

TIME-INVARIANT HOOPS

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# Time-Invariant Hoops

J.C. Sprott

## I. Introduction

Toroidal devices such as Tokapole II with inductively driven rings (or hoops) and plasma currents are relatively easy to construct but are constrained to a ratio of plasma current to hoop current that is determined by the plasma and hoop resistance and inductance. Since the driving electric field will in general penetrate the plasma, it must also penetrate the hoops if the magnetic flux plot produced by the superposition of the plasma and hoop currents is to retain its shape in time. At the beginning of the current pulse the electric field will not have penetrated into the hoop, and the hoop surface will be a magnetic flux surface. If the flux plot is to retain its shape as the field soaks into the hoop, the soak-in must occur in such a way that the hoop surface always lies on a flux surface. In general, this will require that the hoop have a non-uniform electrical conductivity over its cross-section. One such solution in which the hoop consisted of a low conductivity core surrounded by a variable thickness, high conductivity shell was described in PLP 962. That design suffers from mechanical stress limitations. The purpose of this note is to derive a more general class of solution that provides additional free parameters for engineering optimization.

The problem consists of solving Maxwell's equations and Ohm's law with a spatially varying resistivity in the interior of a region with boundary conditions on the magnetic field at the surface. The appropriate equations with displacement currents neglected are

$$\nabla \times \vec{E} = \mu_0 \vec{J} \quad (1)$$

$$\nabla \times \vec{B} = - \frac{\partial \vec{E}}{\partial t} \quad (2)$$

$$\nabla \cdot \vec{B} = 0 \quad (3)$$

$$\vec{E} = \rho \vec{J} \quad (4)$$

which can be combined into a diffusion equation

$$\nabla^2 \vec{B} - \frac{\nabla \rho}{\rho} \times (\nabla \times \vec{B}) = \frac{\mu_0}{\rho} \frac{\partial \vec{B}}{\partial t} \quad (5)$$

subject to the boundary conditions  $B_n = 0$  and  $B_t = f(\vec{r}, t)$ , where  $f(\vec{r}, t)$  is the time-dependent, tangential magnetic field at the surface of a perfectly conducting hoop of the same cross-section.

Rather than solve the full diffusion equation (5), we will solve for the resistive ( $\partial \vec{B} / \partial t = 0$ ) limit and argue that a hoop that has the proper boundary conditions in the inductive ( $t = 0$ ) and resistive ( $t \rightarrow \infty$ ) limits will probably closely approximate the desired solution at intermediate times. As an added condition we can require that the variation of the conductivity at the surface be such that the boundary condition is satisfied asymptotically at early times ( $t \rightarrow 0$ ) when the field has soaked only a small distance into the hoop.

## II. Solution of $\sigma(r,\theta)$ for Circular Cross-Section Hoop

As a specific example, we will consider a hoop of circular cross-section and sufficiently large aspect ratio that the calculation can be done using a cylindrical approximation with the axis of the hoop at  $r=0$ . From  $\vec{B} = \nabla \times \vec{A}$ , we can define a flux function  $\psi = -A_z$ . From equation (1) and the Coulomb gauge ( $\nabla \cdot \vec{A} = 0$ ), the current density is  $\vec{j} = -\nabla^2 \vec{A}$ , or in cylindrical coordinates,

$$j_z = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \psi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \theta^2} \quad (6)$$

The components of the magnetic field are

$$B_\theta = \frac{\partial \psi}{\partial r} \quad B_r = -\frac{1}{r} \frac{\partial \psi}{\partial \theta} \quad (7)$$

Without loss of generality, we can define the flux to be zero at the origin and 1 at the surface of the hoop, which we take to have a dimensionless radius of  $r_0=1$ . We consider only the  $m=0$  and  $m=1$  poloidal modes of the field variation at the surface of the hoop and write the boundary conditions thus:

$$\psi(r=0) = 0 \quad (8)$$

$$\psi(r=1) = 1 \quad (9)$$

$$B_\theta(r=1) = \alpha(1 + \epsilon \sin \theta) \quad (10)$$

$$j_z(r=1) = \beta(1 + \epsilon \sin \theta)^2 \quad (11)$$

In the resistive limit, the electric field  $\vec{E}$  is constant over the cross-section of the hoop, and the solution of the desired conductivity  $\sigma(r, \theta)$  is the same as the solution of  $j_z(r, \theta)$  since  $\vec{j} = \sigma \vec{E}$ . However, solutions in which  $\vec{j}$  is not unidirectional would be unphysical.

As stated, the problem is under-specified, and a variety of solutions is possible. In order to proceed, we will adopt the strategy of choosing a  $\psi$ -function that is sufficiently simple to evaluate analytically and yet sufficiently complicated to allow the boundary conditions to be satisfied with one free parameter that can be varied to exhibit a range of possibilities and to give some freedom in meeting engineering constraints. The solutions thus obtained are not unique but are representative and suggest more realistic possibilities that could actually be built. A function that meets the requirements is

$$\psi(r, \theta) = a(\theta)r + b(\theta)r^2 + c(\theta)r^3 + d(\theta)r^4 \quad (13)$$

From the boundary conditions and the requirement that  $j_z$  be finite and continuous at  $r=0$ , we can derive the coefficients of equation (13):

$$a = \frac{\alpha\epsilon}{2} \sin \theta \quad (14)$$

$$b = 3 + d_0 - \alpha - \frac{\alpha\epsilon^2}{2} \cos 2\theta \quad (15)$$

$$c = \alpha - 2 - 2d_0 + \frac{\alpha\epsilon}{2} \sin \theta + \alpha\epsilon^2 \cos 2\theta \quad (16)$$

$$d = d_0 - \frac{\alpha\epsilon^2}{2} \cos 2\theta \quad (17)$$

where  $d_0$  is a free parameter. Alpha and beta are given by

$$\alpha = \frac{\beta}{2} = \frac{2(3-d_0)}{3-\epsilon^2} \quad (18)$$

Substituting into equation (13) gives

$$\begin{aligned} \psi = & -\frac{\alpha\epsilon}{2} r \sin \theta + (3 - \alpha + d_0 - \frac{\alpha\epsilon^2}{2} \cos 2\theta)r^2 \\ & + (\alpha - 2 - 2d_0 + \frac{\alpha\epsilon}{2} \sin \theta + \alpha\epsilon^2 \cos 2\theta)r^3 \\ & + (d_0 - \frac{\alpha\epsilon^2}{2} \cos 2\theta)r^4 \end{aligned} \quad (19)$$

From equation (6) we obtain

$$\begin{aligned} j_z = & 12 - 4\alpha + 4d_0 + (9\alpha - 18 - 18d_0 + 4\alpha\epsilon \sin \theta \\ & + 5\alpha\epsilon^2 \cos 2\theta)r + (16d_0 - 6\alpha\epsilon^2 \cos 2\theta)r^2 \end{aligned} \quad (20)$$

A useful parameter is the ratio of maximum to minimum value of B at the hoop surface:

$$v = \frac{B_{MAX}}{B_{MIN}} = \frac{1+\epsilon}{1-\epsilon} \quad (21)$$

Contours of  $\psi$  and  $j_z$  (or  $\sigma$ ) over the cross-section of the hoop for  $v=3$  and various values of  $d_0$  are shown in figures 1-9. The solutions span the range from a low conductivity core surrounded by a highly conducting shell ( $d_0 < 0$ ) to a highly conducting core surrounded by a low conductivity shell ( $d_0 > 0$ ). The contours are normalized to a value of unity at the top of the figure ( $\theta = \pi/2$ ) where  $B$  at the surface is maximum and are in units of 0.1.

### III. A Practical, Non-Uniformly Conducting Hoop

Guided by the solutions of the previous section, we are led to consider practical implementations<sup>5</sup> in which the hoop is constructed of two distinct materials of conductivity  $\sigma_0$  and  $\sigma_1$  with one material embedded in a circular cavity in the other but with a center offset from the axis of the hoop by an amount  $\delta$  as shown in figure 10. Such a configuration allows one to calculate the fields and magnetic flux external to the hoop in the resistive limit by the superposition principle with the current concentrated in filaments at the center of the circles. To the fields produced by the hoop must be added a field due to the superposition of all the other currents including the image currents of the hoop in nearby conductors, but it turns out there is an easy way to approximate this field.

For a hoop constructed in this fashion, the average conductivity is

$$\sigma = \sigma_0 + (\sigma_1 - \sigma_0) r_1^2 / r_0^2 \quad (22)$$

Given a hoop radius, a desired average conductivity and two materials,  $r_1$  can be calculated from

$$r_1 = r_0 \sqrt{\frac{\sigma - \sigma_0}{\sigma_1 - \sigma_0}} \quad (23)$$

In terms of the desired total current  $I$  in the hoop, the individual filamentary currents are given by

$$I_0 = \sigma_0 I / \sigma \quad (24)$$

$$I_1 = (\sigma - \sigma_0) I / \sigma \quad (25)$$

Now if we approximate the field due to all the currents external to the hoop by a constant  $B_0$  as shown in figure 10, we can express  $B_0$  in terms of  $v$  as

$$B_0 = \frac{v-1}{v+1} \frac{\mu_0 I_0}{2\pi r_0} + \frac{\mu_0 I_1}{2\pi(v+1)} \left[ \frac{v}{r_0 + \delta} - \frac{1}{r_0 - \delta} \right] \quad (26)$$

If we require that the flux function  $\psi$  have the same value at the surface of the hoop on the high field and low field sides, we obtain the result:

$$B_0 = \frac{\mu_0 I_1}{4\pi r_0} \ln \left( \frac{r_0 + \delta}{r_0 - \delta} \right) \quad (27)$$

Equations (26) and (27) are a pair of simultaneous non-linear equations



which can be solved for  $\delta$  and  $B_0$ . To lowest order in  $\delta/r_0$ , the solutions are

$$\delta \cong \frac{r_0}{2} \frac{\nu-1}{\nu+1} \frac{\sigma}{\sigma-\sigma_0} \quad (28)$$

$$B_0 \cong \frac{\mu_0 I}{4\pi r_0} \frac{\nu-1}{\nu+1} \quad (29)$$

More accurate representations of  $\delta$  and  $B_0$  can be obtained by successive iteration, but the lowest order results are sufficient for most purposes.

In the region external to the hoop, the flux per unit length is given by

$$\psi(x,y) = B_0 y + \frac{\mu_0 I_0}{4\pi} \ln(x^2+y^2) + \frac{\mu_0 I_1}{4\pi} \ln [x^2+(y-\delta)^2] \quad (30)$$

Equation (30) should be reasonably accurate near the surface of the hoop but becomes less so as one moves away from the hoop because the assumption of  $B_0 = \text{constant}$  is progressively less valid. Nevertheless, the flux plot does contain a separatrix and strongly resembles realistic, numerically calculated cases.

Figure 11 shows a flux plot for a perfectly conducting hoop, represented by choosing a very large value for  $\sigma_1$  so that all the current flows on the surface of the hoop. This case has  $\nu=3$ , which for the Tokapole would represent  $I_{\text{plasma}}/I_{\text{hoop}} \sim 0.16$ . Figure 12 shows the same case but in

the resistive limit after the field has completely soaked into a hoop with uniform resistivity. The separatrix is still outside the hoop but only barely. Figure 13 shows a copper-clad stainless steel hoop as described in PLP 962. Figure 14 shows a case with an aluminum core and an inconel shell, illustrating that the same effect can be obtained with the high conductivity material in the interior. Figure 15 shows the opposite case, with the aluminum on the outside. Figures 14 and 15 have been adjusted to give the highest average conductivity allowed by these materials, and so the shell has zero thickness on one side.

To test the sensitivity of the hoop design to the ratio of plasma current to hoop current, the hoop in figure 15 that was optimized for  $v=3$  was run at  $v=2$  (corresponding approximately to the no-plasma limit in Tokapole II), and the result is shown in figure 16. Similarly, if  $v=5$ , the hoop designed for  $v=3$  yields a flux plot as shown in figure 17. This case corresponds to a ratio of  $I_{\text{plasma}}/I_{\text{hoop}}$  about twice the design value in Tokapole II.

