRCUIT D
ωL/R = 2
(3)
(2)
(1)
I1
OPTIMIZED
CIRCUIT E
CROWBAR
time
NO. 340-20 DIETZGEN GRAPH PAPER
20 X 20 PER INCH
DIETZGEN CORPORATION
MADE IN U.S.A.

FIG. 3

$V_c = 2998$

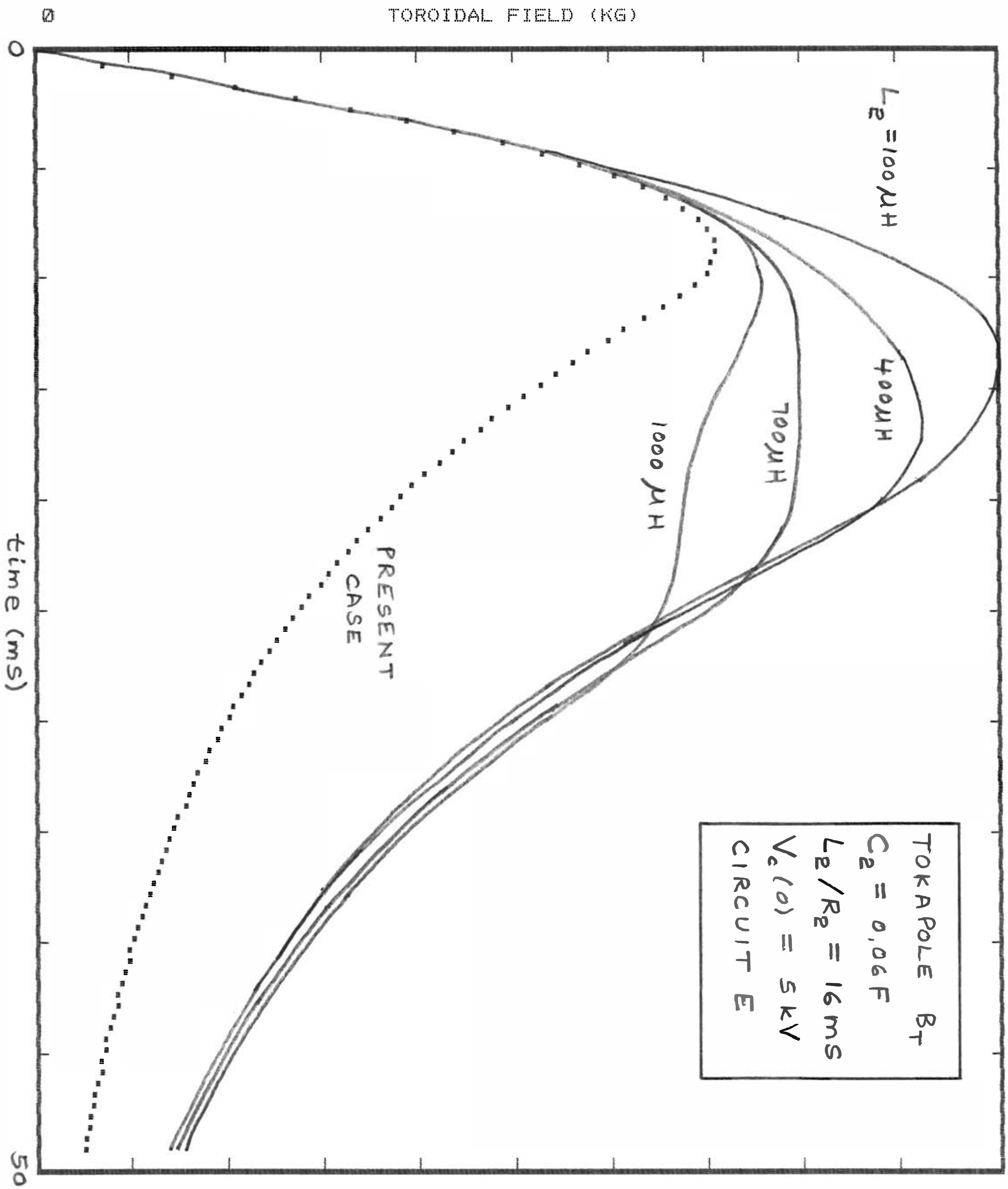
BASELINE
MAX IP = 0
AMP SECONDS = 0
PRES = 0 E-5 TORR

TIME	IP	JSAT	BT	VPG	IHOOP	DIP/DT
2	-2	0	4080	7.5	230	.8
4	-1	0	4160	-3.5	176	.2
6	-1	-10	3920	-2.5	134	.8
8	-1	-10	3440	-2	104	.2
10	-1	0	3040	-1.5	80	0
12	0	-10	2640	-1.5	64	-.8
14	-1	0	2320	-1	50	-1
16	-1	-10	2000	-1	40	-.2
18	-1	0	1840	-.5	32	0
20	0	-10	1600	-.5	26	.2

```
0 AS           TOROIDAL FIELD VS TIME                   BASELINE
               MAXIMUM VALUE: 4240

X   X XX XX XX XX
XX X              XX XX
X                       XX
X                          XX XX
X                                XX XX X
X                                       X XX
X                                            XX XX XX XX
X                                                        XX XX
X
X
X
XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0     2     4     6     8    MSEC   12    14    16    18    20
```

FIG. 4



TOKAPOLE II TOROIDAL FIELD CIRCUIT - 50 MSEC FULL SCALE
MAIN BANK: C = .031 F L = 1.3E-03 H R = .08 OHMS

FIG. 5