Hoops with the high conductivity material on the inside give external flux plots which are essentially the same as those with the high conductivity material on the outside. However, the internal inductance is quite different for the two cases, and this difference can be important because it represents an additional volt-second consumption. This internal inductance can be calculated by equating  $LI^2/2$  to the total magnetic energy inside the hoop, which we obtain by integrating  $B^2$  over the volume of the hoop:

$$L = \frac{\ell}{\mu_0 I^2} \int_0^{r_0} B^2 2\pi r dr \quad (31)$$

The general case is rather difficult to calculate since B has a complicated spatial dependence inside the hoop. However, as an approximation, we can consider the case of  $\delta=0$  (or  $v=1$ ). Then the magnetic field in the resistive limit is given by

$$B = \begin{cases} \frac{\mu_0 I r \sigma_1}{2\pi r_0^2 \sigma} & 0 < r < r_1 \\ \frac{\mu_0 I}{2\pi r \sigma} \left( \sigma - \sigma_0 + \frac{r^2}{r_0^2} \sigma_0 \right) & r_1 < r < r_0 \end{cases} \quad (32)$$

Substituting into equation (31) and integrating gives

$$L = \frac{\mu_0 \ell}{8\pi \sigma^2} \left[ 2(\sigma - \sigma_0)^2 \ln \left( \frac{\sigma_1 - \sigma_0}{\sigma - \sigma_0} \right) + \sigma_0^2 + (\sigma_1^2 - \sigma_0^2) \frac{(\sigma - \sigma_0)^2}{(\sigma_1 - \sigma_0)^2} + 4\sigma_0(\sigma - \sigma_0) - \frac{4(\sigma - \sigma_0)^2 \sigma_0}{\sigma_1 - \sigma_0} \right] \quad (33)$$

which apparently cannot be simplified significantly. However, for the usual case where one of the conductivities is negligibly small, equation (33) can be simplified considerably. If  $\sigma_0=0$  and  $\lambda_1=\sigma_1/\sigma$ , then

$$L = \frac{\mu_0 \ell}{8\pi} (2 \ln \lambda_1 + 1) \quad (34)$$

If  $\sigma_1=0$  and  $\lambda_0=\sigma_0/\sigma$ , then

$$L = \frac{\mu_0 \ell}{8\pi} [2(\lambda_0-1)^2 \ln \left( \frac{\lambda_0}{\lambda_0-1} \right) - 2\lambda_0 + 3] \quad (35)$$

These inductances, normalized to the internal inductance of a hoop of constant resistivity ( $L_0=\mu_0 \ell/8\pi$ ), are plotted in figure 18. The advantage of placing the high conductivity material on the outside is readily apparent.

In a practical design situation, the procedure would be as follows:

1) Find a hoop size and position that is acceptable in the inductive limit for the highest ratio of plasma current to hoop current imagined using standard flux plot codes.

2) From the flux plots, determine the ratio of currents in the various hoops, the forces, and the variation of B around each hoop (v).

3) Choose an average conductivity such that the L/R time of the hoops is approximately equal to the L/R time of the plasma (see PLP 962) and adjust the ratio of hoop resistivities slightly so that the resistive division of currents among the hoops is the same as the inductive division.

4) Choose two hoop materials whose conductivities bridge the desired average and whose strength is sufficient to withstand the peak magnetic forces.

5) Determine the required values of  $r_1$  and  $\delta$  as described above.

6) Iterate as required to yield an optimal solution.

#### IV. Design Example: Improved Tokapole Hoops

As described in PLP 962, Tokapole hoops of the same size as the present hoops but with an average resistivity of  $\sim 7 \mu\Omega\text{-cm}$  and total current of 500 kA should permit plasma currents of  $\sim 80$  kA. An inductive flux plot for  $I_p/I_h = 80/492$  kA with hoops of the present size and in their present position is shown in figure 19. Although the hoop position could be adjusted to give a more nearly square flux plot, it is probably better not to do so because it would increase the stress in the outer hoops, make startup and low plasma current operation more difficult, and hinder comparison with the past seven years of operation.

The ratio of inner to outer hoop currents for figure 19 is  $158/88 = 1.80$  which coincidentally is within a fraction of a percent of the ratio of hoop major radii. Thus the resistive division of current in the hoops will be correct if they have the

same average resistivity. If they are made of the same two materials, they will also have the same internal inductance per unit length. The average private flux around the hoops is  $\sim 24$  mWebers which is close to the  $\sim 35$  mWebers of figure 11.

The variation of magnetic field around the minor circumference of each hoop is shown in figure 20. Shown are the upper hoops with the poloidal angle measured counter-clockwise from the vertical. By Fourier analyzing the curves in figure 20, as shown in figure 21, the amplitude  $c$  and phase  $\phi$  of the various modes  $m$  can be determined. From these, we derive the following values for a total hoop current of 500 kA:

	<u>Inner Hoop</u>	<u>Outer Hoop</u>
$\alpha$	1.265 tesla	0.701 tesla
$\epsilon$	0.535	0.478
$\nu$	3.30	2.83
$\phi$	54.8°	-37.7°
error	11%	20%

The Fourier representation as in equation (10) for  $m=0$  and  $m=1$  is plotted in figure 20.

If we design hoops using a core of Inconel 718 ( $125 \mu\Omega\text{-cm}$ )

and a shell of 2024-T6 aluminum ( $4.5 \mu\Omega\text{-cm}$ ), the lowest average resistivity for which we can get a physical solution ( $r_1 - \delta < r_o$ ) is  $8.5 \mu\Omega\text{-cm}$ . For this value the results of the previous section give:

	<u>Inner Hoop</u>	<u>Outer Hoop</u>
$r_o$	2.5 cm	2.5 cm
$r_1$	1.747 cm	1.747 cm
$\delta$	-0.752 cm	-0.672 cm
$B_o$	3444 gauss	1701 gauss
$I_o$	304 kA	168 kA
$I_1$	-143 kA	-79 kA
$I$	161 kA	89 kA
$L_{int}$	0.0465 $\mu\text{H}$	0.0837 $\mu\text{H}$
$L_{int}/R$	478 $\mu\text{sec}$	478 $\mu\text{sec}$

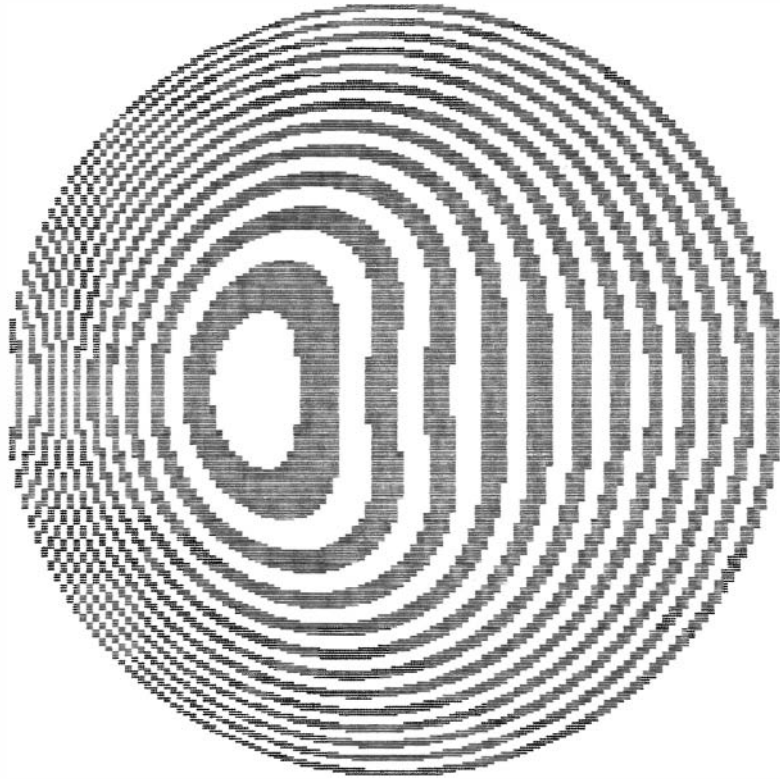
A complete flux plot for the above case is shown in figure 22. The results are consistent with figures 19 and 15. Such hoops are estimated to be capable of sustaining currents 1.21 times the present hoops with the same margin of safety. Other material combinations that might provide even higher currents are being considered.

Acknowledgements: I am grateful to Stewart Prager for critically reading an early version of the manuscript and to Cisse Spragins for providing the full Tokapole flux plots.



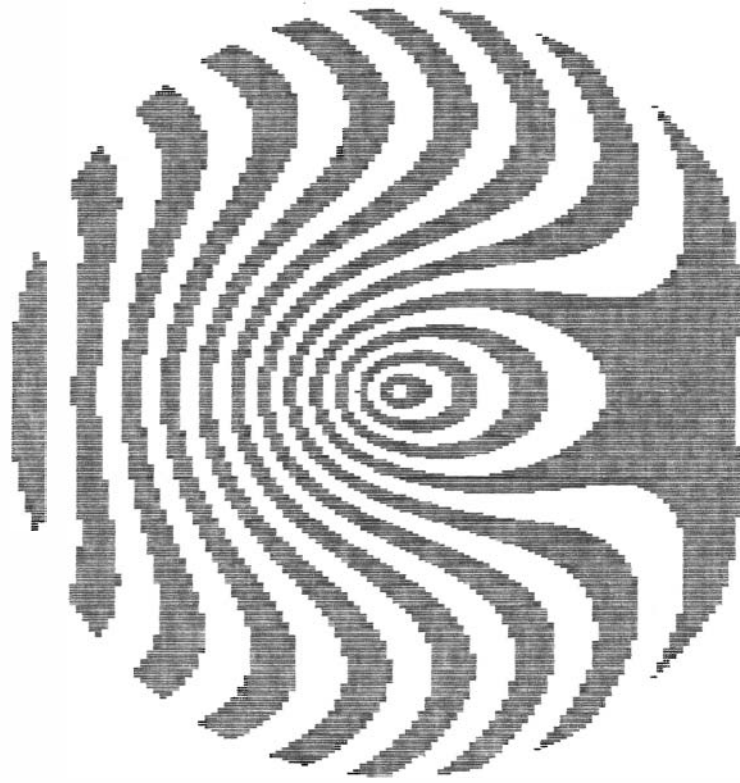
$\nu = 3$      $do = -.8$

MAGNETIC FLUX



Psi min = -.1743642  
Psi max = 1.009091  
Psi opp = .9999999

CONDUCTIVITY

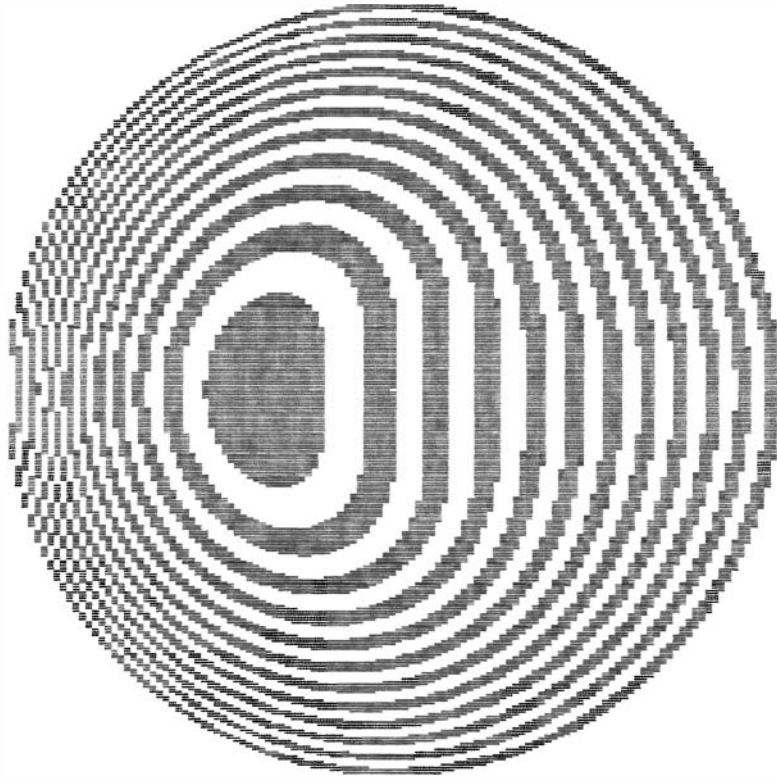


Sigma min = -.1812865  
Sigma max = 1  
Sigma opp = .1111112

Figure 1

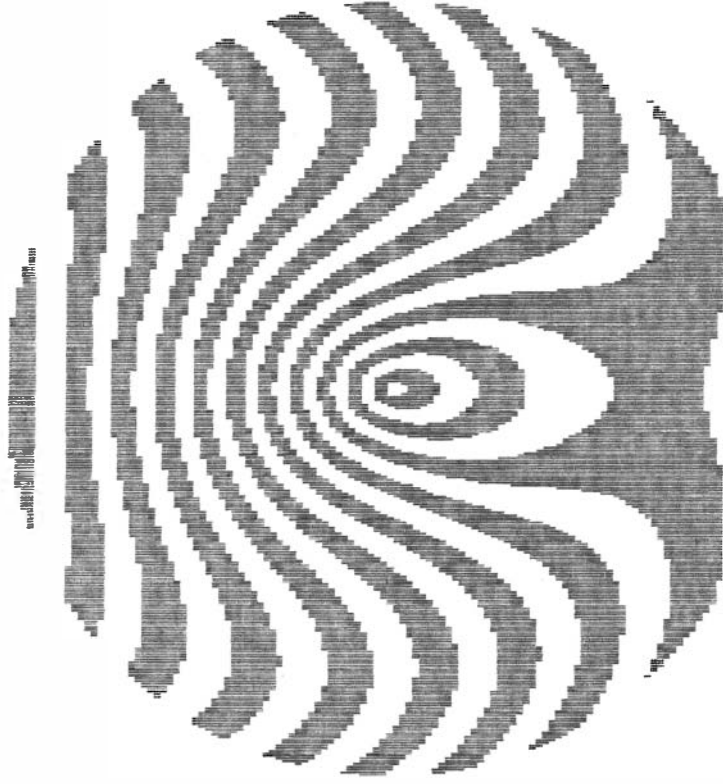
$\nu = 3$        $do = -.6$

MAGNETIC FLUX



Psi min = -.1420875  
Psi max = 1.000612  
Psi opp = .9999998

CONDUCTIVITY

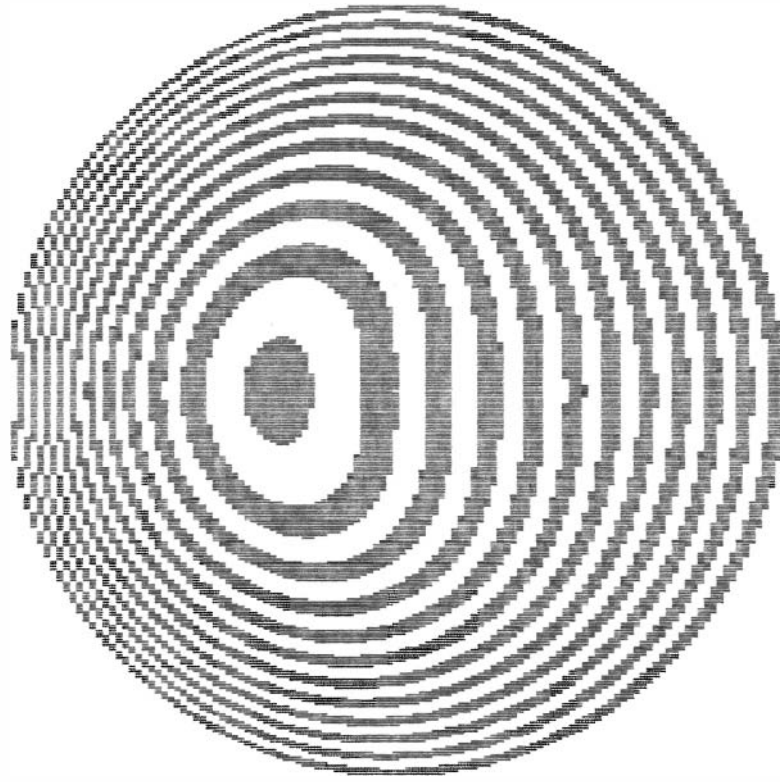


Sigma min = -7.407405E-02  
Sigma max = 1  
Sigma opp = .1111111

Figure 2

$v = 3$        $do = -.4$

MAGNETIC FLUX



Psi min = .1125546  
Psi max = 1.000134  
Psi opp = .99999998

CONDUCTIVITY

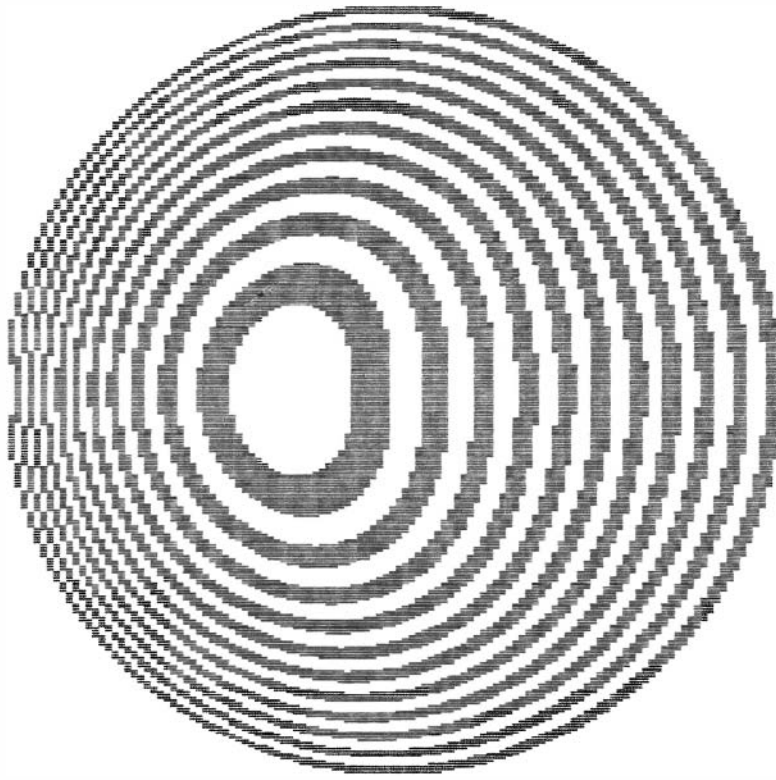


Sigma min = 4.575162E-02  
Sigma max = 1  
Sigma opp = .11111111

Figure 3

$\nu = 3$        $d_0 = -.2$

### MAGNETIC FLUX



Psi min = -8.659752E-02  
Psi max = 1.007656  
Psi opp = 1

### CONDUCTIVITY

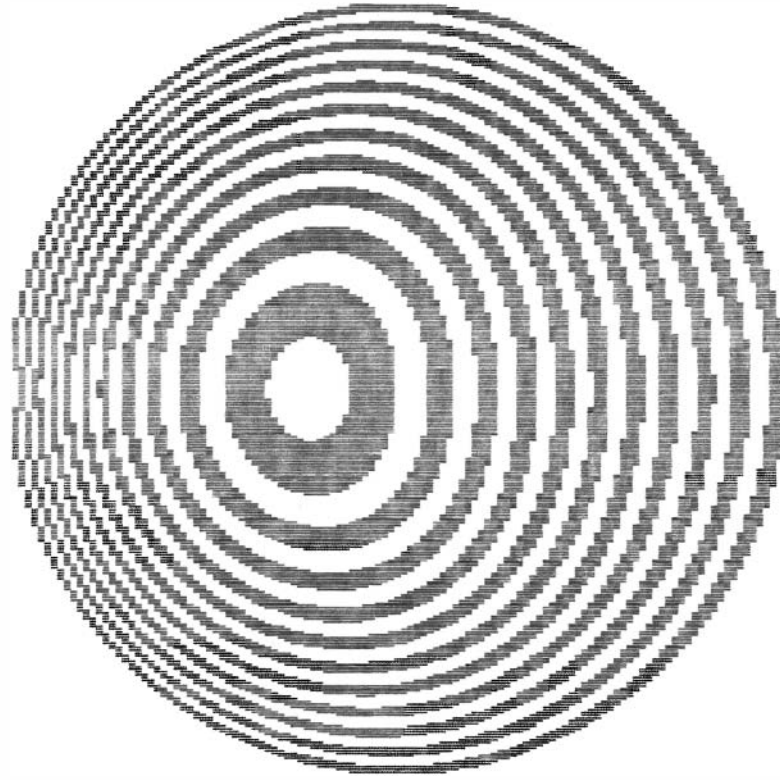


Sigma min = .1111112  
Sigma max = 1  
Sigma opp = .1111112

Figure 4

$\nu = 3$        $d_0 = 0$

MAGNETIC FLUX



Psi min = -6.476061E-02  
Psi max = 1.007177  
Psi opp = 1

CONDUCTIVITY

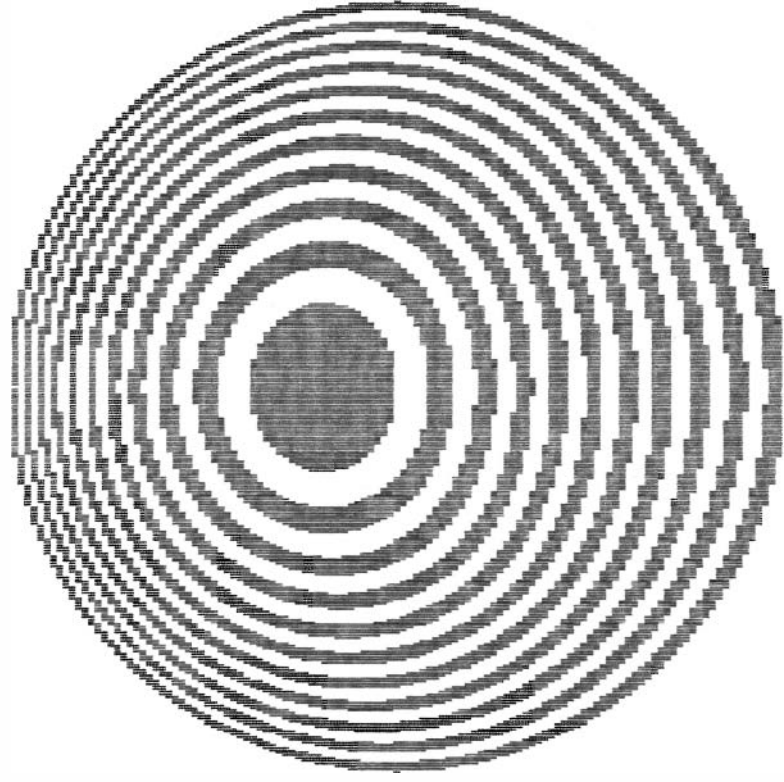


Sigma min = .1018519  
Sigma max = 1  
Sigma opp = .1111112

Figure 5

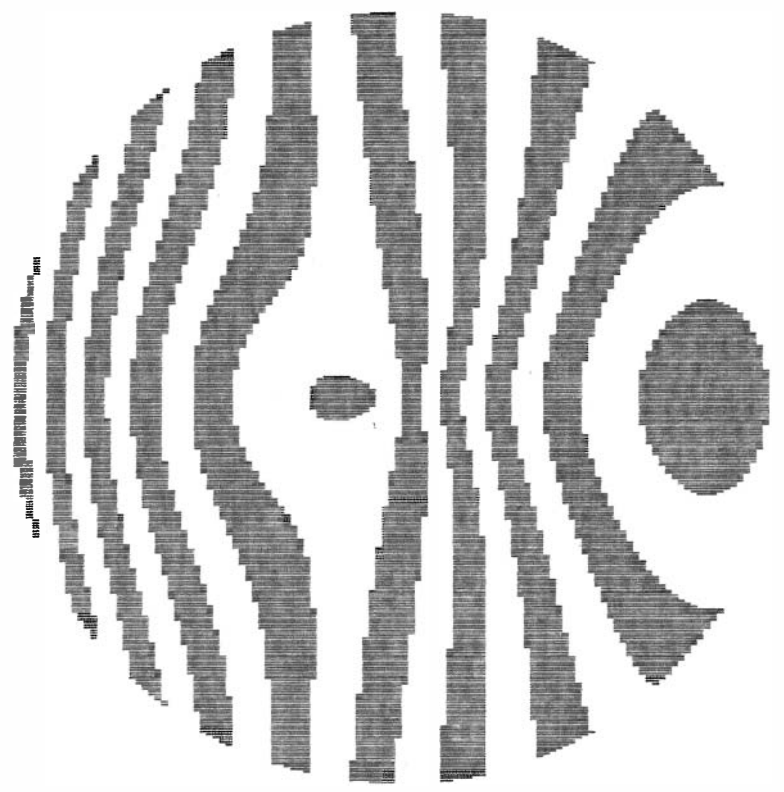
$\nu = 3$        $d_0 = .2$

### MAGNETIC FLUX



Psi min = -.0473939  
Psi max = 1.006698  
Psi opp = 1

### CONDUCTIVITY



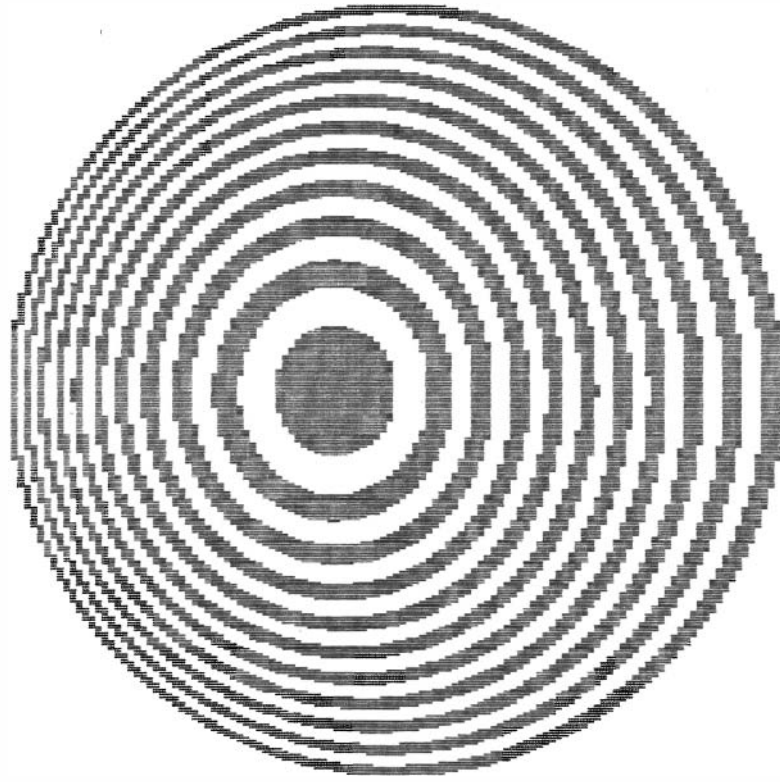
Sigma min = 8.124781E-02  
Sigma max = 1  
Sigma opp = .1111111

Figure 6



$\nu = 3$        $d_0 = .4$

### MAGNETIC FLUX



Psi min = -3.438239E-02  
Psi max = 1.00622  
Psi opp = 1

### CONDUCTIVITY

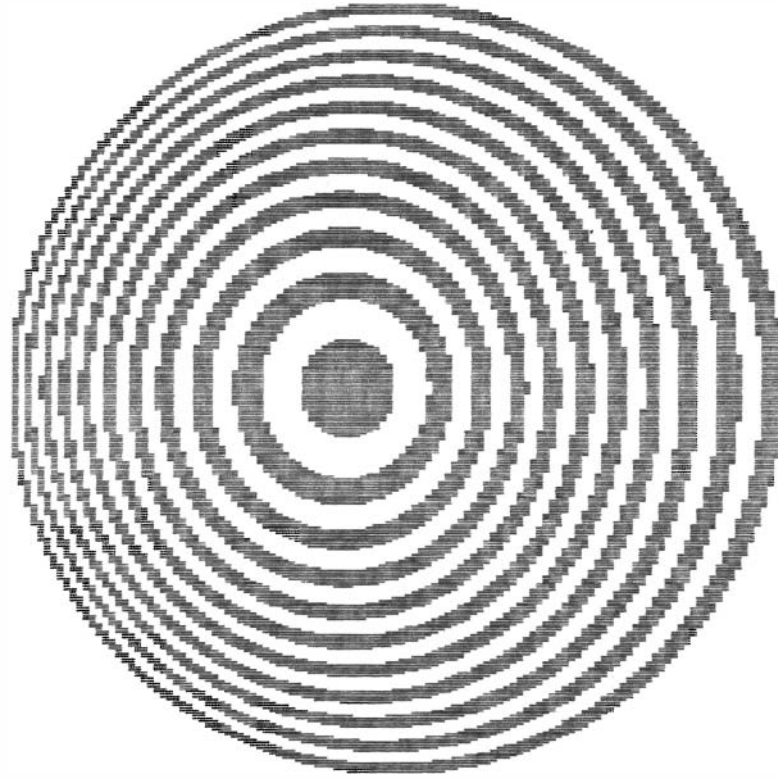


Sigma min = .0565122  
Sigma max = 1  
Sigma opp = .1111109

Figure 7

$\nu = 3$        $d_0 = .6$

### MAGNETIC FLUX



Psi min = -2.485516E-02  
Psi max = 1.005741  
Psi opp = 1

### CONDUCTIVITY



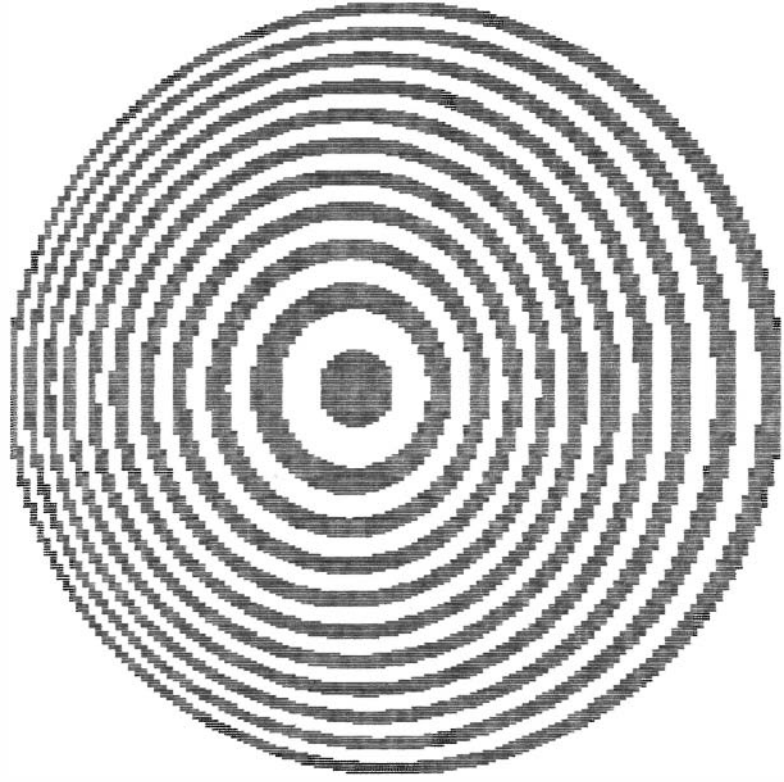
Sigma min = 2.728412E-02  
Sigma max = 1  
Sigma opp = .1111112

Figure 8



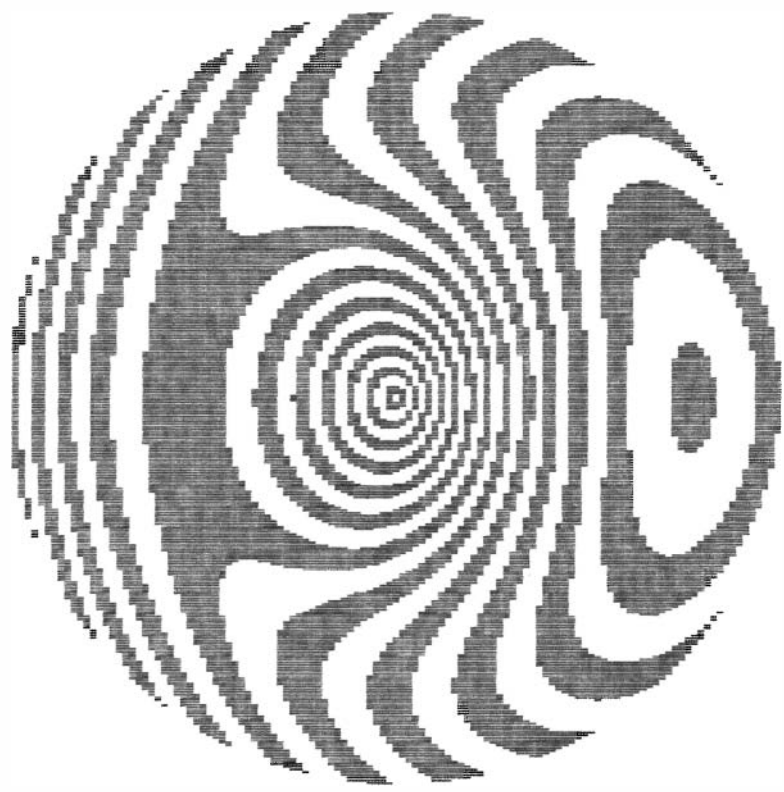
$\nu = 3$        $d_0 = .8$

### MAGNETIC FLUX



Psi min = -.0179  
Psi max = 1.005263  
Psi opp = 1

### CONDUCTIVITY



Sigma min = -7.283819E-03  
Sigma max = 1.222222  
Sigma opp = .1111113

Figure 9

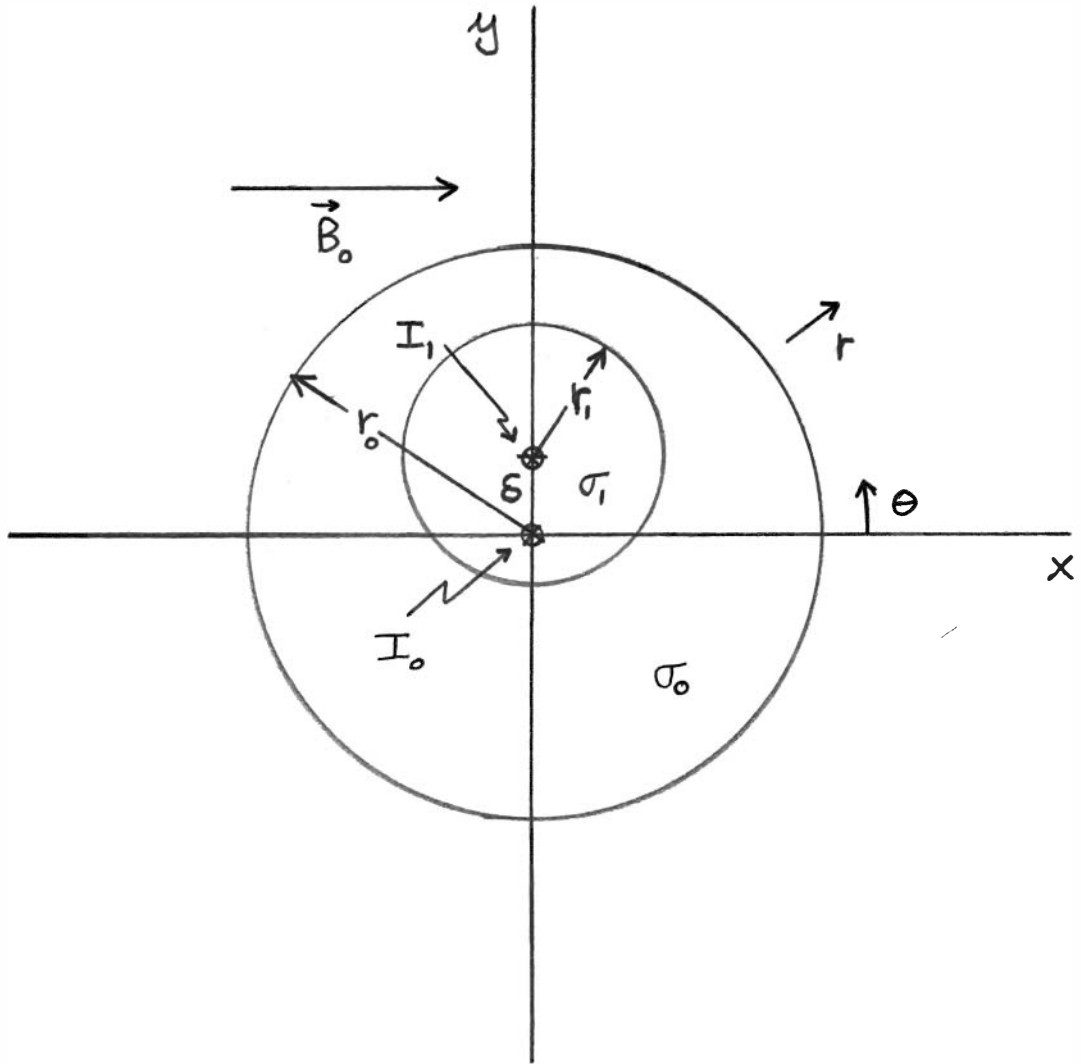
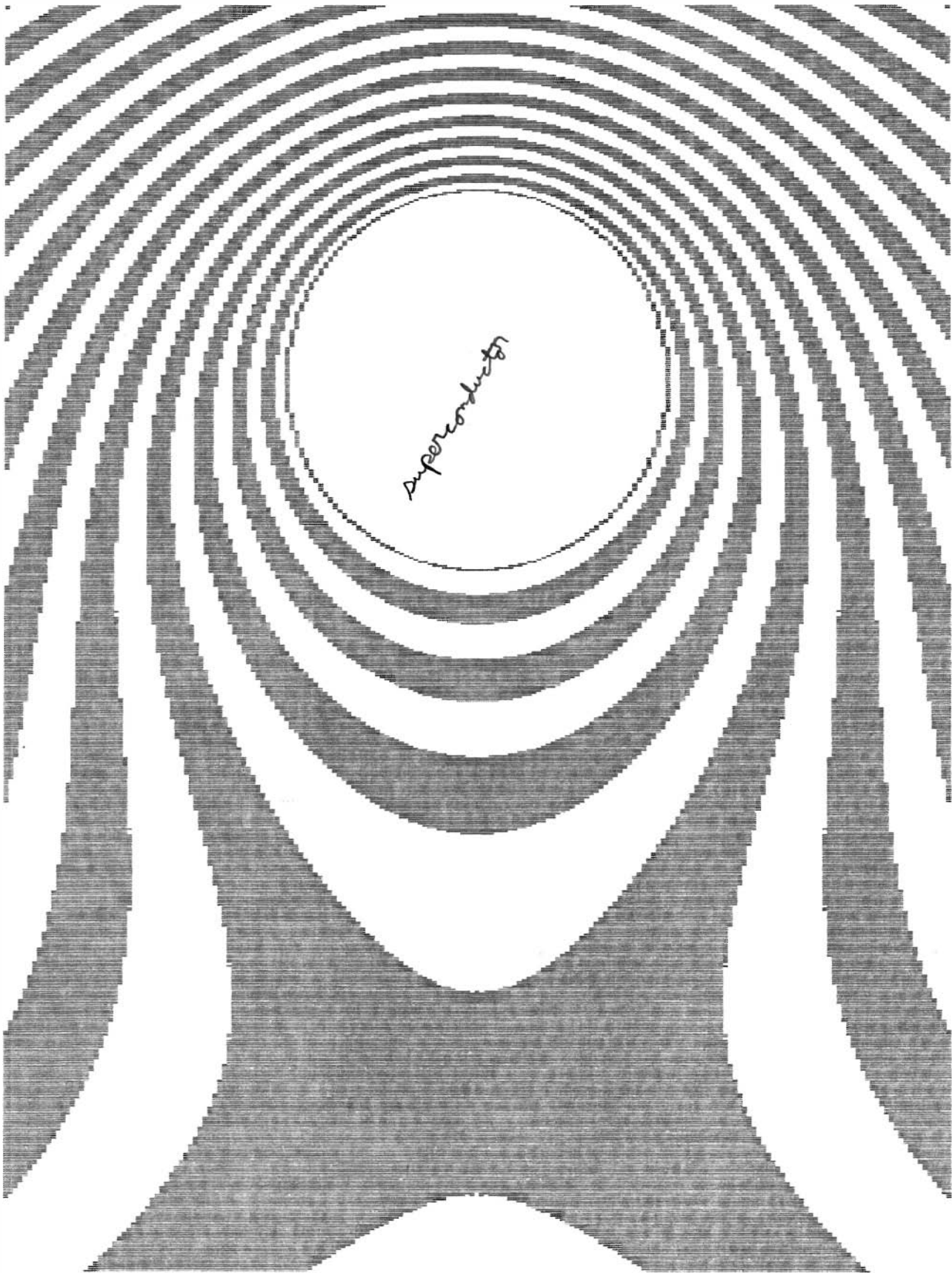
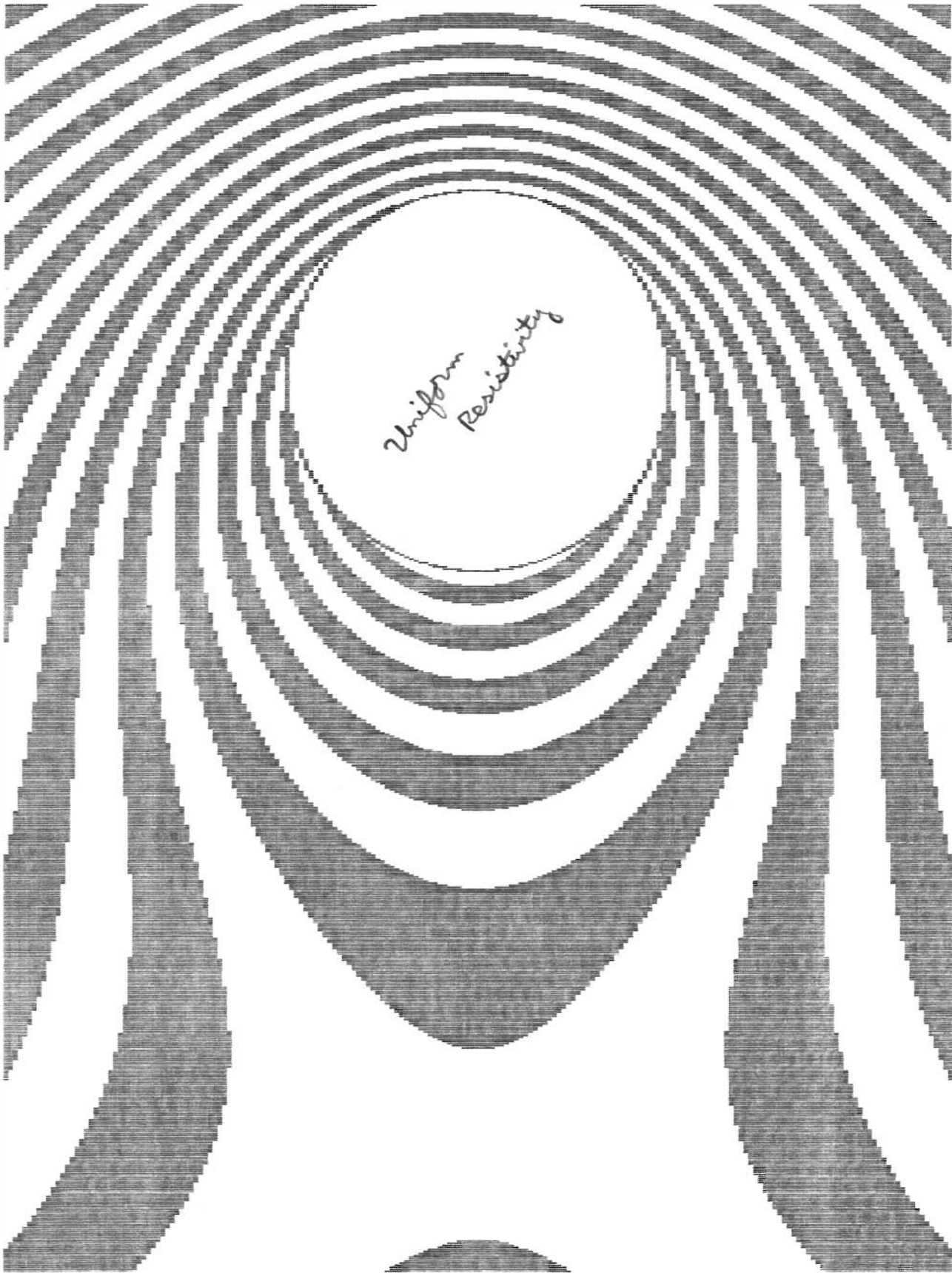


Figure 10



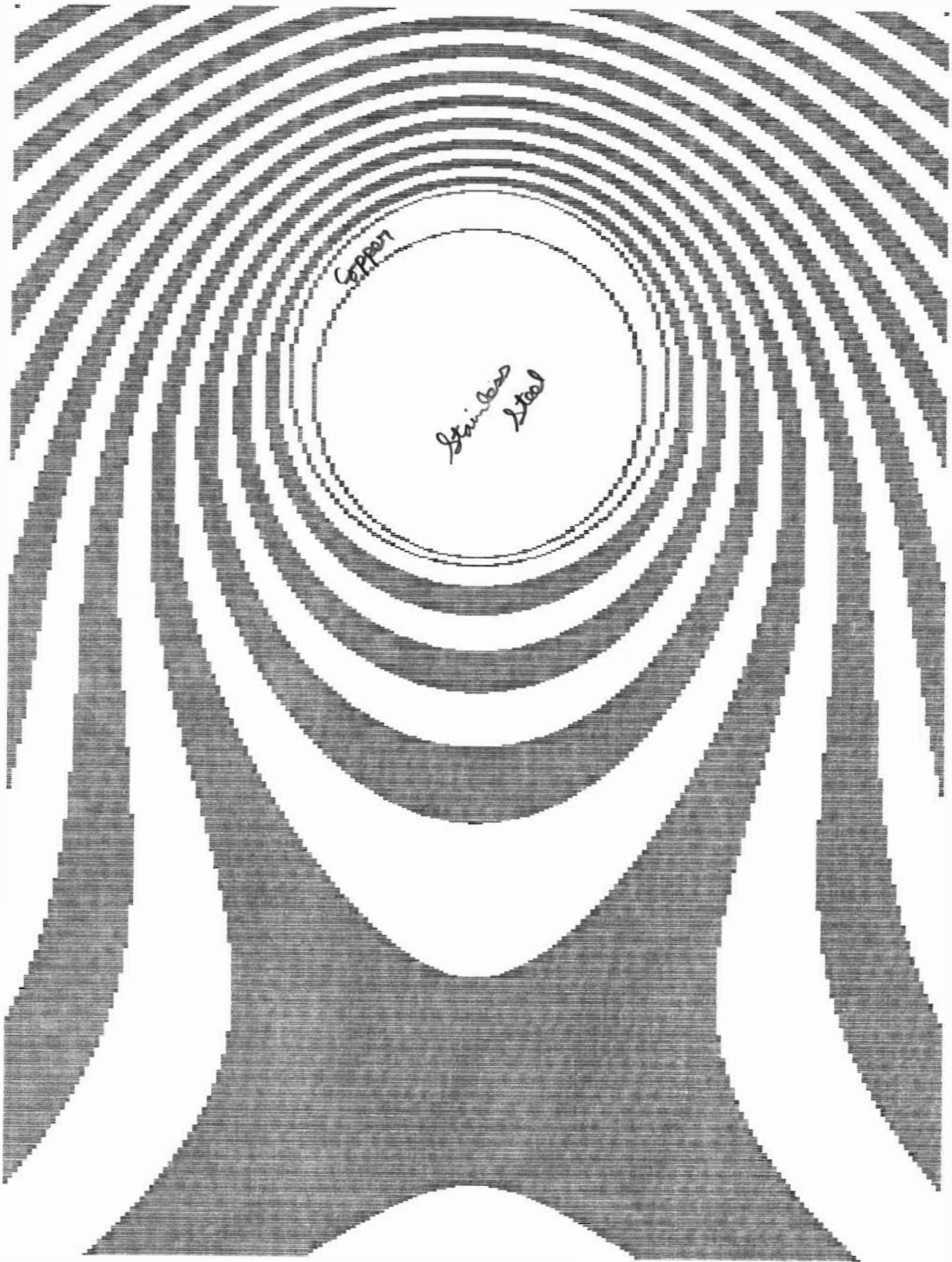
INDUCTIVE HOOP WITH  $\nu = 3$

Figure 11



*Uniform  
Resistivity*

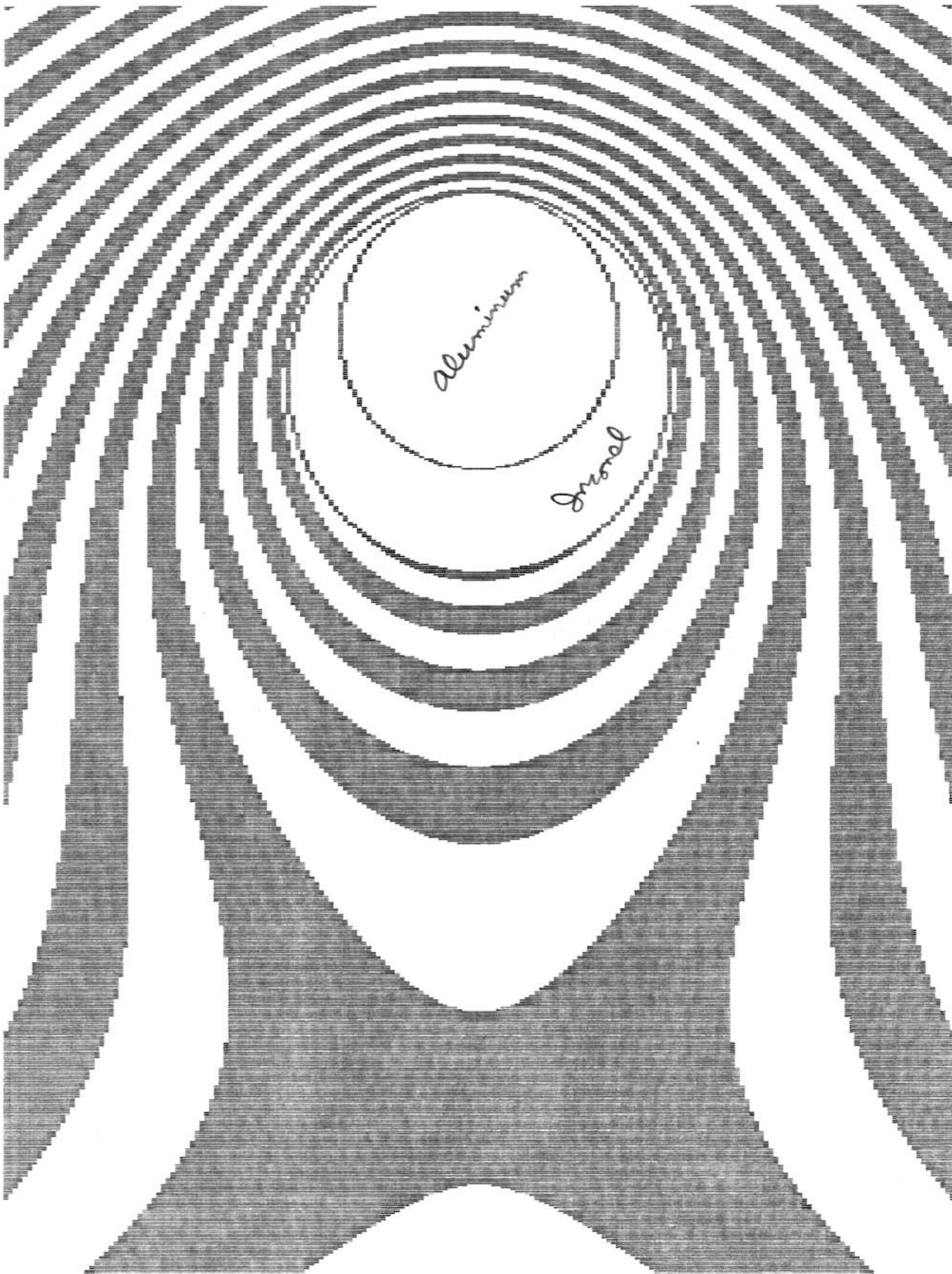
UNIFORMLY RESISTIVE HOOP WITH  $\nu = 3$   
Figure 12



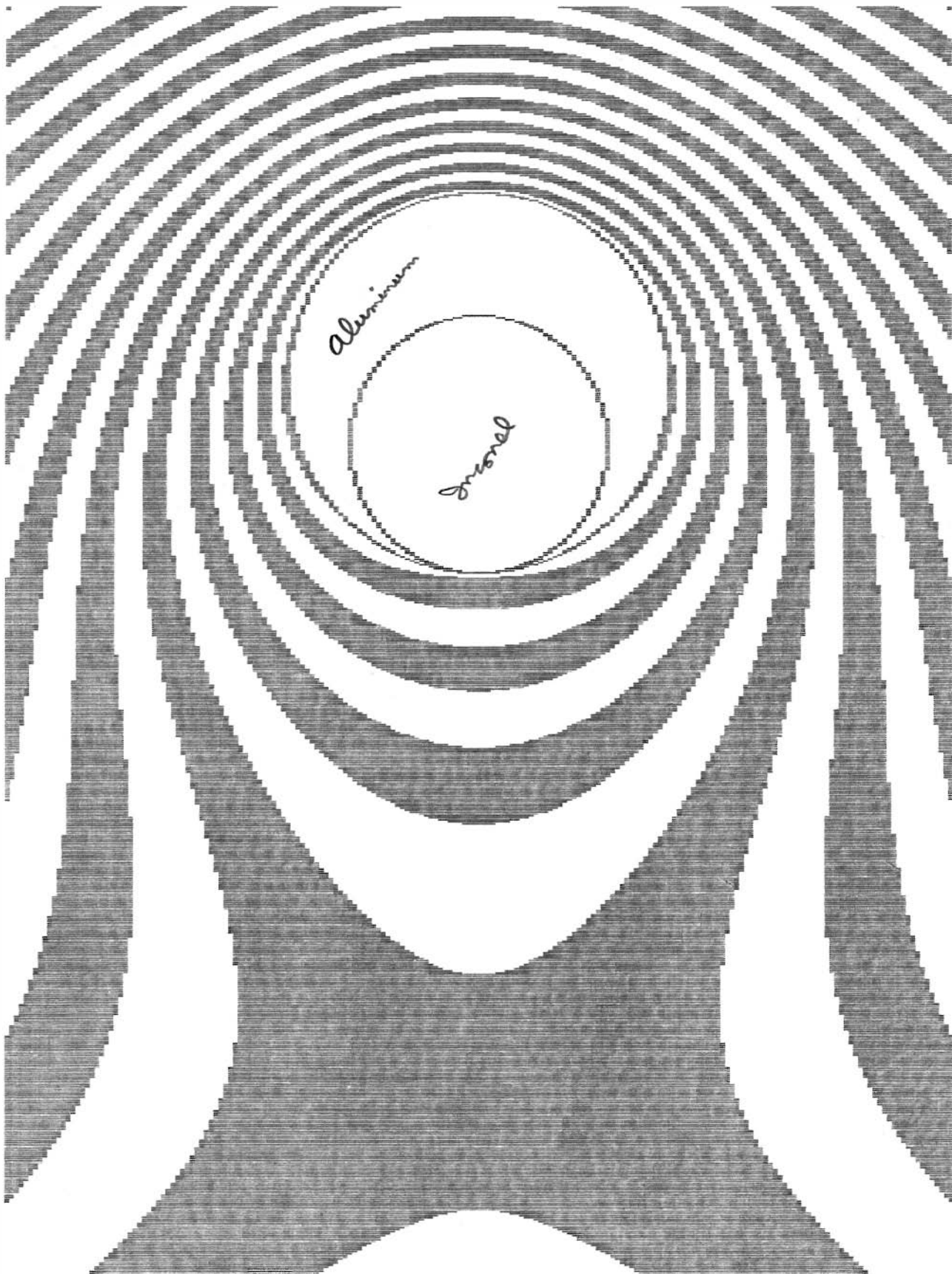
RESISTIVE HOOP (1.7/70/6.8) WITH  $\nu = 3$

Figure 13

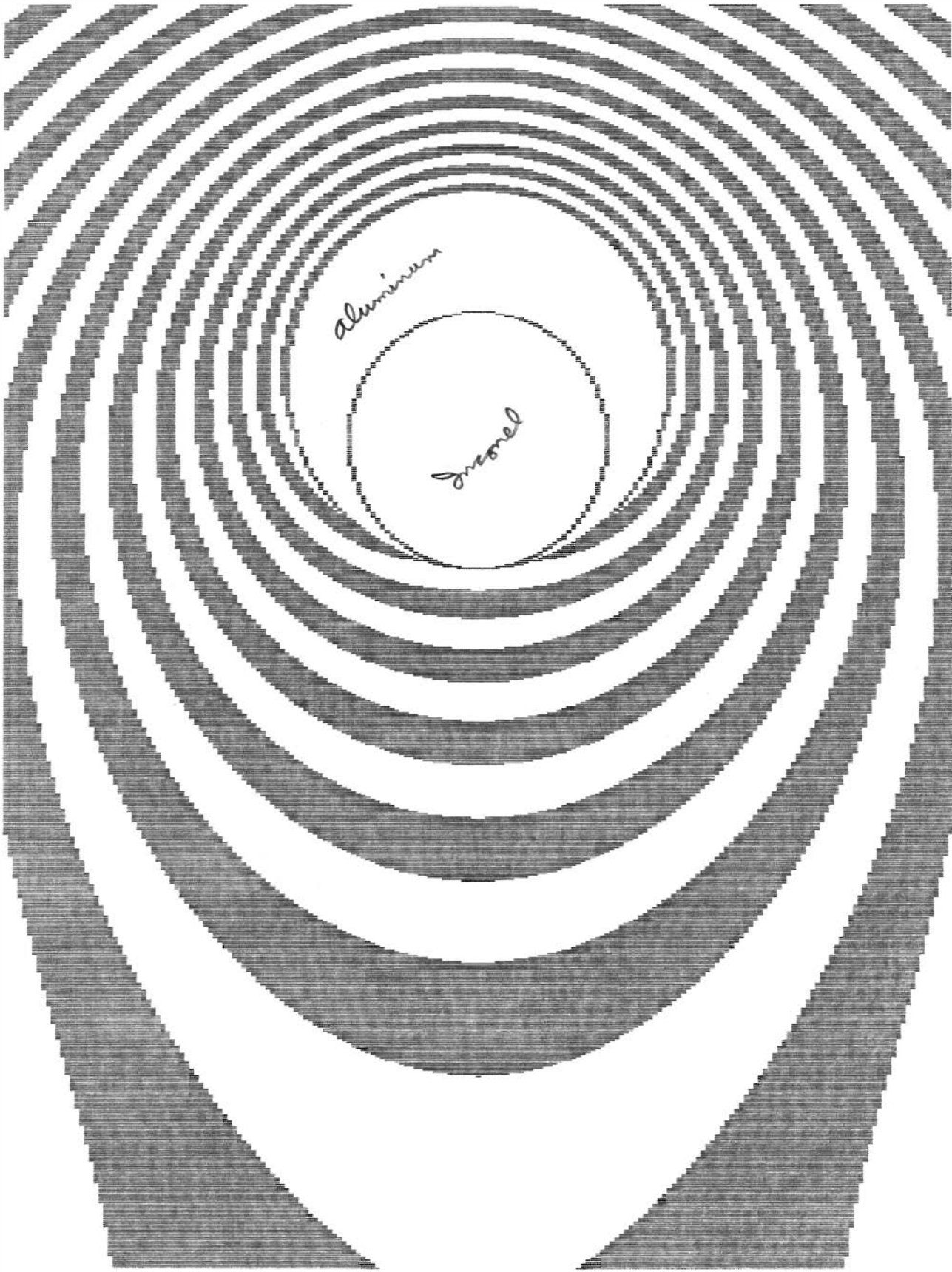




RESISTIVE HOOP (125/4.5/8.2) WITH  $\nu = 3$   
Figure 14

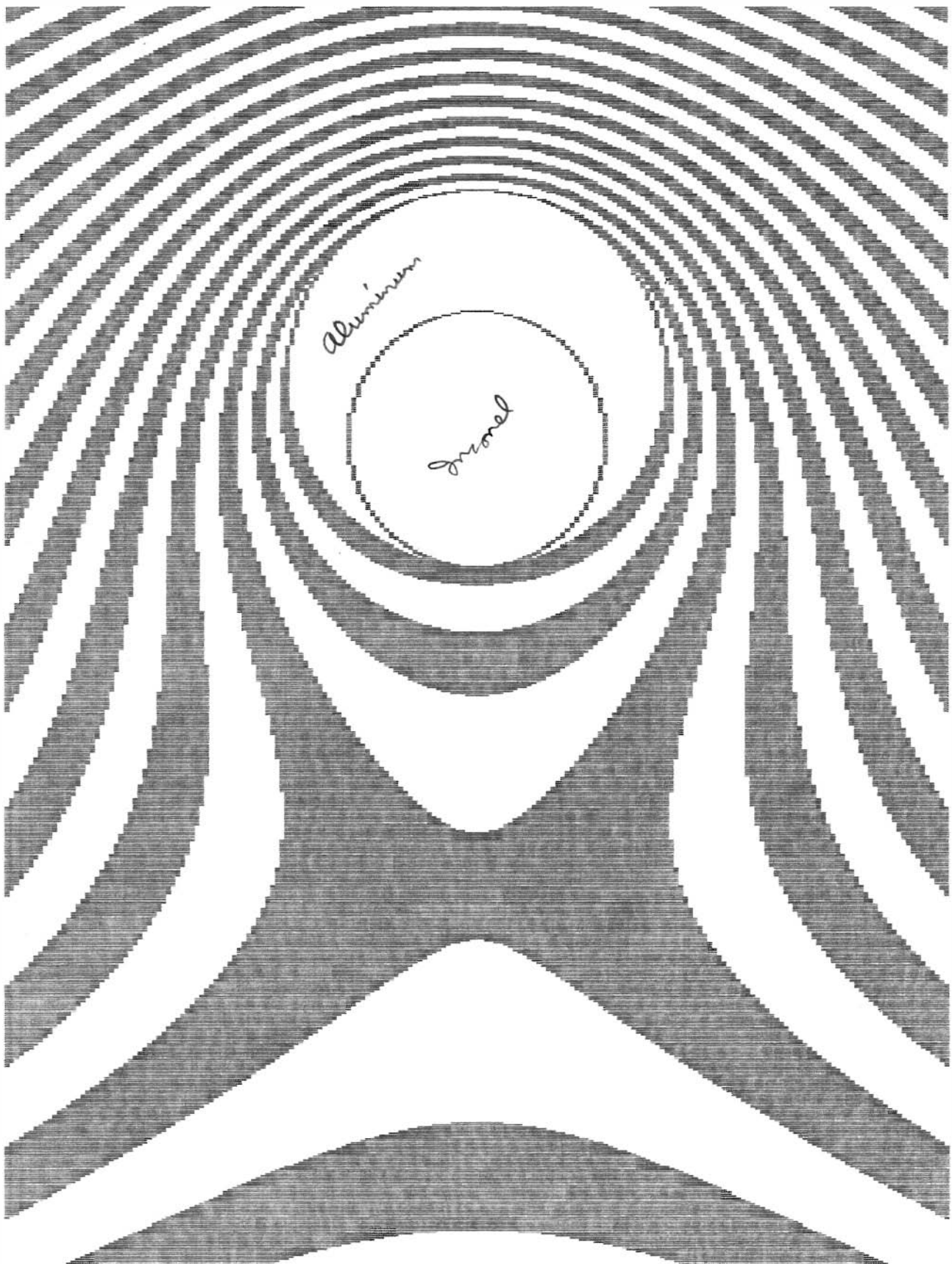


RESISTIVE HOOP (4.5/125/8) WITH  $\nu = 3$   
Figure 15



RESISTIVE HOOP (4.5/125/8) OPTIMIZED FOR  $\nu = 3$  BUT WITH  $\nu = 2$   
Figure 16





RESISTIVE HOOP (4.5/125/8) OPTIMIZED FOR  $\nu = 3$  BUT WITH  $\nu = 5$

Figure 17

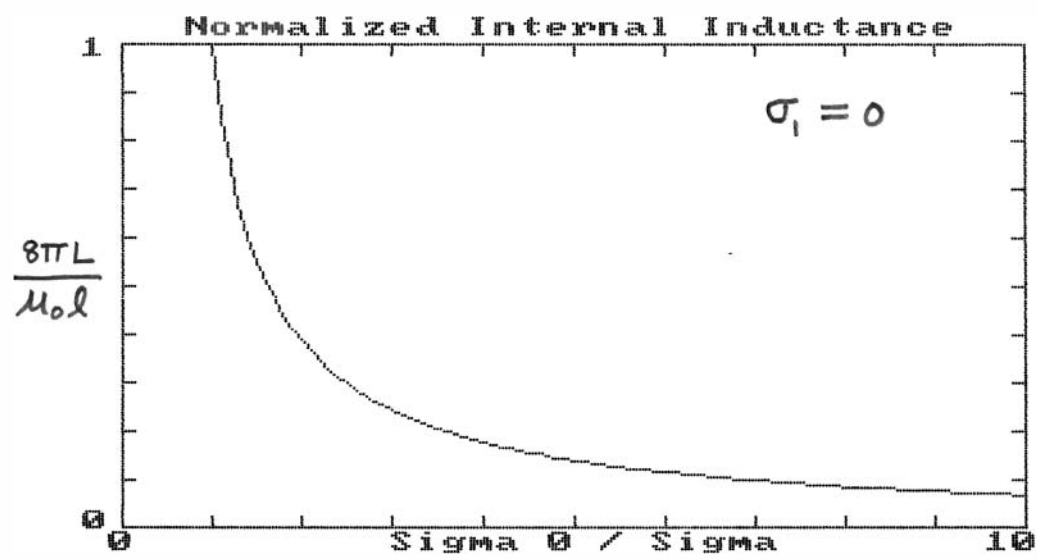
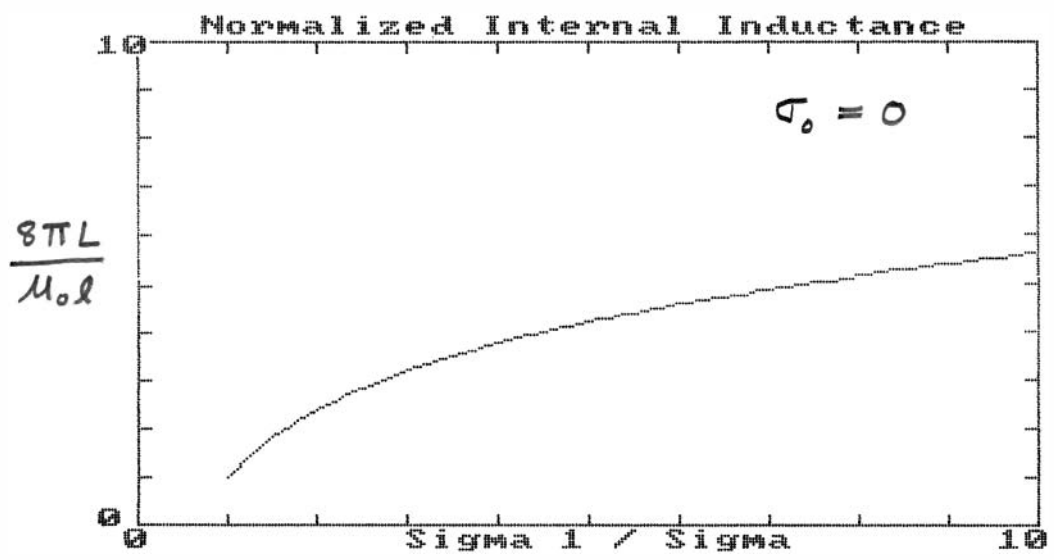


Figure 18

inner hoop major diam. 71.46?  
 71.707 cm  
 outer " " " " 128.774 cm  
 129?

$$\frac{D_I}{D_O} = .55684$$

$$\frac{I_O}{I_I} = .55692$$

present case  $I_H \text{ total} = 495 \text{ K amp}$   
 $I_0 = 80 \text{ KA}$

only .014% off

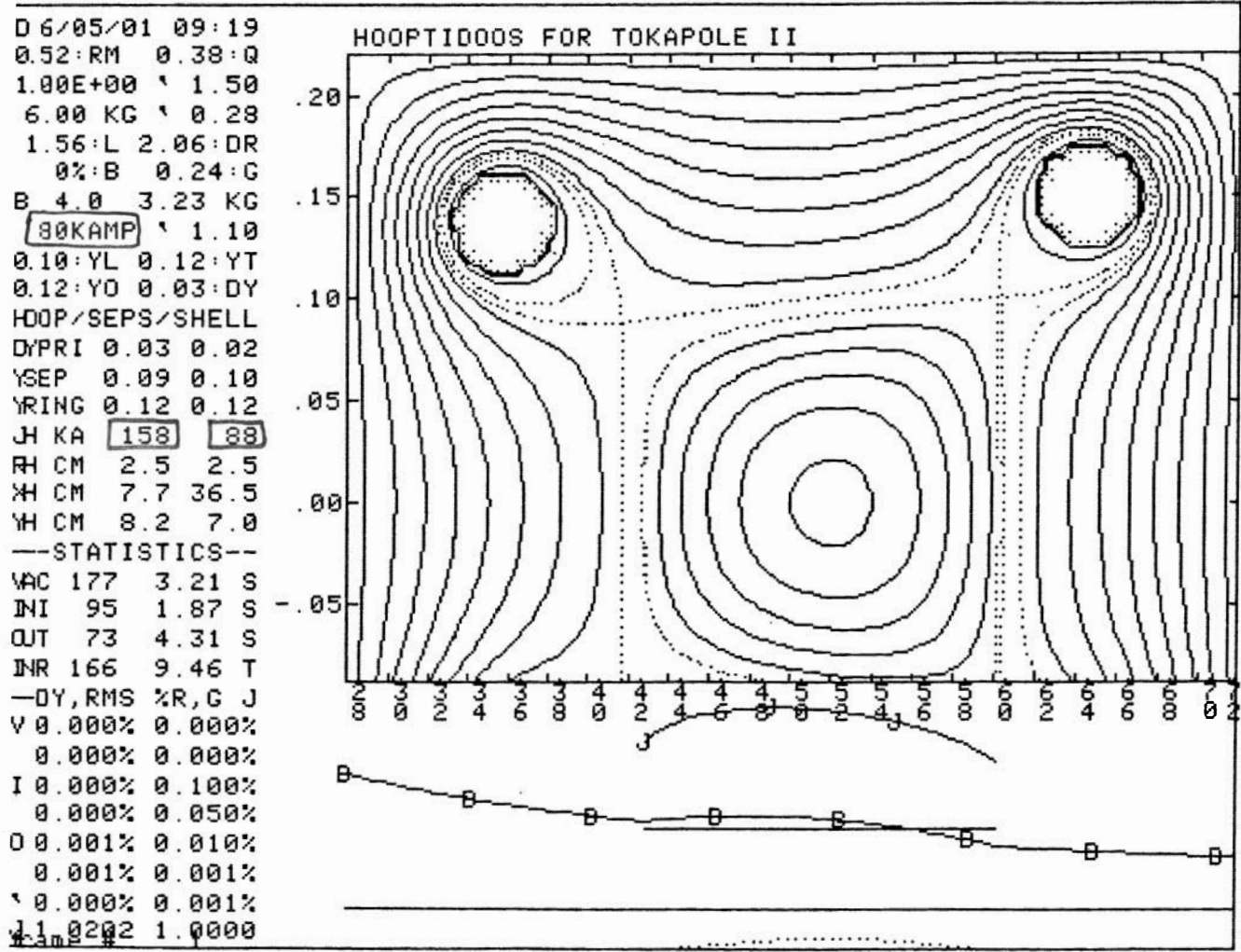


Figure 19

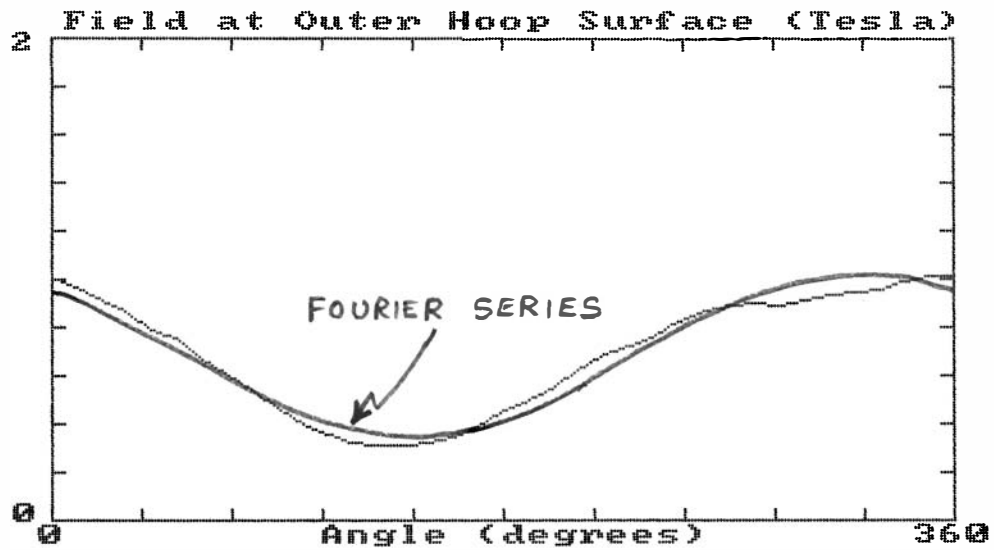
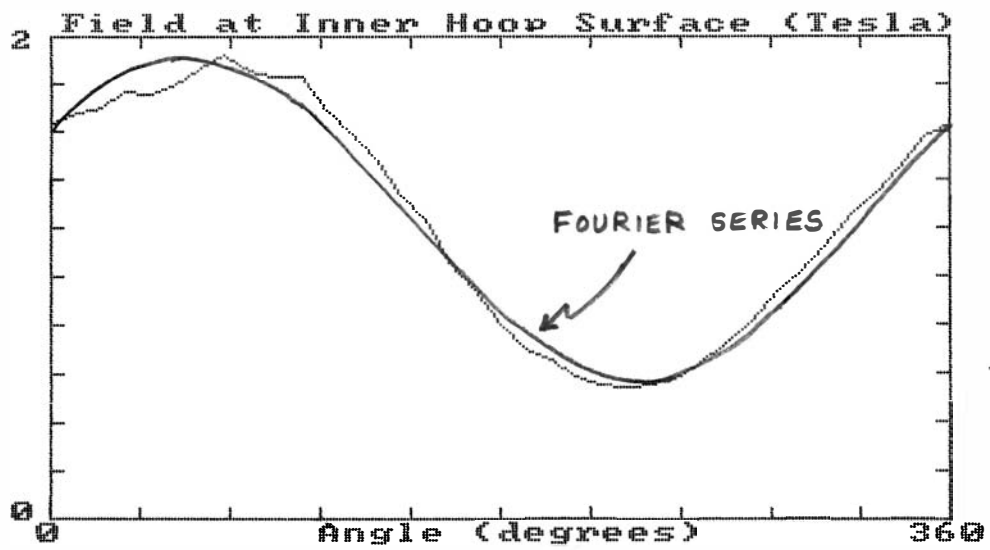


Figure 20

## INNER HOOP

m	a	b	c	Ø (deg)
0	2.48942	0	2.48942	0
1	.3832822	.5440395	.6654955	54.83485
2	-3.445208E-02	-6.250882E-02	7.137436E-02	61.1384
3	1.543633E-02	-2.038316E-02	.0255686	-52.86305
4	7.825018E-03	-3.082322E-03	8.410209E-03	-21.49978
5	9.713844E-03	1.457274E-03	9.822546E-03	8.531904
6	-4.02471E-04	-1.382607E-03	1.439994E-03	73.76995
7	1.771135E-03	2.026812E-03	2.691632E-03	48.85133
8	-3.01275E-03	6.819906E-04	3.088976E-03	-12.75497
9	-6.031941E-04	-4.404999E-03	4.446106E-03	82.20275
10	3.616192E-03	-8.007448E-03	8.786128E-03	-65.6959
11	6.909614E-03	-1.045882E-03	6.988321E-03	-8.607308
12	1.332448E-03	1.510535E-03	2.014232E-03	48.58438
13	-5.520309E-03	9.176736E-04	5.596064E-03	-9.438306
14	4.964153E-04	-2.425583E-03	2.475859E-03	-78.43366
15	1.453736E-03	4.223817E-03	4.466987E-03	71.00776
16	-1.594614E-03	-2.79413E-04	1.618909E-03	9.938637
17	-1.251583E-03	-2.805985E-04	1.282652E-03	12.63647
18	-2.054457E-03	-2.04799E-07	2.054457E-03	5.711541E-03

## OUTER HOOP

m	a	b	c	Ø (deg)
0	1.378911	0	1.378911	0
1	.2605977	-.2014055	.3293559	-37.69899
2	2.105624E-02	5.923057E-02	6.286195E-02	70.4299
3	.011228	2.382218E-03	1.147794E-02	11.97867
4	1.110442E-02	-4.172542E-03	1.186248E-02	-20.59395
5	5.288375E-03	-4.61189E-03	7.016868E-03	-41.09105
6	1.446656E-03	1.92585E-03	2.408675E-03	53.08693
7	2.215194E-03	1.863412E-03	2.894718E-03	40.07041
8	4.509034E-03	-4.030165E-03	6.047612E-03	-41.79027
9	-1.927456E-03	-.0013244	2.338615E-03	34.49386
10	3.698656E-04	3.270022E-04	4.936912E-04	41.48024
11	-1.538709E-03	-1.54228E-03	2.177178E-03	45.10366
12	-1.689332E-04	2.137319E-04	2.72433E-04	-51.67723
13	1.32119E-03	-1.373961E-03	1.906125E-03	-46.12172
14	9.860154E-04	4.255649E-04	1.073933E-03	23.34506
15	4.396658E-04	1.32284E-03	1.393991E-03	71.61499
16	-4.564469E-04	1.750979E-03	1.809495E-03	-75.38925
17	-7.600606E-04	1.051138E-03	1.297144E-03	-54.12992
18	-5.495495E-04	-1.383876E-07	5.495495E-04	1.442823E-02

Figure 21

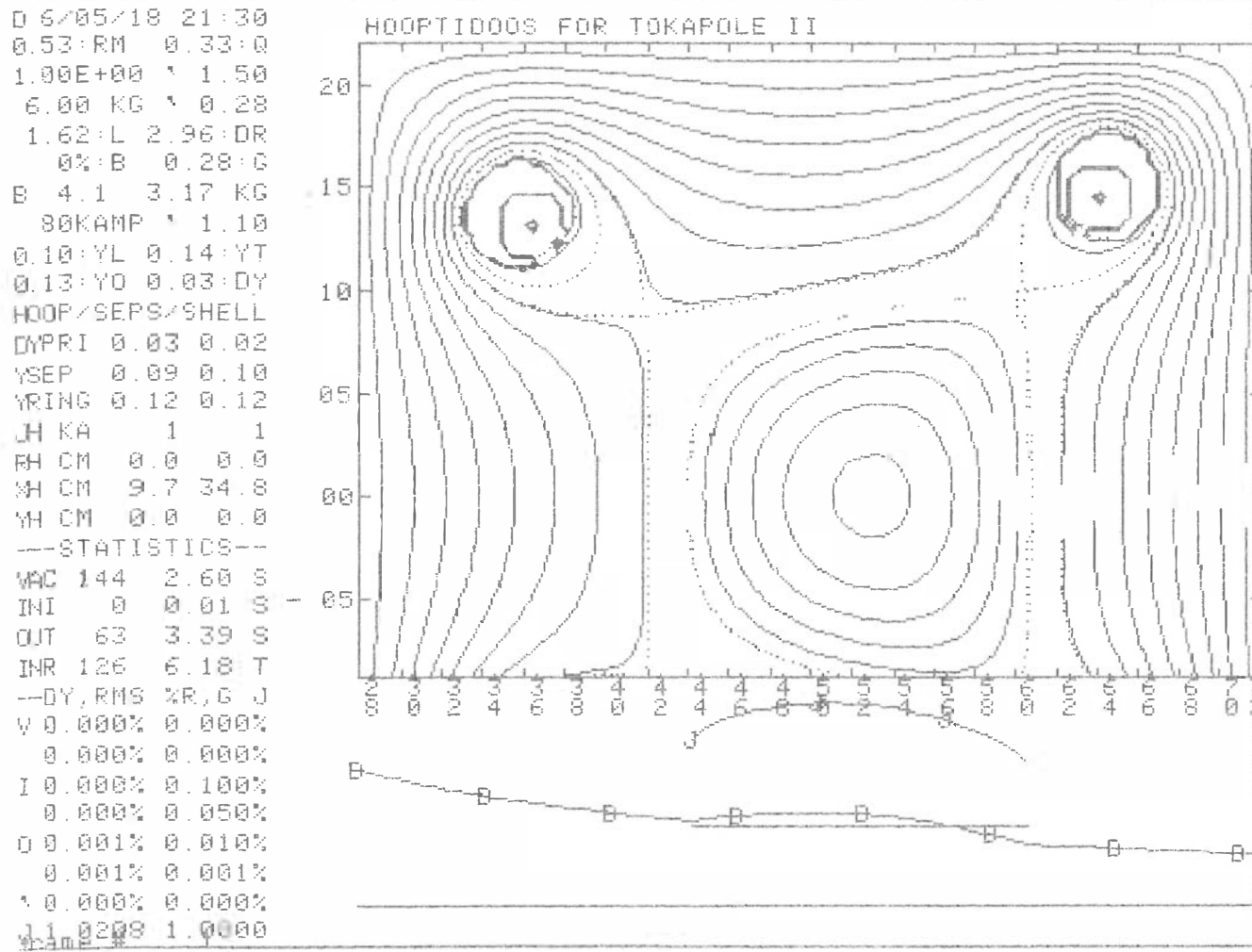


Figure 22